Coherent pion production

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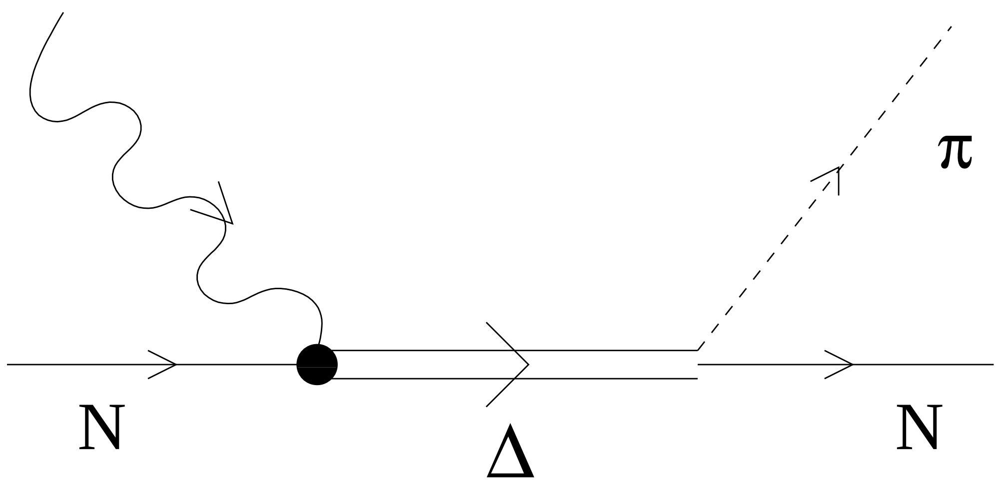
- “Weak Pion Production off the Nucleon”, Phys. Rev. D 76, 033005 (2007)
E.Hernández, J. Nieves, M. Valverde
- “Theoretical study of neutrino-induced coherent pion production off nuclei at T2K and MiniBooNE energies”, Phys. Rev. D 79, 013002 (2009)
J.E. Amaro, E.Hernández, J. Nieves, M. Valverde
- “Neutrino Induced Coherent Pion Production off Nuclei and PCAC”, Phys. Rev. D 80, 013003 (2009)
E.Hernández, J. Nieves, M.J. Vicente-Vacas

Plan of the talk

- Part I: Is it possible to constraint the $C_5^A \;\; N \rightarrow \Delta$ axial form factor from coherent production data?.
- Part II: A Critique of the use of the Rein-Sehgal model for low energy neutrinos.

Delta Pole Term

The dominant contribution for weak pion production at intermediate energies is given by the Δ pole mechanism



$N \to \Delta$ weak current I

$$\langle \Delta^+ ; p_\Delta = p + q \,| j^\mu_{cc+}(0) | n; p \rangle = \cos\theta_C \bar{u}_\alpha(\vec{p}_\Delta) \Gamma^{\alpha\mu}(p,q)\, u(\vec{p})$$

$$\begin{aligned}
\Gamma^{\alpha\mu}(p,q) \\
&= \left[\frac{C_3^V}{M} \left(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu \right) + \frac{C_4^V}{M^2} \left(g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu \right) + \frac{C_5^V}{M^2} \left(g^{\alpha\mu} q \cdot p - q^\alpha p^\mu \right) + C_6^V g^{\mu\alpha} \right] \gamma_5 \\
&\quad + \left[\frac{C_3^A}{M} \left(g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu \right) + \frac{C_4^A}{M^2} \left(g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu \right) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^\mu q^\alpha \right]
\end{aligned}$$

$N \to \Delta$ weak current II

- Vector form factors: determined from the analysis of photo and electroproduction

(O. Lalakulich *et al.*, Phys. Rev. D74, 014009 (2006))

$$C_3^V = \frac{2.13}{(1-q^2/M_V^2)^2} \cdot \frac{1}{1-\frac{q^2}{4M_V^2}}, \qquad C_4^V = \frac{-1.51}{(1-q^2/M_V^2)^2} \cdot \frac{1}{1-\frac{q^2}{4M_V^2}},$$

$$C_5^V = \frac{0.48}{(1-q^2/M_V^2)^2} \cdot \frac{1}{1-\frac{q^2}{0.776M_V^2}}, \qquad C_6^V = 0\ (CVC), \qquad M_V = 0.84\,\mathrm{GeV}$$

- Axial form factors: use Adler model which assumes

$$C_4^A(q^2) = -\frac{C_5^A(q^2)}{4}, \qquad C_3^A(q^2) = 0$$

and take (E.A. Paschos *et al.*, Phys. Rev. D69, 014013 (2004))

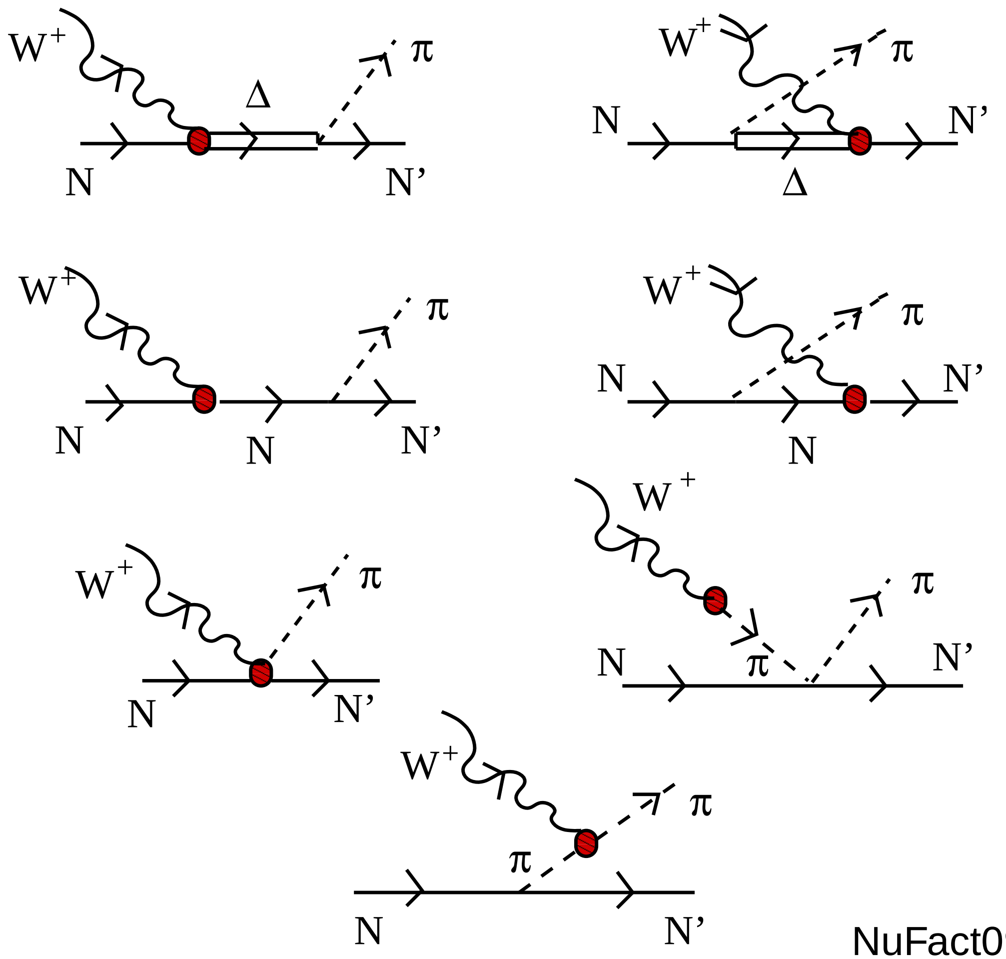
$$C_5^A(q^2) = \frac{1.2}{(1-q^2/M_{A\Delta}^2)^2} \cdot \frac{1}{1-\frac{q^2}{3M_{A\Delta}^2}}, \qquad C_6^A(q^2) = C_5^A(q^2)\frac{M^2}{m_\pi^2-q^2}, \quad \text{with } M_{A\Delta} = 1.05\,\mathrm{GeV}$$

