Advanced Scaling FFAG

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Scaling FFAG

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- FFAG : Fixed Field Alternating Gradient
  - Scaling: zero-chromaticity: constant tunes

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- FFAG : Fixed Field Alternating Gradient
  - Scaling: zero-chromaticity: constant tunes
- Scaling conditions for zero chromaticity
  - Orbit similarity for different beam momentum
  - Constant field index for any orbits

Scaling condition I
circular ring

Circular ring

- Betatron eqs.

$$\frac{d^2x}{d\theta^2}+\frac{r^2}{\rho^2}\left(1-K\rho^2\right)x=0$$

$$\frac{d^2z}{d\theta^2}+\frac{r^2}{\rho^2}\left(K\rho^2\right)z=0$$

- Scaling condition: zero-chromaticity

$$\begin{cases}\dfrac{d\left(r^2/\rho^2\right)}{dp}=0\\ \dfrac{d\left(K\rho^2\right)}{dp}=0\end{cases}\longrightarrow\begin{cases}r\propto\rho\\ \dfrac{r}{B}\left[\dfrac{\partial B_z}{\partial x}\right]_{z=0}=k\end{cases}$$

  - B-field

$$B_z=B_0\left(\frac{r}{r_0}\right)^k f(\theta)$$

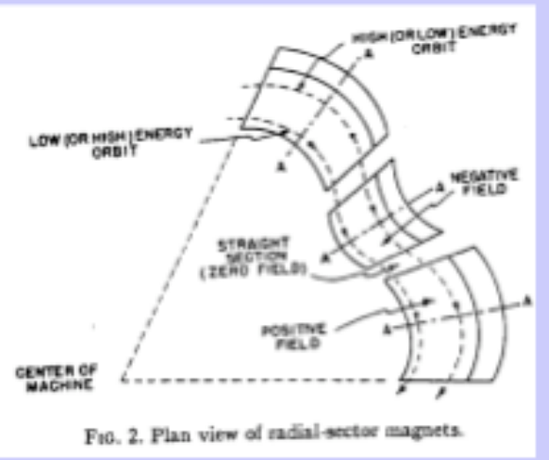


FIG. 2. Plan view of radial-sector magnets.

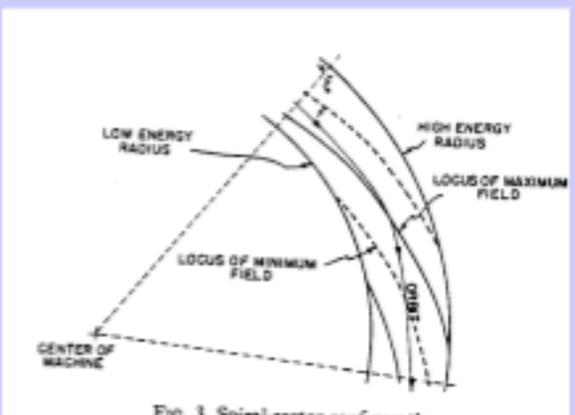


FIG. 3. Spiral-sector configuration.

Scaling FFAG ring

Scaling FFAG ring

- Pro/
  - Fixed field & Strong focusing
  - Zero chromaticity
    - constant betatron tunes → no-resonance crossing
  - Large acceptance (longitudinal & transverse)

Scaling FFAG ring

- Pro/
  - Fixed field & Strong focusing
  - Zero chromaticity
    - constant betatron tunes → no-resonance crossing
  - Large acceptance (longitudinal & transverse)
- Con/
  - Relative large dispersion:Orbit excursion is large.
    - Large aperture magnet
    - Large aperture rf cavity → Low frequency
  - Short straight section
    - Injection/Extraction difficulties → Kicker/Septum needs large apertures.
    - Available space for rf cavity is limited.

