


... up to now, in spite of reasonable efforts,

- **NO** any unambiguous experimental confirmation in favour of nonvanishing *em* properties,

available experimental data in the field does not rule out possibility that have “ZERO” *em* properties.

- ... However, in course of recent development of knowledge on *mixing and oscillations*,
Neutrino mass

\[ m_\nu \neq 0 \]

Neutrino magnetic moment

\[ \mu_\nu \neq 0 \]

...Massive neutrino electromagnetic properties...
Theory (Standard Model with $\nu_R$)

\[ M_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \times 10^{-19} \mu_B \left( \frac{m_{\nu_e}}{\text{TeV}} \right), \quad \mu_B = \frac{e}{2m_e} \]

Lee Shrock, 1977; Fujikawa Shrock, 1980

In the Standard Model: \( m_\nu = 0 \),
there is no $\nu_R \Rightarrow$

$\nu$ magnetic moment \( \mu_\nu = 0 \).

Thus, \( \mu_\nu \neq 0 \rightarrow \text{beyond the SM} \).
... puzzling

something that is tiny or probably even does not exist at all...
Pauli himself wrote to Baade:

“Today I did something a physicist should never do. I predicted something which will never be observed experimentally...”.

now we know that it is neutrino $\nu$  

E.Fermi, 1933

W. Pauli, 1930

and probably $m_\nu \neq 0$ in plasma and beyond SM (?)

now we know that $\mu_\nu \neq 0$ ?!

very important player (astrophysics, cosmology etc. . .)
... we very much hope that

✓ electromagnetic properties

will not follow the presentiment of Pauli
Outline (short list)

- electromagnetic properties - theory
- magnetic moment - experiment
- constraints on electromagnetic properties
- electromagnetic interactions (ν-σ processes)
0. Introduction

1. magnetic moment in experiments
2. New experimental result on $\mu$
3. electromagnetic properties - theory
   3.1 vertex function
   3.2 $\mu$ (arbitrary masses)
   3.3 relationship between $m$ and $\mu$
   3.4 vertex function in case of flavour mixing
   3.5 dipole moments in case of mixing
   3.6 $\mu$ in left-right symmetry models
   3.7 astrophysical bounds on $\mu$
   3.8 millicharge (Red Gaits cooling etc)
   3.9 charge radius and anapole moment
   3.10 electromagnetic properties in matter and e.m.f.

4. Effects of electromagnetic properties
   3.11 radiative decay, Ch radiation and Spin Light of in matter
   3.12 radiative $2\times\gamma$ - decay
   3.13 spin-flavour oscillations

5. Direct-Indirect influence of e.m.f. on

6. Conclusion
…please see also posters…

1. Alexander Grigoriev,
   “Neutrino motion in moving matter: the method of modified Dirac equation”

2. Alexey Lokhov,
   “Spin light of neutrino transitions between different mass states in matter”

3. Ilya Balantsev,
   “On the problem of relativistic particle motion in magnetic field and dense matter”

4. Alexey Kouzakov,
   “Weak decay processes in external electromagnetic fields”
electromagnetic vertex function

\[ < \psi(p') | J_{\mu}^{EM} | \psi(p) > = \bar{u}(p') \Lambda_{\mu}(q, l) u(p) \]

Matrix element of electromagnetic current is a Lorentz vector

\( \Lambda_{\mu}(q, l) \) should be constructed using

- matrices: \( \hat{1}, \gamma_5, \gamma_{\mu}, \gamma_5 \gamma_{\mu}, \sigma_{\mu\nu} \),
- tensors: \( g_{\mu\nu}, \epsilon_{\mu\nu\sigma\gamma} \),
- vectors: \( q_{\mu} \) and \( l_{\mu} \)

\[ q_{\mu} = p'_{\mu} - p_{\mu}, \quad l_{\mu} = p'_{\mu} + p_{\mu} \]
Electromagnetic gauge invariance (2)  
(requirement of current conservation)

\[ \partial_\mu j^\mu = 0 \]

\[ f_1(q^2) q^2 + f_2(q^2) q^2 \gamma_5 + 2m f_4(q^2) \gamma_5 = 0, \]

\[ f_1(q^2) = 0, \quad f_2(q^2) q^2 + 2m f_4(q^2) = 0 \]

\[ \Lambda_\mu(q) = f_Q(q^2) \gamma_\mu + f_M(q^2) i \sigma_{\mu\nu} q^\nu + f_E(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 + f_A(q^2) (q^2 \gamma_\mu - q_\nu \gamma_5) \]