where $C_5^A(0) = 1.2$ from the off-diagonal GT relation

Background Terms

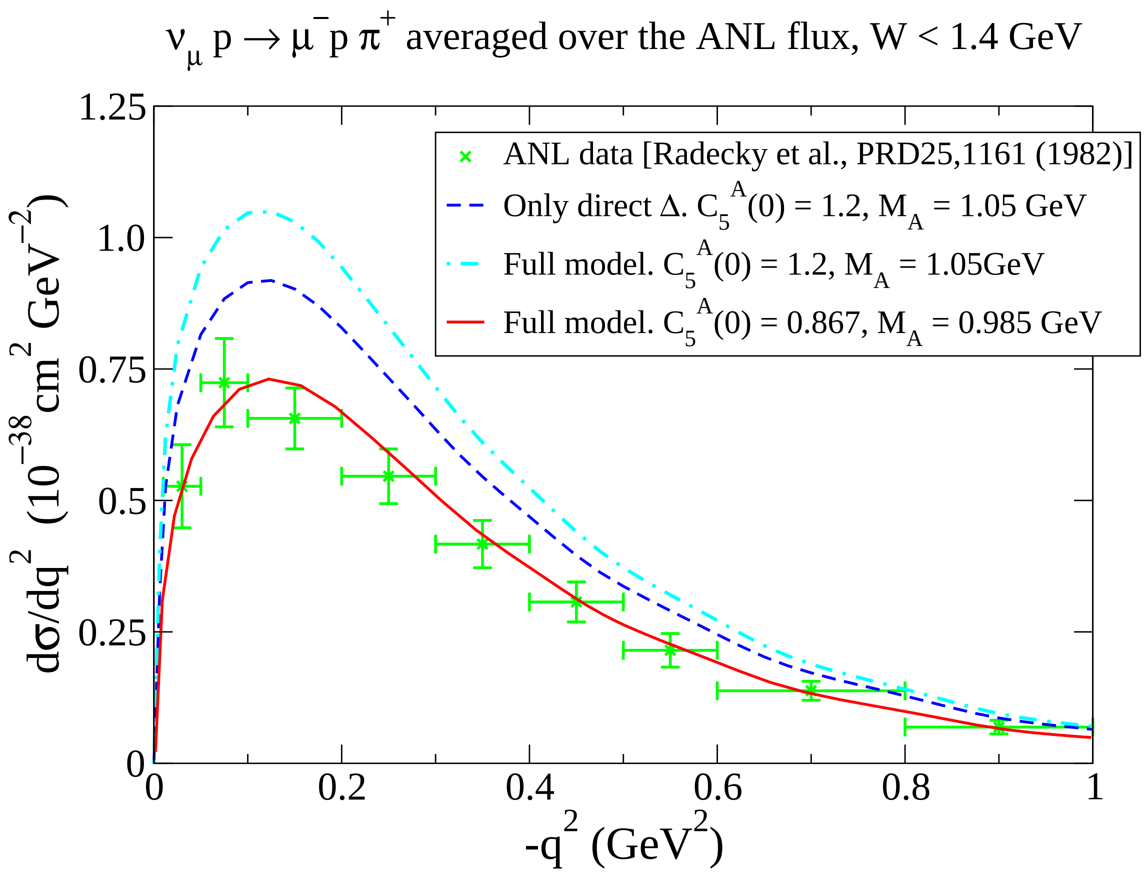
We shall also include background terms required by chiral symmetry. To that purpose we use a SU(2) non-linear σ model Lagrangian.

- No freedom in coupling constants
- We supplement it with well known form factors



$\nu_\mu p \to \mu^- p \pi^+$ reaction I

Flux averaged q^2-differential $\nu_\mu p \to \mu^- p\pi^+$ cross section $\int_{M+m_\pi}^{1.4\,\mathrm{GeV}} dW \frac{d\,\overline{\sigma}_{\nu_\mu \mu^-}}{dq^2 dW}$

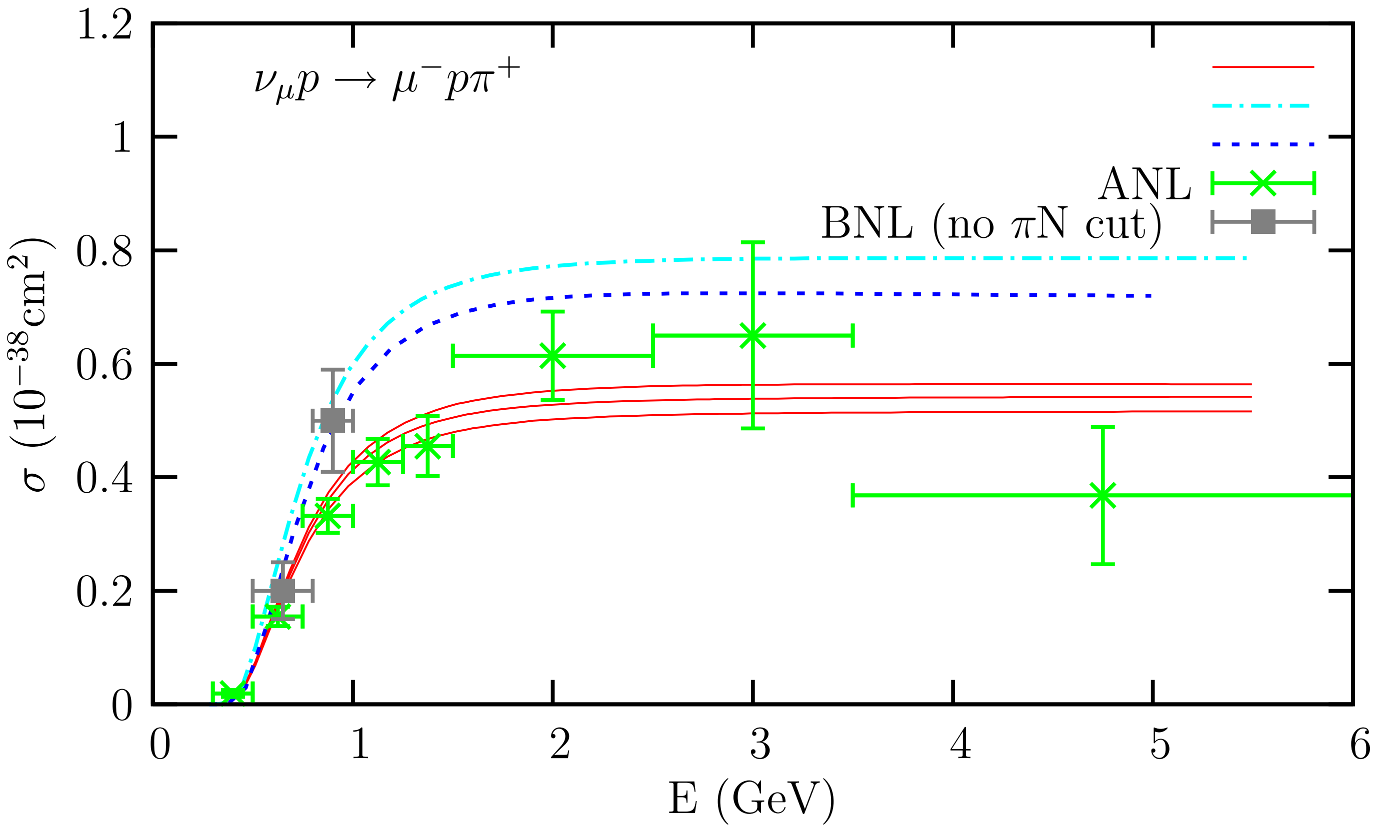


$$C_5^A(q^2) = \frac{C_5^A(0)}{(1-q^2/M_{A\Delta}^2)^2} \cdot \frac{1}{1-\frac{q^2}{3 M_{A\Delta}{}^2}}$$