Scaling FFAG linear line

- Is it possible to make a linear FFAG straight line?
  - keeping a scaling law: zero chromaticity
  - reducing dispersion: dispersion suppressor
  - making a good match with ring: insertion
- Magnetic field configuration for FFAG linear line?
  - Obviously not: $B = B_0 \left(\frac{r}{r_0} \right)^k f(\theta)$

Scaling condition II
linear (straight) transport line

- Betatron eqs.

$$\frac{d^2x}{dy^2}+\frac{1}{\rho^2}\left(1-K\rho^2\right)x=0$$

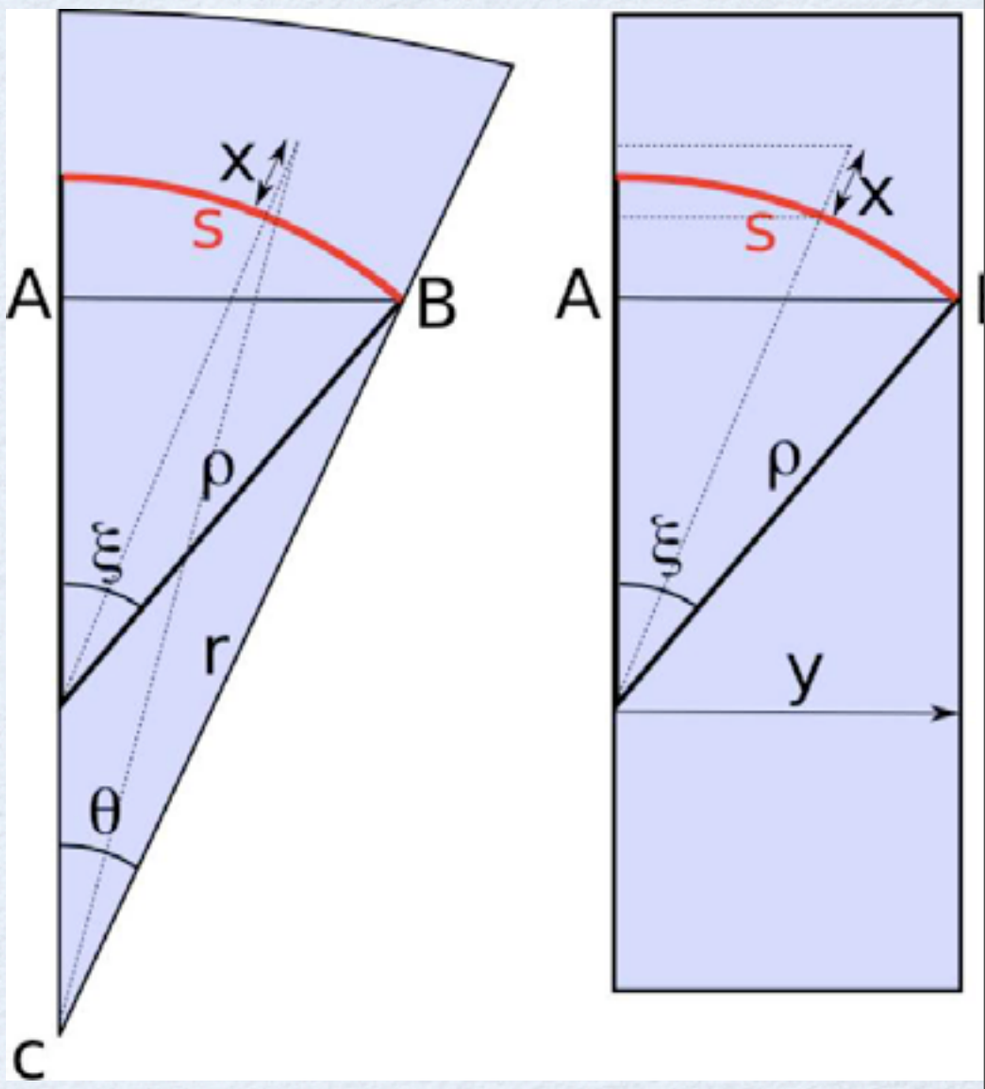
$$\frac{d^2z}{dy^2}+\frac{1}{\rho^2}\left(K\rho^2\right)z=0$$