4 Form Factors

\( \text{... consistent with Lorentz-covariance (1) + electromagnetic gauge invariance (2) } \)
Matrix element of electromagnetic current between neutrino states

\[ \langle \nu(p')|J_\mu^{EM}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p) \]

where vertex function generally contains 4 form factors

\[ \Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu\gamma_5)\gamma_5 \]

1. electric dipole
2. magnetic
3. electric
4. anapole

Hermiticity and discrete symmetries of EM current \( J_\mu^{EM} \) put constraints on form factors

Dirac \( \checkmark \)
1) CP invariance + hermiticity \( \Rightarrow f_E = 0 \),
2) at zero momentum transfer only electric charge \( f_Q(0) \) and magnetic moment \( f_M(0) \) contribute to \( H_{int} \sim J_\mu^{EM}A^\mu \),
3) hermiticity itself \( \Rightarrow \) three form factors are real: \( \text{Im}f_Q = \text{Im}f_M = \text{Im}f_A = 0 \).

Majorana \( \checkmark \)
1) from CPT invariance (regardless CP or CP ),
\[ f_Q = f_M = f_E = 0 \]

...as early as 1939, W.Pauli...

EM properties \( \rightarrow \) a way to distinguish Dirac and Majorana \( \checkmark \)
In general case matrix element of $J_{\mu}^{EM}$ can be considered between different initial $\psi_i(p)$ and final $\psi_j(p')$ states of different masses $p^2 = m_i^2, p'^2 = m_j^2$:

$$< \psi_j(p') | J_{\mu}^{EM} | \psi_i(p) > = \bar{u}_j(p') \Lambda_{\mu}(q) u_i(p)$$

and

$$\Lambda_{\mu}(q) = \left( f_Q(q^2)_{ij} + f_A(q^2)_{ij} \gamma_5 \right) (q^2 \gamma_{\mu} - q_{\mu} q) + f_M(q^2)_{ij} i\sigma_{\mu\nu} q^\nu + f_E(q^2)_{ij} \sigma_{\mu\nu} q^\nu \gamma_5$$

form factors are matrices in mass eigenstates space.

Dirac $\checkmark$ (off-diagonal case $i \neq j$)

1) hermiticity itself does not apply restrictions on form factors,

2) CP invariance + hermiticity $f_Q(q^2), f_M(q^2), f_E(q^2), f_A(q^2)$ are relatively real (no relative phases).

Majorana $\checkmark$

1) CP invariance + hermiticity $\mu_{ij}^M = 2\mu_{ij}^D$ and $\epsilon_{ij}^M = 0$ or $\mu_{ij}^M = 0$ and $\epsilon_{ij}^M = 2\epsilon_{ij}^D$

... quite different EM properties ...
magnetic moment ?
Dipole magnetic $f_M(q^2)$ and electric $f_E(q^2)$ are most well studied and theoretically understood among form factors

...because even in the limit $q^2 \to 0$ they may have nonvanishing values

\[ \mu_\nu = f_M(0) \quad \nu \text{  magnetic moment} \]

\[ \epsilon_\nu = f_E(0) \quad \nu \text{  electric moment} \]
magnetic moment in experiments

Samuel Ting
(wrote on the wall at Department of Theoretical Physics of Moscow State University):

“Physics is an experimental science”
Studies of $\nu-e$ scattering - most sensitive method of experimental investigation of $\mu$-$\nu$

Cross-section:

$$\frac{d\sigma}{dT}(\nu + e \rightarrow \nu + e) = \left( \frac{d\sigma}{dT} \right)_{\text{SM}} + \left( \frac{d\sigma}{dT} \right)_{\mu\nu},$$

where the Standard Model contribution

$$\left( \frac{d\sigma}{dT} \right)_{\text{SM}} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V^2 + g_A^2) + (g_V^2 - g_A^2) \left( 1 - \frac{T}{E_\nu} \right)^2 + (g_A^2 - g_V^2) \frac{m_e T}{E_\nu^2} \right],$$

$T$ is the electron recoil energy and

$$\mu_{\nu j}^2 = \sum_{j=\nu_e, \nu_\mu, \nu_\tau} |\mu_{ij} - \epsilon_{ij}|^2,$$