Results suggest a refit of C_5^A

$$C_5^A(0) = 0.867 \pm 0.075, \quad M_{A\Delta} = 0.985 \pm 0.082\,\mathrm{GeV}$$

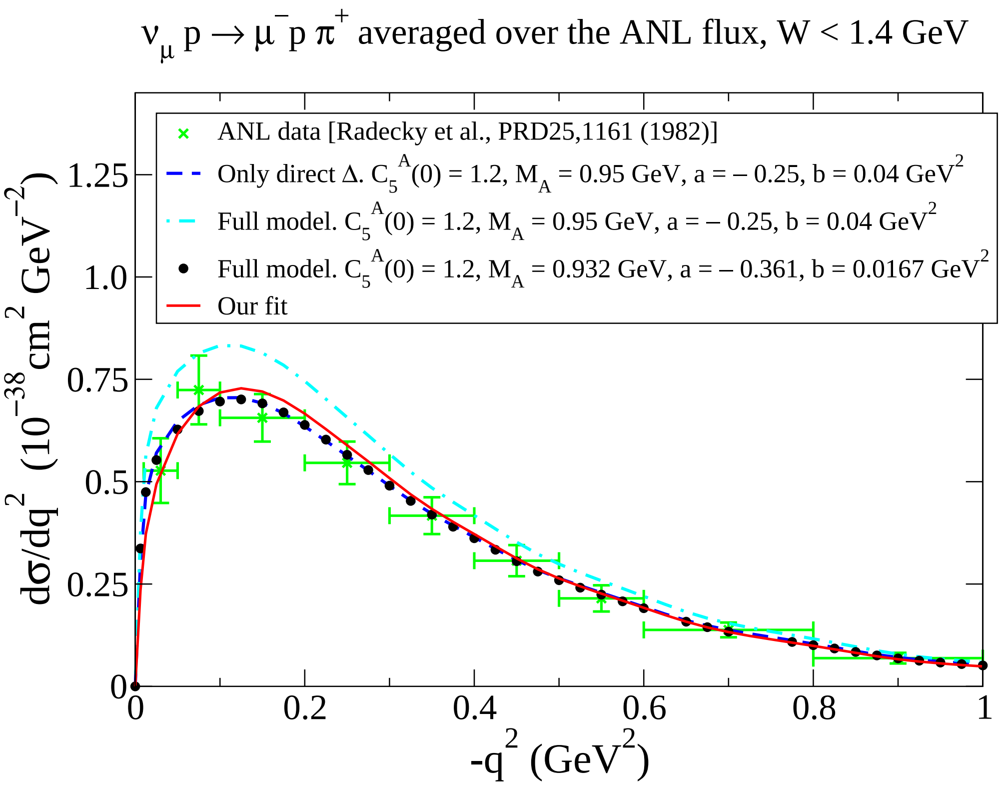
$\nu_\mu p \rightarrow \mu^- p \pi^+$ reaction II



But mind BNL data [T. Kitagaki et al., Phys. Rev. D34, 2554 (1986)] for which $C_5^A(0) = 1.2$ would be preferred

$\nu_\mu p \to \mu^- p \pi^+$ reaction III

A different $C_5^A(q^2)$ parameterization is possible

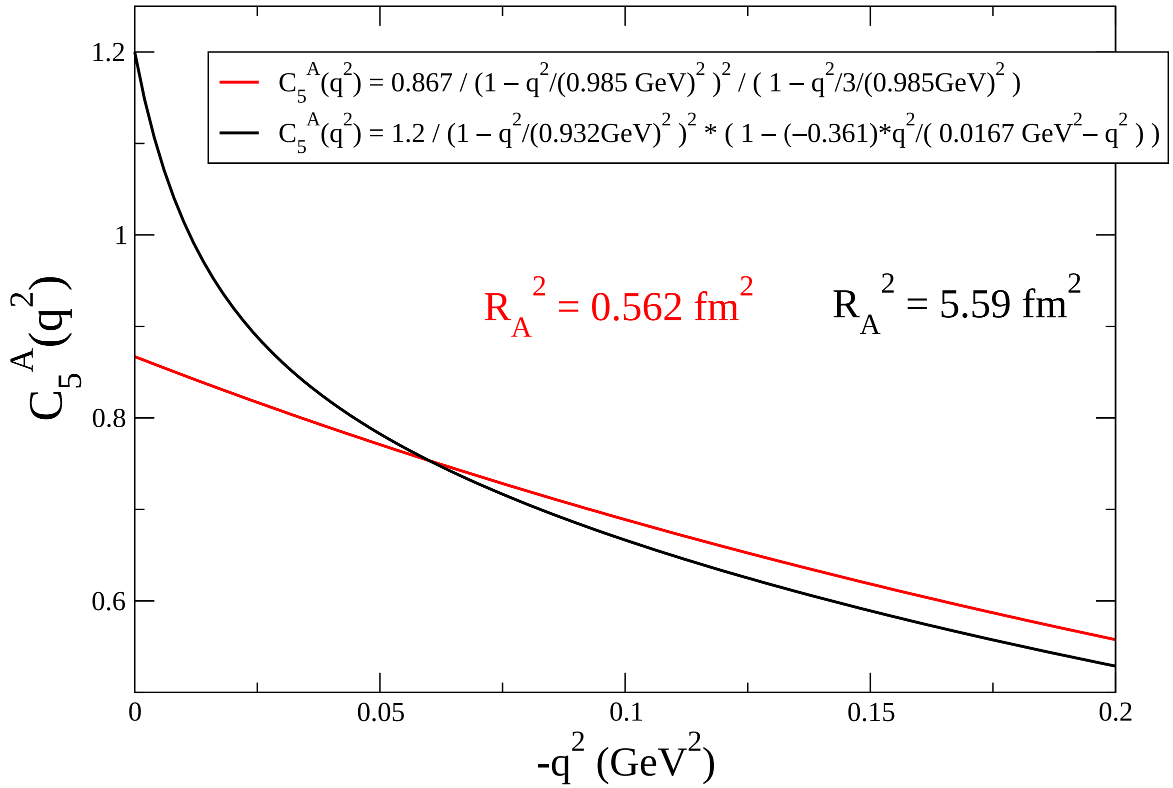


$$C_5^A(q^2) = \frac{1.2\cdot(1-\frac{a\,q^2}{b-q^2})}{(1-q^2/M_{A\Delta}{}^2)^2}$$

Leitner et al. [Phys. Rev. C 79, 034601 (2009)] find $a = -0.25$, $b = 0.04\,GeV^2$, $M_{A\Delta} = 0.95\,GeV$ when only direct Δ is included

With background terms included one needs $a = -0.361$, $b = 0.0167\,GeV^2$, $M_{A\Delta} = 0.932\,GeV$