- Scaling conditions: zero-chromaticity

$$\begin{cases}\dfrac{d\left(1/\rho^2\right)}{dp}=0\\ \dfrac{d\left(K\rho^2\right)}{dp}=0\end{cases}\longrightarrow\begin{cases}\rho=const.\\ \dfrac{1}{B}\left[\dfrac{\partial B_z}{\partial x}\right]_{z=0}=\dfrac{n}{\rho}\end{cases}$$

  - Magnetic field

$$B_z=B_0\exp\left[\frac{n}{\rho}x\right]$$



$$\left[\lim_{r_0\to\infty}\left(\frac{r}{r_0}\right)^k=\lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{x}{r_0}k}=\lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{n}{\rho}x}=\exp\left(\frac{n}{\rho}x\right)\right]$$

Scaling linear line

- Example (JB. Lagrange)
  - Perfect scaling(zero-chromatic) FFAG linear transport line
  - proton 80-200MeV

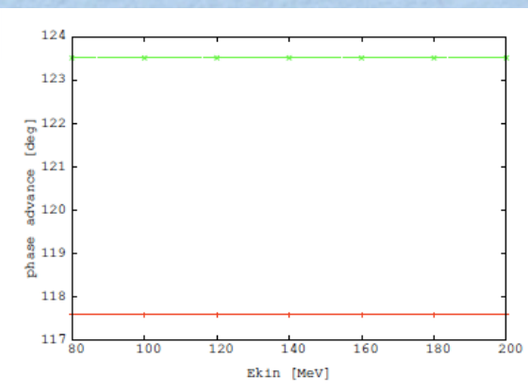
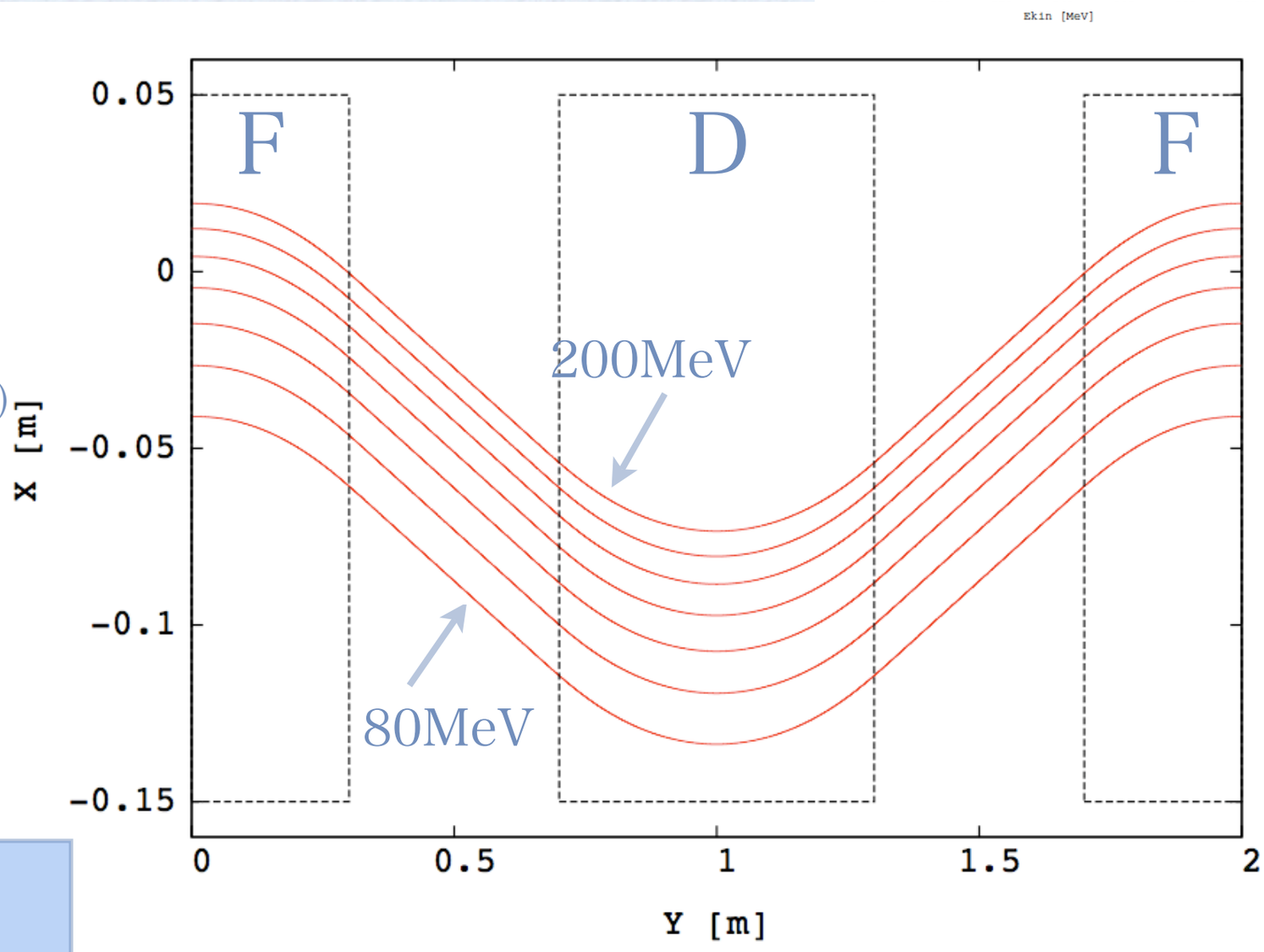