$g_V = \begin{cases} 2 \sin^2 \theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2 \sin^2 \theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases}$

$g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases}$

for anti-neutrinos $g_A \rightarrow -g_A$

to incorporate charge radius:

$$g_V \rightarrow g_V + \frac{2}{3} M_W^2 \langle r^2 \rangle \sin^2 \theta_W$$
**Effective magnetic moment in experiments**

(for neutrino produced as $\nu_l$ with energy $E_\nu$
and after traveling a distance $L$)

\[
\mu^2_{\nu}(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_iL} \mu_{ji} \right|^2
\]

where

- **neutrino mixing matrix**
- **magnetic and electric moments**

Observable $\mu_{\nu}$ is an effective parameter that depends on neutrino flavour composition at the detector.

Implications of $\mu_{\nu}$ limits from different experiments (reactor, solar $^8\text{B}$ and $^7\text{Be}$) are different.
Magnetic moment contribution is dominated at low electron recoil energies and \( \left( \frac{d\sigma}{dT} \right)_{\mu_\nu} > \left( \frac{d\sigma}{dT} \right)_{SM} \) when

\[
\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m^4} \mu^2
\]

… the lower the smallest measurable electron recoil energy is, the smaller values of \( \mu^2 \) can be probed in scattering experiments …

from A. Starostin

Bernabeu, Papavassiliou, Passera, PLB 2005
MUNU experiment at Bugey reactor (2005)

\[ \mu_\nu \leq 9 \times 10^{-11} \mu_B \]

TEXONO collaboration at Kuo-Sheng power plant (2006)

\[ \mu_\nu \leq 7 \times 10^{-11} \mu_B \]

GEMMA (2007)

\[ \mu_\nu \leq 5.8 \times 10^{-11} \mu_B \]

GEMMA I 2005 - 2007

BOREXINO (2008)

\[ \mu_\nu \leq 5.4 \times 10^{-11} \mu_B \]

...was considered as the world best constraint...

Montanino, Picariello, Pulido, PRD 2008

based on first release of BOREXINO data
GEMMA (2005-2008)
Germanium Experiment on measurement
of Magnetic Moment of Antineutrino
JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant

\[ \mu_\nu < 3.2 \times 10^{-11} \mu_B \]

...till Jan 2010 best limit on magnetic moment

A. Beda, E. Demidova, A. Starostin, V. Brudanin, V. Egorov, D. Medvedev, M. Shirchenko, A. Starostin, Ts. Vylov,
arXiv:09.06.1926 , June 10, 2009,

A. Beda, V. Brudanin, E. Demidova, V. Egorov, G. Gavrilov, M. Shirchenko, A. Starostin, Ts. Vylov,

www.icas.ru (13th Lomonosov Conference)
... quite recent progress due to denotement of new detection channel of Atomic Ionization effect:

\[ \nu + (A, Z) \rightarrow \nu' + (A, Z)^+ + e^- \]
\[ \downarrow \text{recombination} \]
\[ (A, Z) + \gamma \]

H. Wong et al., arXiv: 1001.2074, 13 Jan 2010, reported at the Neutrino 2010 Conference (Athens, June 2010)
...best world limits on $\nu$ effective magnetic moment:

$\mu_\nu < 1.3 \times 10^{-11} \mu_B$

H. Wong et al.,
arXiv: 1001.2074, 13 Jan 2010,

Neutrino 2010 Conference (Athens, June 2010)

$\mu_\nu < 5.0 \times 10^{-12} \mu_B$

A. Beda et al.
(GEMMA Coll.),
arXiv: 1005.2736, 16 May 2010

(Atomic Ionization effect accounted for)

GEMMA-2 (Phase III) expected for 2011:

$\mu_\nu \sim 1.0 \times 10^{-12} \mu_B$
... a bit of electromagnetic properties theory
The most general study of the massive neutrino vertex function (including electric and magnetic form factors) in arbitrary $R_5$ gauge in the context of the SM + $SU(2)$-singlet $Y_R$ accounting for masses of particles in polarization loops.
M. Dvornikov, A. Studenikin

Phys. Rev. D 63, 013004 (2004),

"Electric charge and magnetic moment of massive neutrino;"
JETP 126 (2004), N 8, 1

"Electromagnetic form factors of a massive neutrino."

\[ \Lambda_\mu (q) = f_q(q^2) \sigma_\mu + f_M(q^2) i \sigma_{\mu\nu} q^\nu - f_E(q^2) i \sigma_{\mu\nu} q^\nu \gamma_5 - f_A(q^2)(q^2 \gamma_\mu - q_\mu \gamma_5) \gamma_5 \]

charge

magnetic moment

electric moment

anapole moment

R-gauge

and

\( q^2 \neq 0 \)
Magnetic moment dependence on neutrino mass

\[ \mu_\nu = \mu_\nu(m_\nu) \]
magnetic moment

(for arbitrary neutrino mass, heavy neutrino...)