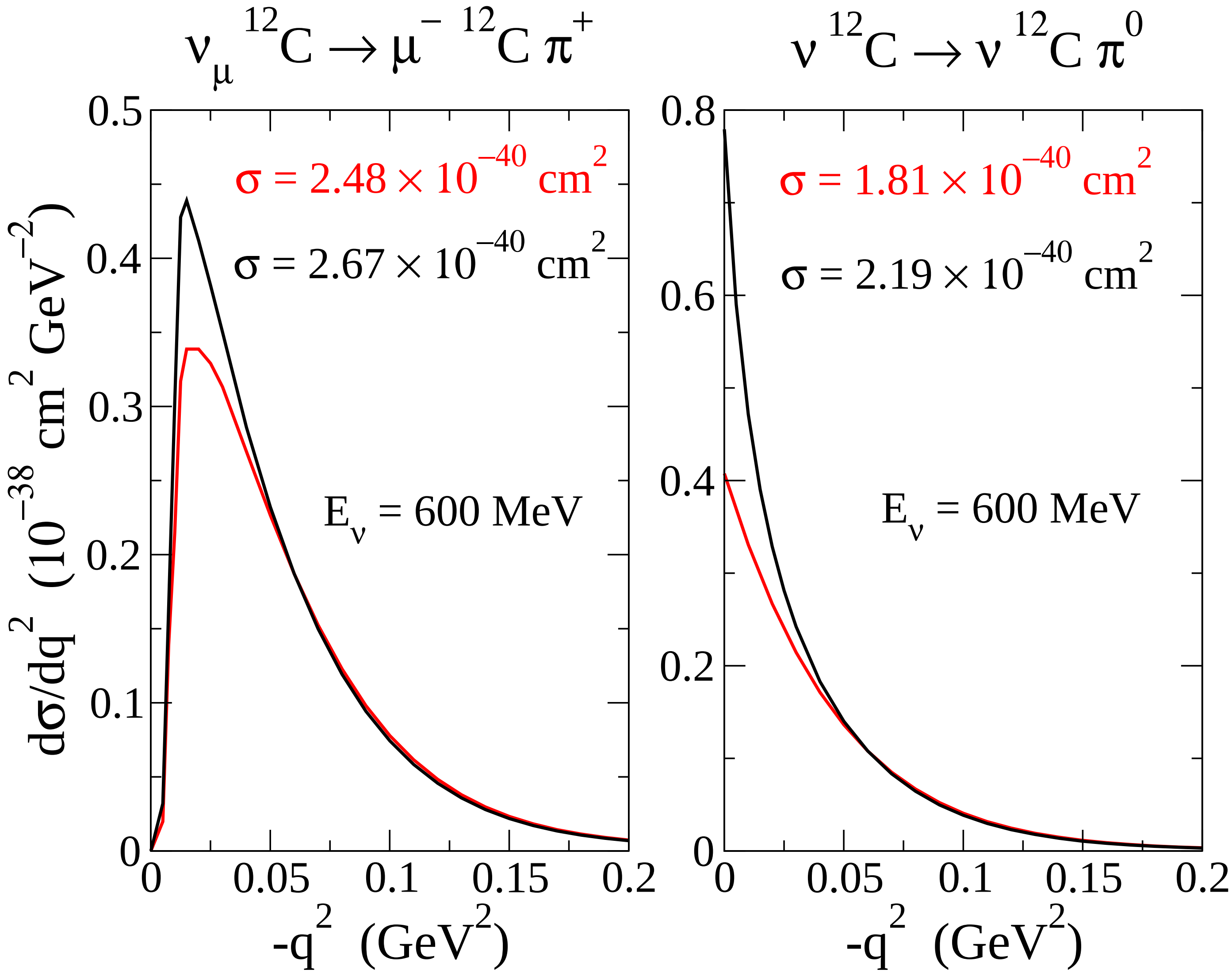
$C_5^A(q^2)$ comparison



$$R_A^2 = -\frac{6}{C_5^A(0)} \frac{dC_5^A(q^2)}{d(-q^2)}\bigg|_{q^2=0}$$

$$C_5^A(q^2) = \frac{C_5^A(0)}{1 - \frac{q^2}{M_{A\Delta}^2}} \cdot \frac{1}{1 - \frac{q^2}{3M_{A\Delta}^2}} \qquad C_5^A(q^2) = \frac{1.2 \cdot \left(1 - \frac{aq^2}{b-q^2}\right)}{1 - \frac{q^2}{M_{A\Delta}^2}}$$

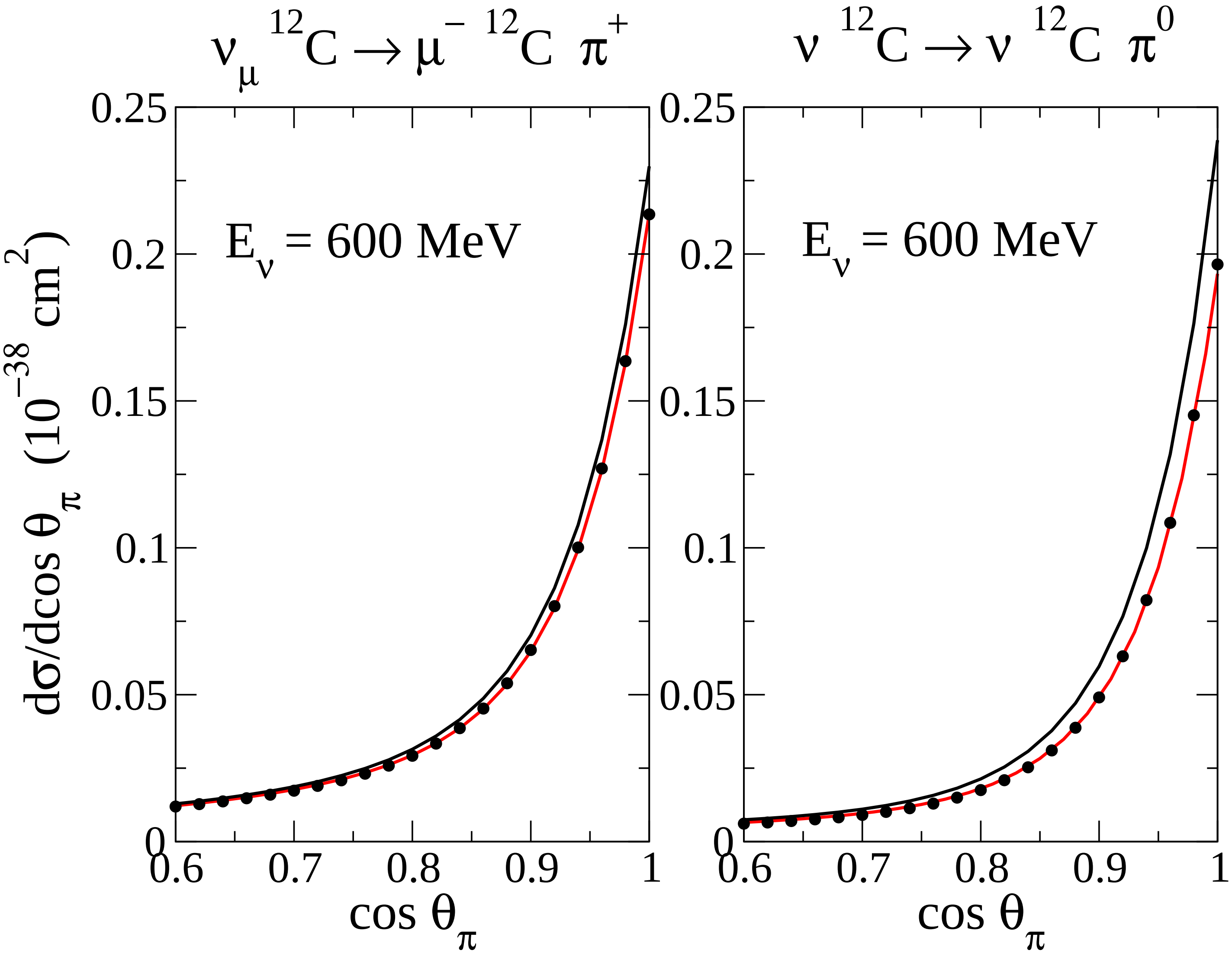
$\frac{d\sigma}{dq^2}$ for coherent production



$$C_5^A(q^2) = \frac{C_5^A(0)}{1 - \frac{q^2}{M_{A\Delta}^2}} \cdot \frac{1}{1 - \frac{q^2}{3M_{A\Delta}^2}}$$

$$C_5^A(q^2) = \frac{1.2 \cdot \left(1 - \frac{a q^2}{b - q^2}\right)}{1 - \frac{q^2}{M_{A\Delta}^2}}$$