Table 1: Tracking parameters

Length of the magnets	60 cm
Drift	40 cm
Kinetic energy range	80 to 200 MeV (proton)
Field index	17
Local curvature radius	2.1 m
Step size	1 mm
Phase advances:	
horizontal μ_x	104.8 deg.
vertical μ_z	112.5 deg.



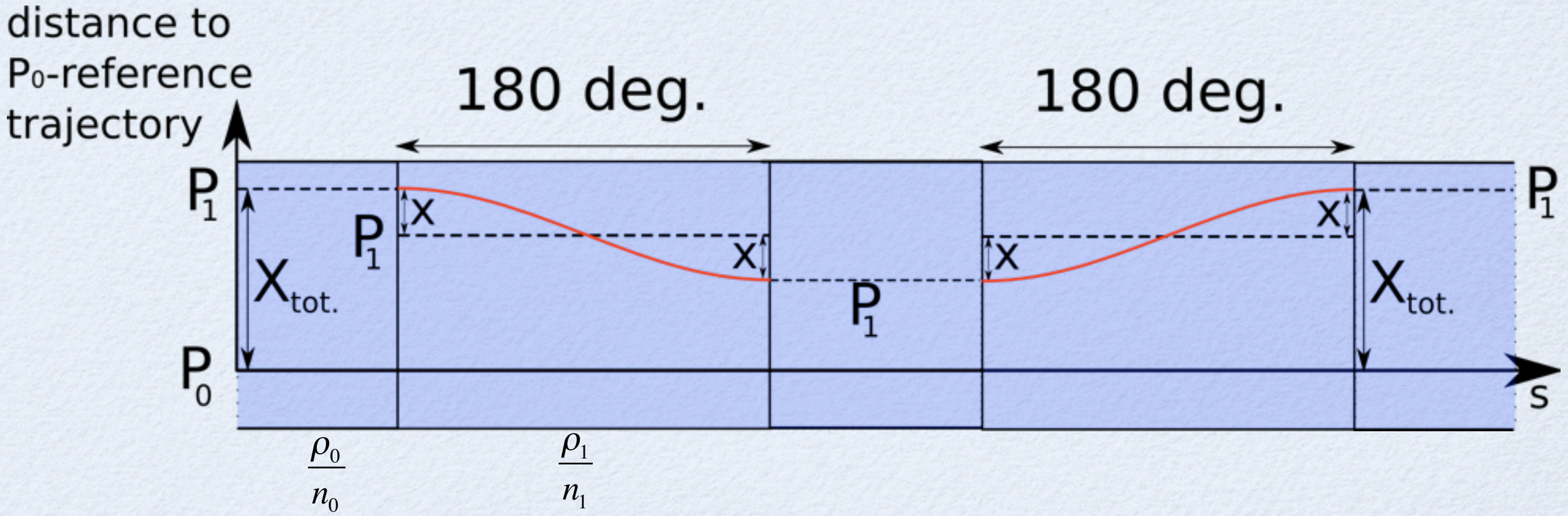
B-field

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

Dispersion suppressor

- Dispersion suppressor
  - successive π-cells

$$X_{tot} = X_1 - X_0 = \frac{1}{n/\rho}\ln\left(\frac{P_1}{P_0}\right) \qquad x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$