- LEP data

only 3 light $\nu$s coupled to $Z^0$.

for any additional neutrino

$m_\nu \geq 45$ Gev
\[ \mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \left( \frac{3}{4(1-a)^3} \right) \left( 2 - 7a + 6a^2 - 2a^2 \ln a - a^3 \right), \quad a = \left( \frac{m_e}{M_W} \right)^2 \]

\[ \mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_e \]

**Light**

**Intermediate**

**Heavy**

\[ \mu_\nu \sim 10^{-19} \mu_0 \left( \frac{m_\nu}{1 \text{ eV}} \right) \]

\[ m_e \ll m_\nu \ll M_W \]

\[ m_e \ll m_\nu \ll M_W \]

\[ m_e \ll M_W \ll m_\nu \]


... in case of mixing...
### Neutrino (beyond SM) dipole moments (+ transition moments)

#### Dirac neutrino

\[
\begin{align*}
\mu_{ij} & = \frac{eG_F m_i}{8\sqrt{2}\pi^2} \left( 1 \pm \frac{m_j}{m_i} \right) \sum_{l=e, \mu, \tau} f(r_l) U_{lj} U_{li}^* \\
\epsilon_{ij} & = \frac{m_i - m_j}{2m_W} 
\end{align*}
\]

- \( m_i, m_j \ll m_\ell, m_W \)
- \( f(r_l) \approx \frac{3}{2} \left( 1 - \frac{1}{2} r_l \right), \quad r_l \ll 1 \)

#### Majorana neutrino

- \( i \neq j \)

\[
\mu_{ij}^M = 2\mu_{ij}^D \quad \text{and} \quad \epsilon_{ij}^M = 0 
\]

\[
\mu_{ij}^M = 0 \quad \text{and} \quad \epsilon_{ij}^M = 2\epsilon_{ij}^D 
\]

- transition moments vanish because unitarity of \( U \) implies that its rows or columns represent orthogonal vectors
- transition moments are suppressed, Glashow-Iliopoulos-Maiani cancellation,
- for diagonal moments there is no GIM cancellation

... depending on relative CP phase of \( \nu_i \) and \( \nu_j \)
... A remark on electric charge of \( \nu \) ...

\( \nu \) neutrality \( Q=0 \) is attributed to gauge invariance + anomaly cancellation constraints

\[ SU(2)_L \times U(1)_Y \]

... General proof:

- In SM:
  \[ Q = I_3 + \frac{Y}{2} \]

- In SM (without \( \nu_R \)) triangle anomalies cancellation constraints \( \rightarrow \) certain relations among particle hypercharges \( Y \), that is enough to fix all \( Y \) so that they, and consequently \( Q \), are quantized

\( Q=0 \) is proven also by direct calculation in SM within different gauges and methods

... However, strict requirements for \( Q \) quantization may disappear in extensions of standard \( SU(2)_L \times U(1)_Y \) EW model if \( \nu_R \) with \( Y \neq 0 \) are included: in the absence of \( Y \) quantization electric charges \( Q \) gets dequantized

imposed in SM of electroweak interactions


millicharged \( \nu \)
Neutrino–photon couplings

\[ \nu \rightarrow \gamma \text{ decay, Cherenkov radiation} \]

\[ \gamma \text{ decay in plasma} \]

\[ \nu \rightarrow \nu \text{ scattering} \]

\[ \nu_L \rightarrow \nu_R \text{ spin precession} \]

\[ e/N \rightarrow e/N \text{ scattering} \]

\[ \text{external source} \]
3.9 The tightest astrophysical bound on $\mu_\nu$ comes from cooling of red giant stars by plasmon decay $\gamma^* \rightarrow \nu \bar{\nu}$