$\frac{d\sigma}{d\cos\theta_\pi}$ for coherent production

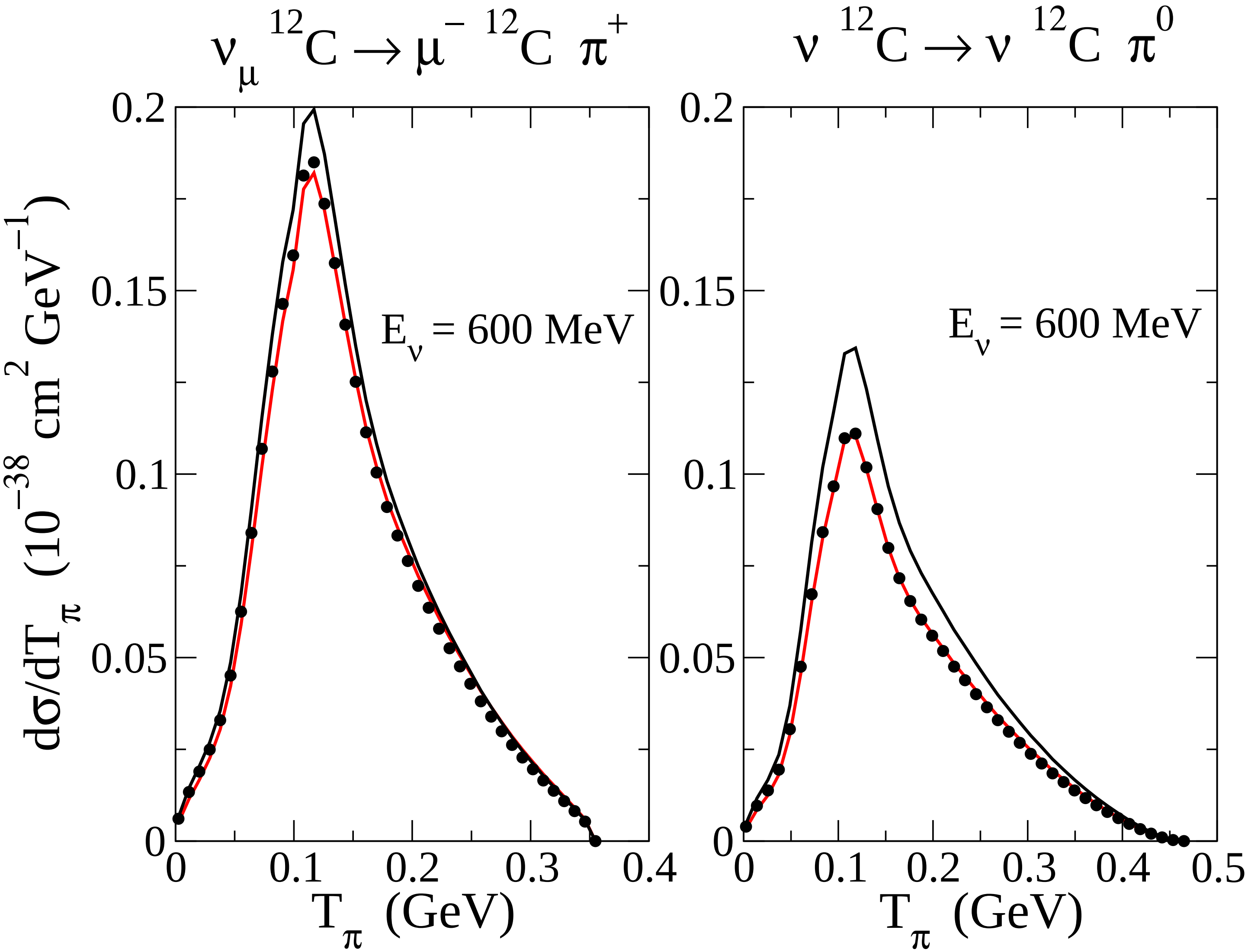


$$C_5^A(q^2) = \frac{C_5^A(0)}{1-\frac{q^2}{M_{A\Delta}^2}} \cdot \frac{1}{1-\frac{q^2}{3M_{A\Delta}^2}}$$

$$C_5^A(q^2) = \frac{1.2\cdot(1-\frac{a q^2}{b-q^2})}{1-\frac{q^2}{M_{A\Delta}^2}}$$

Shape is not affected!

$\frac{d\sigma}{dT_\pi}$ for coherent production

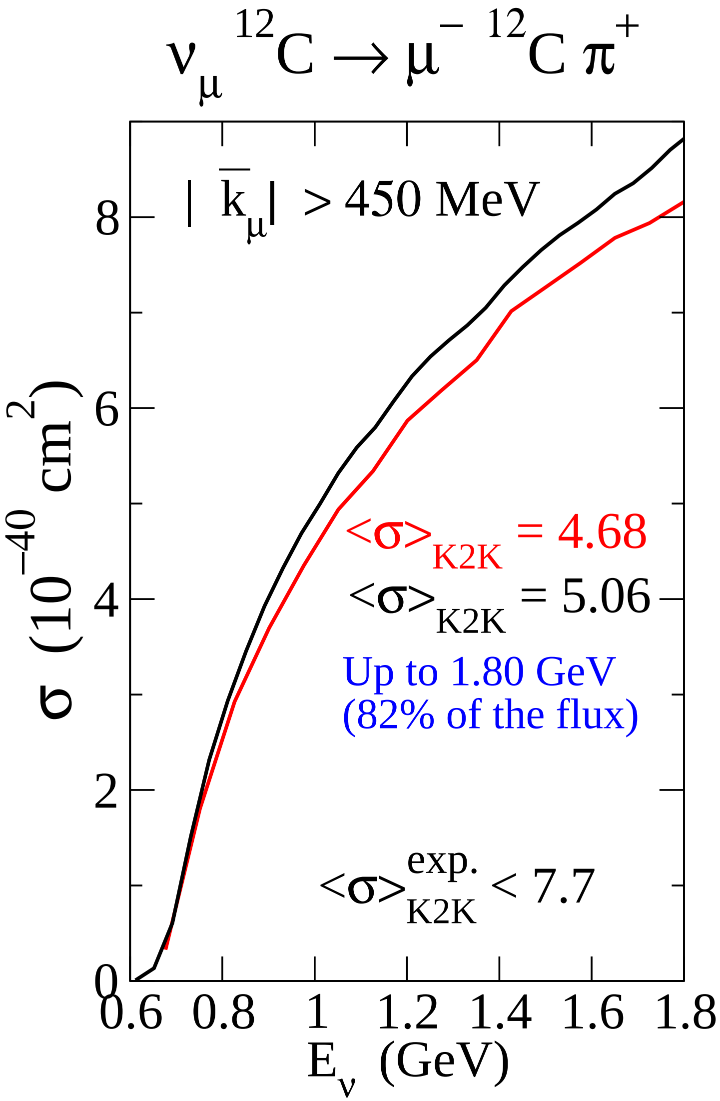


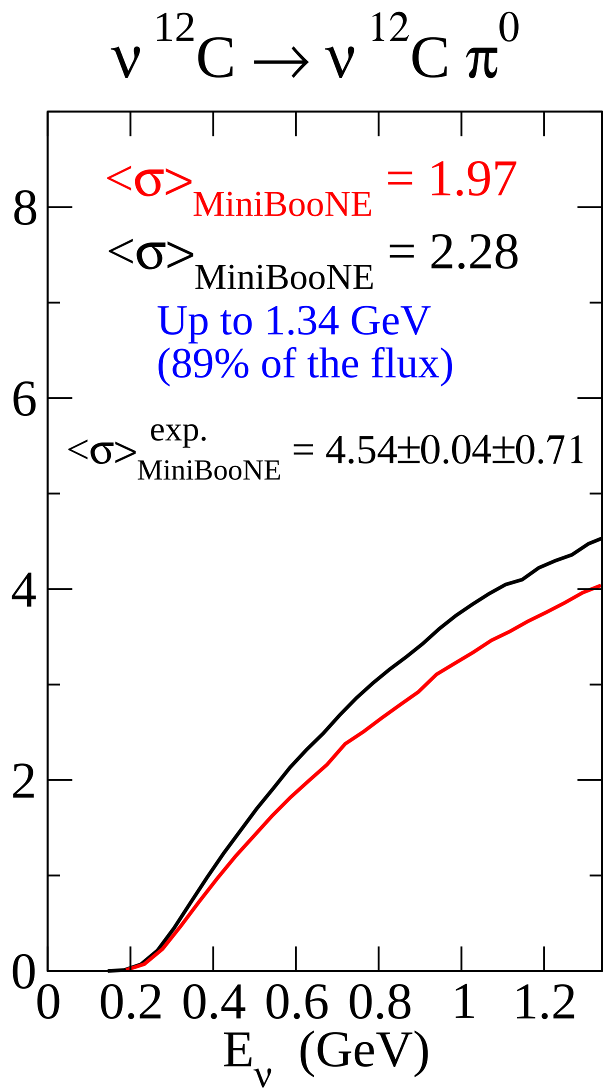
$$C_5^A(q^2) = \frac{C_5^A(0)}{1-\frac{q^2}{M_{A\Delta}^2}} \cdot \frac{1}{1-\frac{q^2}{3M_{A\Delta}^2}}$$

$$C_5^A(q^2) = \frac{1.2\cdot(1-\frac{a\,q^2}{b-q^2})}{1-\frac{q^2}{M_{A\Delta}^2}}$$

Shape is very slightly affected!