Insertion Matching

btw. ring & straight line

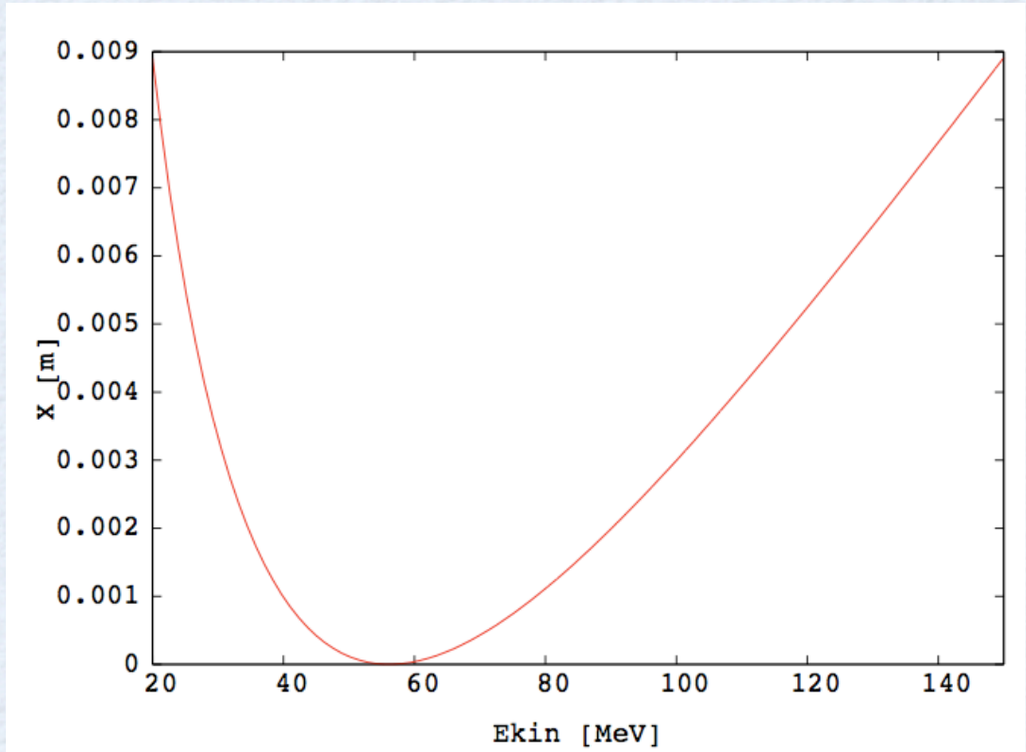
- Matching condition

$$\left(1+\frac{x}{r_m}\right)^{k+1} = \exp\left(\frac{n}{\rho}x\right)$$

ring　　linear line

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

← 1st order



Example: 150MeV p-FFAG ring(KURRI) with insertion

higher order error: $\sim \frac{1}{k}x$