$\mathcal{L}_{int} = \frac{1}{2} \sum_{a,b} \left( \mu_{a,b} \bar{\psi}_a \sigma_{\mu \nu} \psi_b + \epsilon_{a,b} \bar{\psi}_a \sigma_{\mu \nu} \gamma_5 \psi_b \right)$

Matrix element

$|M|^2 = M_{\alpha \beta} p^\alpha p^\beta$, $M_{\alpha \beta} = 4 \mu^2 (2 k_\alpha k_\beta - 2 k^2 \epsilon^*_\alpha \epsilon_\beta - k^2 g_{\alpha \beta})$,

Decay rate

$\Gamma_{\gamma \rightarrow \nu \bar{\nu}} = \frac{\mu^2}{24\pi} \frac{\left( \omega^2 - k^2 \right)^2}{\omega}$

= 0 in vacuum $\omega = k$

In the classical limit $\gamma^*$ - like a massive particle with $\omega^2 - k^2 = \omega_{pl}^2$

Energy-loss rate per unit volume

$Q_\mu = g \int \frac{d^3k}{(2\pi)^3} \omega f_{BE} \Gamma_{\gamma \rightarrow \nu \bar{\nu}}$

distribution function of plasmons
Magnetic moment plasmon decay enhances the Standard Model photo-neutrino cooling by photon polarization tensor

more fast cooling of the star.

In order not to delay helium ignition (≤5% in $Q$)

... best astrophysical limit on magnetic moment...

G. Raffelt, PRL 1990

$$Q_\mu = g \int \frac{d^3k}{(2\pi)^3} \omega f_{BE} \Gamma \gamma \to \nu \bar{\nu}$$
New mechanism of electromagnetic radiation
Spin light of neutrino in matter and electromagnetic fields
Quasi-classical theory of spin light of neutrino in matter and gravitational field

We predict the existence of a new mechanism of the electromagnetic process stimulated by the presence of matter, in which a neutrino with non-zero magnetic moment emits light.

\[ \text{Spin light of neutrino in matter} \quad \nu_L \rightarrow \nu_R \]

\[ \gamma \]

\text{(quantum approach)}

A.Studenikin, A.Ternov, PLB 2004
A.Grigoriev, Studenikin, Ternov, PLB 2005
Conclusion
\( e.m. \) **vertex function** \( \rightarrow 4 \) form factors

- **charge** dipole magnetic and electric

\[
\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5
\]

\[
f_A(q^2)(q^2\gamma_\mu - q_\mu \gamma_4)\gamma_5 \text{ anapole}
\]

- **EM properties** \( \rightarrow \) a way to distinguish **Dirac** and **Majorana**

- **Standard Model with** \( \nu_R \) \( (m_\nu \neq 0) \):
  \[
  \mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_e \sim 3 \times 10^{-10}\mu_B \left( \frac{m_\nu}{3\text{eV}} \right)
  \]

- **In extensions of SM**
  - **Limits from reactor** \( \nu \)-\( e \) scattering experiments (2010):
    \[
    \mu_\nu < 1.3 \times 10^{-11} \mu_B
    \]
    \[
    \mu_\nu < 5.0 \times 10^{-12} \mu_B
    \]
    - **H.Wong et al.** (Kuo-Sheng Reactor Lab)
    - **A.Beda et al.** (GEMMA Collab.)

- **Limits from astrophysics** (star cooling) (1990):
  \[
  \mu^2 \leq 3 \times 10^{-12} \mu_B
  \]
  - **G.Raffelt**

Due to smallness of neutrino-mass-induced magnetic moments, any indication for non-trivial electromagnetic properties of, that could be obtained within reasonable time in the future, would give evidence for interactions beyond extended Standard Model.
\( \mu_\nu \) is presently known to be in the range

\[
10^{-20} \mu_B \leq \mu_\nu \leq 10^{-11} \mu_B
\]

\( \mu_\nu \) provides a tool for exploration possible physics beyond the Standard Model

Due to smallness of neutrino-mass-induced magnetic moments,

\[
\mu_{ii} \approx 3.2 \times 10^{-19} \left( \frac{m_i}{1 \text{ eV}} \right) \mu_B
\]

any indication for non-trivial electromagnetic properties of \( \nu \), that could be obtained within reasonable time in the future, would give evidence for interactions beyond extended Standard Model
… situation with electromagnetic properties

is better that it was for in the time of W. Pauli, 1930

… once will be observed experimentally

… our hopes

… are important in astrophysics

… there is a need for further theoretical and experimental studies