Total cross sections for coherent production





$$C_5^A(q^2) = \frac{C_5^A(0)}{1-\frac{q^2}{M_{A\Delta}^2}} \cdot \frac{1}{1-\frac{q^2}{3M_{A\Delta}^2}}$$

$$C_5^A(q^2) = \frac{1.2\cdot(1-\frac{a\,q^2}{b-q^2})}{1-\frac{q^2}{M_{A\Delta}^2}}$$

Deuteron effects

ANL and BNL data were measured in deuterium

- Deuteron effects were estimated by L. Alvarez-Ruso et al [Phys. Rev. C 59, 3386 (1999)] to be of the order of 8% for $E_\nu = 1.6$ GeV even for $-q^2 < 0.1$ GeV
- A recent fit to ANL data by Graczyk et al (arXiv:0907.1886) including deuteron effects and using a dipole parameterization for C_5^A finds $C_5^A(0) = 1.13 \pm 0.15$, $M_{A\Delta} = 0.936 \pm 0.077$ GeV.

  No background terms were included.

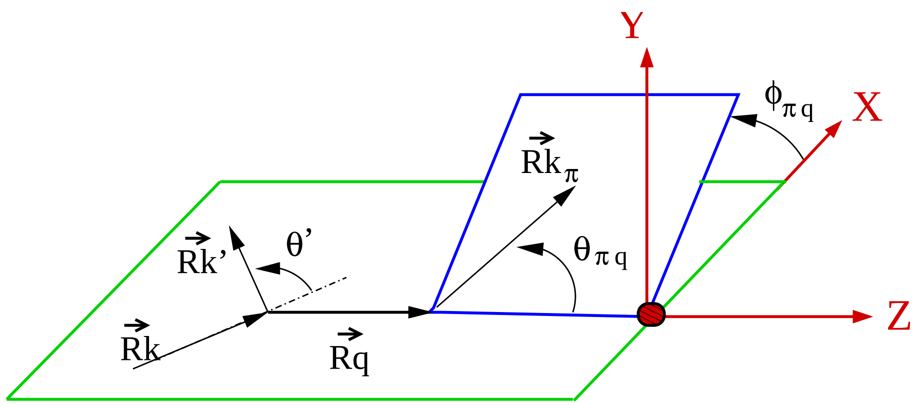
Conclusions part I

- Background terms have to be included in the analysis of the reaction at the nucleon level (We know they are not relevant for coherent production).
- If ANL data is correct then, either you reduce $C_5^A(0)$ or you have to live with a very large axial radius.
- If BNL data is correct you can have both $C_5^A(0) = 1.2$, as given by the off-diagonal Goldberger-Treiman relation, and an axial radius of $0.7 \sim 0.9$ fm.
- Deuteron effects could also play a role in decreasing the cross section and thus affecting the determination of C_5^A.
- Coherent reactions do not help much in obtaining information on C_5^A.
  Note however that recent MiniBooNE data is consistent with a dipole parameterization having $C_5^A(0) \approx 1.2$ and $M_{A\Delta} \approx 1.$ GeV.
- New data **at the nucleon level** would be welcome.

PCAC models I

For NC processes

$$\nu_l(k) + \mathcal{N}_{gs} \to \nu_l(k') + \mathcal{N}_{gs} + \pi^0(k_\pi)\,.$$



$$q = k - k',\ y = q^0/E,\ t = (q - k_\pi)^2$$

$$\frac{d\sigma}{dq^2\,dy\,dt\,d\phi_{\pi q}} = \frac{G^2}{16\pi^2} E\,\kappa \left(-\frac{q^2}{|\vec{q}\,|^2}\right) \left(\frac{u^2}{2\pi}\frac{d\sigma_L}{dt} + \frac{v^2}{2\pi}\frac{d\sigma_R}{dt} + 2\,\frac{uv}{2\pi}\frac{d\sigma_S}{dt} + \frac{d\mathcal{A}}{dt\,d\phi_{\pi q}}\right),$$

where $$\kappa = q^0 + \frac{q^2}{2\mathcal{M}}, \qquad u, v = \frac{E + E' \pm |\vec{q}\,|}{2E}, \qquad \int \frac{d\mathcal{A}}{dt\,d\phi_{\pi q}}\,d\phi_{\pi q} = 0$$

$\sigma_{R,L,S}$ stand for cross sections for right, left and scalar polarized intermediate vector mesons. $\mathcal{A}$ is not a proper cross section and it contains all the dependence on $\phi_{\pi q}$. In the $q^2 \to 0$ limit only the σ_S term survives

PCAC models II

Equation

$$\frac{d\sigma}{dq^2 dy\, dt\, d\phi_{\pi q}} = \frac{G^2}{16\pi^2} E\,\kappa \left(-\frac{q^2}{|\vec{q}\,|^2}\right) \left(\frac{u^2}{2\pi}\frac{d\sigma_L}{dt} + \frac{v^2}{2\pi}\frac{d\sigma_R}{dt} + 2\,\frac{uv}{2\pi}\frac{d\sigma_S}{dt} + \frac{d\mathcal{A}}{dt\, d\phi_{\pi q}}\right),$$

should be the starting point to evaluate differential cross sections with respect to θ_π, the angle made by the pion and the incoming neutrino. As shown in Hernandez et al. [Phys. Rev D 80,013003 (2009)]

$$\cos\theta_\pi = \hat{k}\cdot\hat{k}_\pi = \frac{|\vec{k}'|}{|\vec{q}\,|}\sin\theta'\sin\theta_{\pi q}\cos\phi_{\pi q} + \frac{|\vec{k}\,| - |\vec{k}'|\cos\theta'}{|\vec{q}\,|}\cos\theta_{\pi q}\,,$$

The incoming neutrino energy and the variables q^2 and y determine $|\vec{k}'|$, $|\vec{q}\,|$ and θ', while, within the $E_\pi = q^0$ approximation, t fixes $\theta_{\pi q}$ [$t = -\vec{q}^{\,2} - \vec{k}_\pi^2 + 2|\vec{q}\,||\vec{k}_\pi|\cos\theta_{k_\pi q}$].