→ smaller for larger ring

Advanced scaling FFAG

$$\frac{k+1}{r_m} = \frac{n}{\rho}$$

$$B_z = B_0 \exp\left[\frac{n}{\rho} x\right]$$

linear straight

insertion/
matching

dispersion suppressor

ring

$$B = B_0 \left(\frac{r}{r_0}\right)^k$$

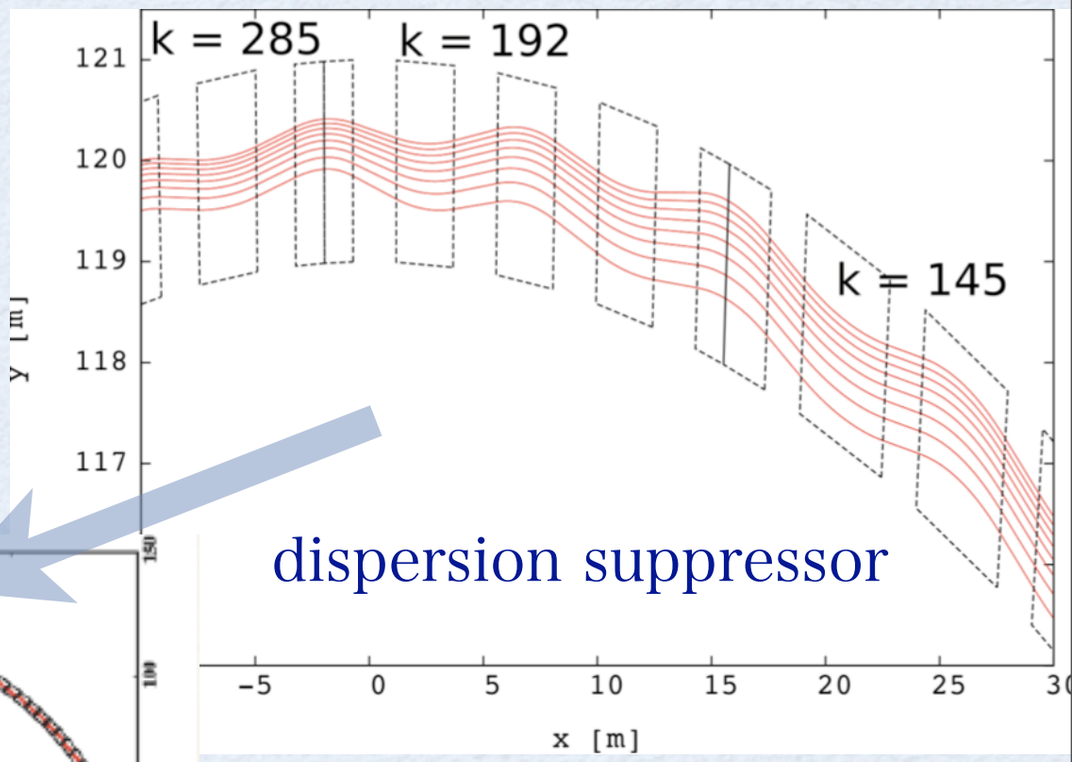
$$x = \ln\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_0}{n_0} - \frac{\rho_1}{n_1}\right)$$

Applied to muon accelerator

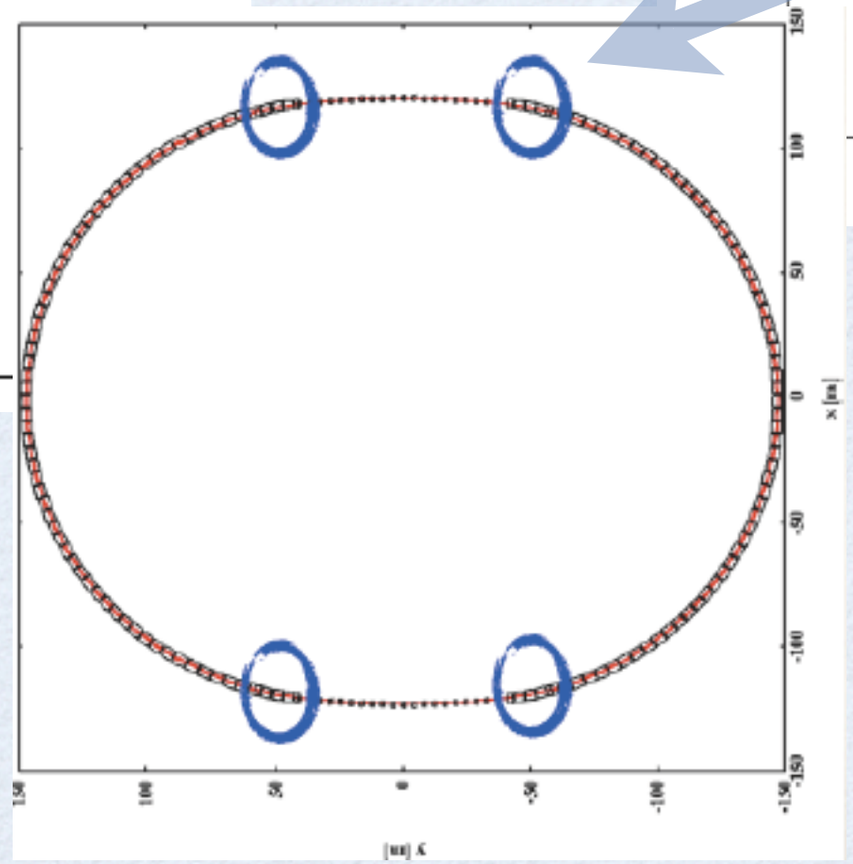
- 3-10 GeV Muon accelerator in Neutrino Factory
  - T.Planche - next talk

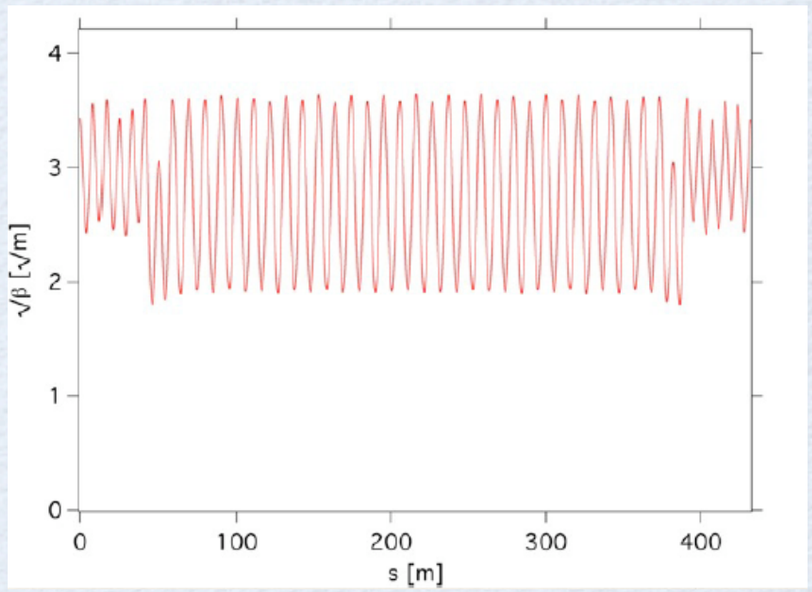
Table 1: 3 to 10 GeV Muon Ring Parameters

Lattice type	scaling FFAG - double beam
Mean radius	120 m
Number of cells	72
Field index k	145
Packing factor	0.7
B_{max}	2.6 T
Horiz. phase adv. per cell	93.2 deg.
Verti. phase adv. per cell	30.2 deg.
Mean RF frequency	$\sim$ 400 MHz
RF peak voltage	1.6 GV/turn
Number of RF cavities	72



dispersion suppressor





Summary

- Advanced scaling FFAG scheme has been developed.
- Scaling linear system requires,
  - Scaling law
  - Insertion/Matching
  - Dispersion suppressor
- Race-track FFAG ring is in reality.
  - Muon acceleration → Neutrino Factory
    - option replacing RLA (T. Planche)
  - Applications