Knowledge of $\phi_{\pi q}$ is needed unless $q^2 = 0 \Longrightarrow \theta' = 0$

PCAC models III

Integrating on $\phi_{\pi q}$ one arrives at (T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962))

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G^2}{16\pi^2} E\,\kappa \left(-\frac{q^2}{|\vec{q}|^2}\right) \; (u^2 \frac{d\sigma_L}{dt} + v^2 \frac{d\sigma_R}{dt} + 2uv \frac{d\sigma_S}{dt})\,,$$

This is the actual starting point for PCAC-based models

At $q^2 = 0$ only σ_S contributes and one has

$$q^2\, \frac{d\sigma_S}{dt} = -\frac{\pi}{\kappa}\left(|\vec{q}|^2 H_{00} + q^0|\vec{q}|\,(H_{0z}+H_{z0}) + (q^0)^2\, H_{zz}\right) \stackrel{q^2=0}{\equiv} q_\mu q_\nu H^{\mu\nu} = q_\mu \mathcal{J}^\mu_{NC} (q_\nu \mathcal{J}^\nu_{NC})^*$$

Since the vector NC is conserved we are left with the divergence of the axial part. Using PCAC

$$\begin{aligned} q_\mu \mathcal{J}^\mu_{NC} = \langle \mathcal{N}_{gs}\pi^0(k_\pi)|q_\mu A^\mu_{NC}|\mathcal{N}_{gs}\rangle_{q^2=0} \quad &= \quad -2\mathrm{i} f_\pi T\left(\mathcal{N}_{gs}\pi^0(k_\pi) \leftarrow \pi^0(q)\mathcal{N}_{gs}\right)\Big|_{q^2=0} \\ \Longrightarrow\; q^2\, \frac{d\sigma_S}{dt}\Big|_{q^2=0} \quad &= \quad -4\frac{E_\pi}{\kappa}\, f_\pi^2\;\; \frac{d\sigma(\pi^0\mathcal{N}_{gs} \to \pi^0\mathcal{N}_{gs})}{dt}\Big|_{q^2=0} \end{aligned}$$

and then, neglecting the nucleus recoil $(q^0 = E_\pi)$, one can further write

$$\frac{d\sigma}{dq^2 dy\, dt}\Big|_{q^2=0} = \frac{G^2 f_\pi^2}{2\pi^2}\,\frac{E\,u\,v}{|\vec{q}|}\;\; \frac{d\sigma(\pi^0\mathcal{N}_{gs} \to \pi^0\mathcal{N}_{gs})}{dt}\Big|_{q^2=0,\,E_\pi=q^0}\;.$$

PCAC models IV. The Rein-Sehgal model

To go on-shell one eliminates the $q^2 = 0$ restriction on the $\pi\mathcal{N}$ cross section. Besides, for $q^2 \neq 0$ a form factor is added.The final answer would be

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G^2 f_\pi^2}{2\pi^2} \frac{E\,u\,v}{|\vec{q}\,|} G_A^2 \left. \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \to \pi^0 \mathcal{N}_{gs})}{dt} \right|_{E_\pi = q^0}$$

with

$$G_A = 1/(1 - q^2/m_A^2)$$

This is the Berger-Sehgal model for NC coherent π^0 production [Phys. Rev. D79,053003 (2009)].

In the Rein-Sehgal model [Nuc. Phys. B223, 29 (1983)] one further uses

$$\begin{aligned} \frac{Euv}{|\vec{q}\,|} &\to \frac{1-y}{y} \quad (\text{exact for } \; q^2 = 0) \\ \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \to \pi^0 \mathcal{N}_{gs})}{dt} &= |F_A(t)|^2 \, F_{\text{abs}} \left. \frac{d\sigma(\pi^0 N \to \pi^0 N)}{dt} \right|_{t=0} \end{aligned}$$

with $F_A(t) = \int d^3\vec{r}\, e^{i(\vec{q} - \vec{k}_\pi)\cdot\vec{r}} \rho(\vec{r})$ the nucleus form factor and F_{abs} a $t-$independent absorption factor that takes into account the distortion of the final pion.

Improvements on the Rein-Sehgal model

One can easily eliminate two approximations made in the RS model

- $$|F_A(t)|^2\,F_{abs} \longrightarrow \left|\int d^3\vec{r}\,e^{i(\vec{q}-\vec{k}_\pi)\cdot\vec{r}}\,\rho(\vec{r})\Gamma(b,z)\right|^2$$

  where

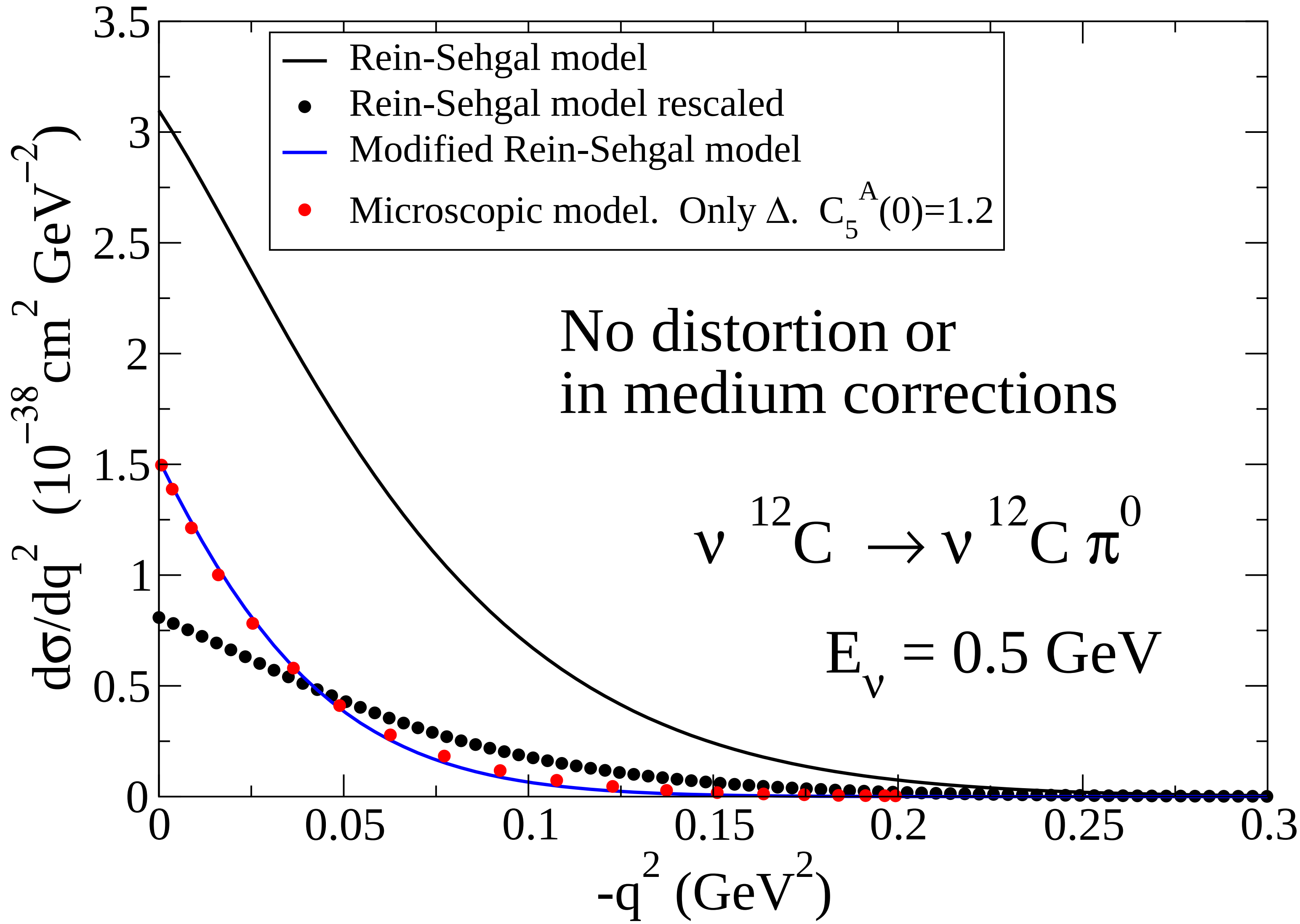
  $$\Gamma(b,z) = \exp\left(-\frac{1}{2}\sigma_{inel}\int_z^\infty dz'\rho(\sqrt{b^2+z'^2})\right)$$

- $$\left.\frac{d\sigma(\pi^0 N\to\pi^0 N)}{dt}\right|_{t=0} \longrightarrow \frac{d\sigma_{nsp}(\pi^0 N\to\pi^0 N)}{dt}$$

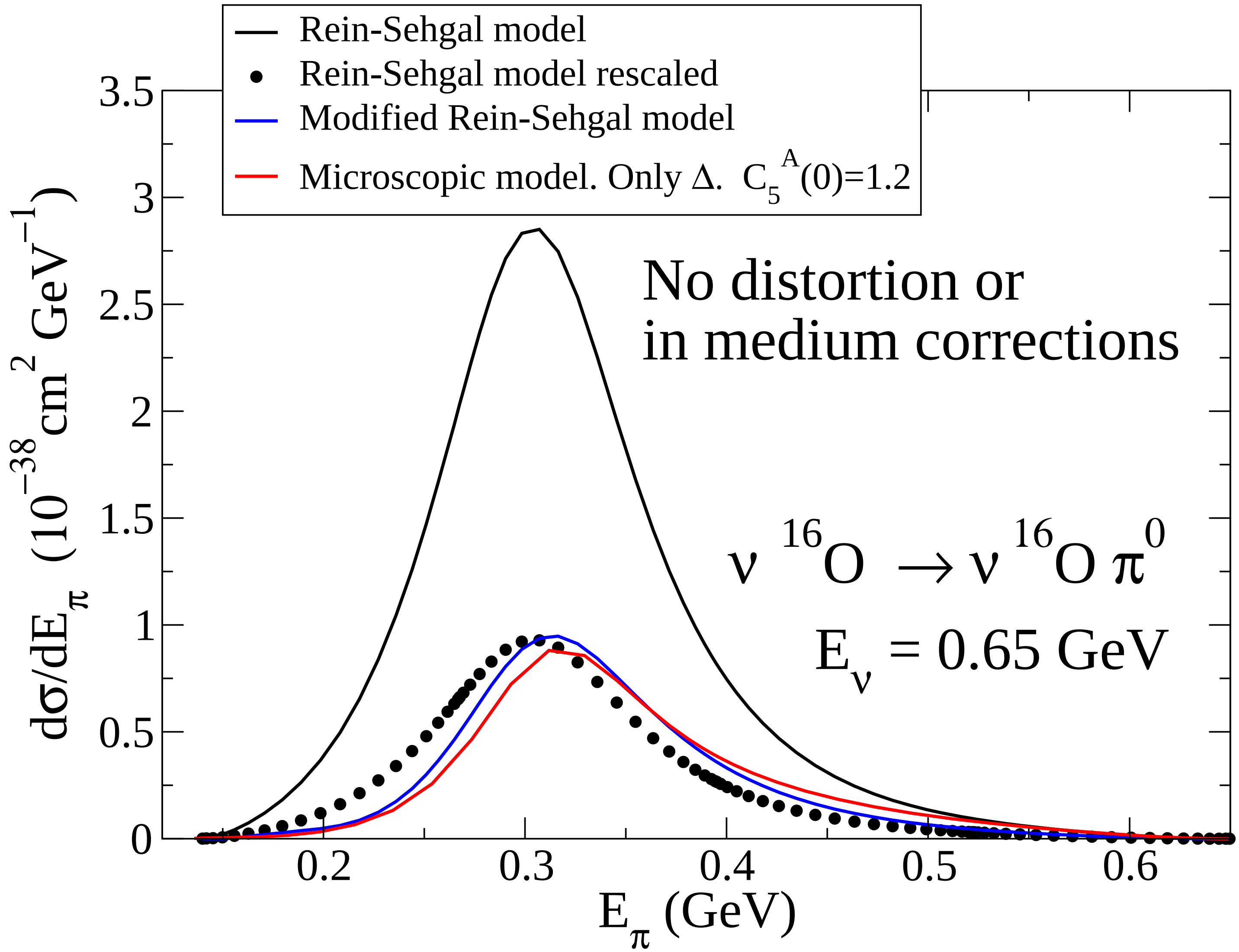
Besides

- We will also use $\frac{Euv}{|\vec{q}\,|}$ instead of $\frac{1-y}{y}$

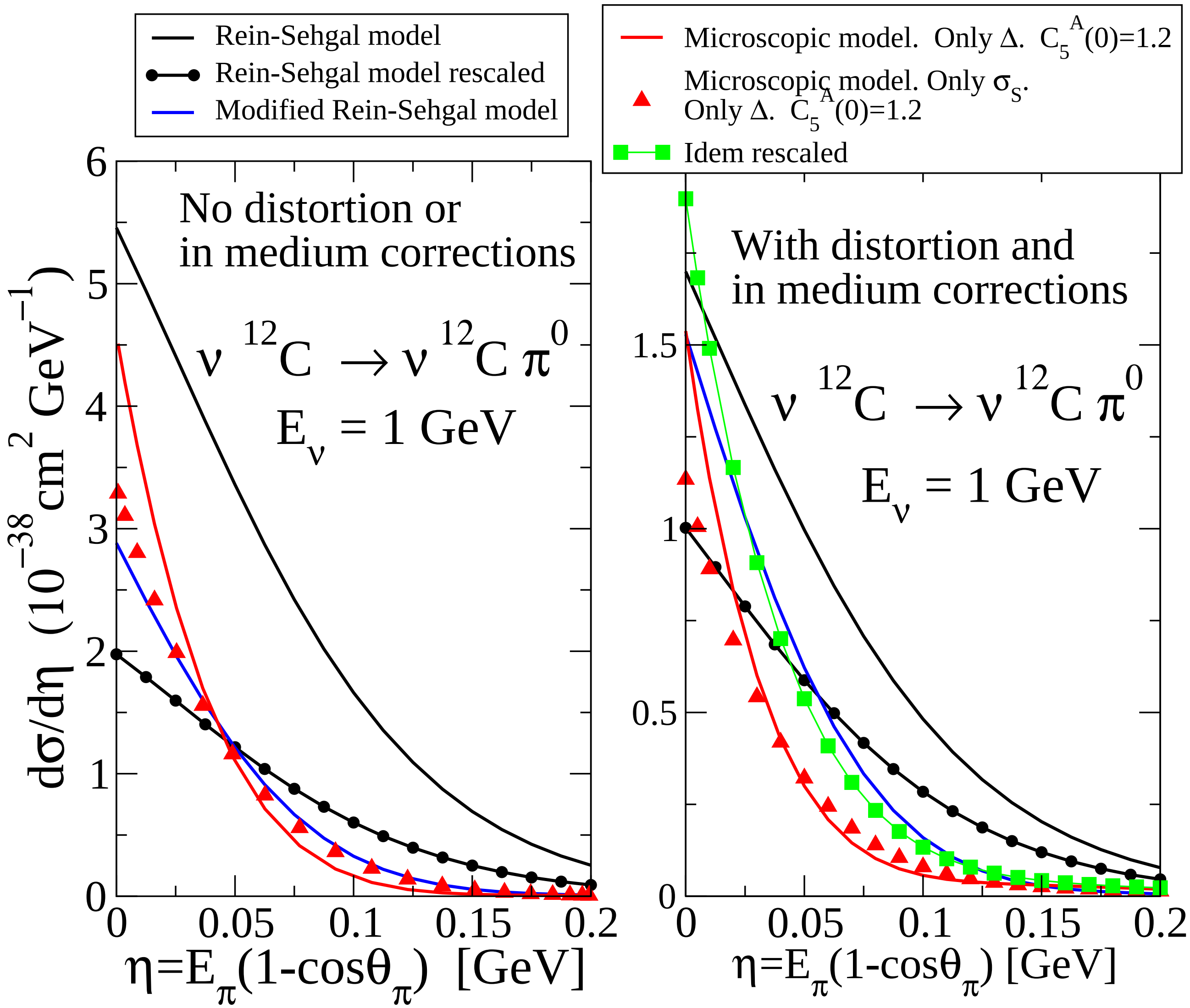
Model comparison. No distortion I



Model comparison. No distortion II



Model comparison



Due to neglected terms $\sigma_{R,L}$ and $\mathcal{A}(\phi_{\pi q})$

Conclusions part II

- It is not justified the use of the Rein-Sehgal model for low energy neutrinos.
  - It is a good model for high energy neutrinos and heavier nuclei for which the nucleus form factor selects $t \approx 0 \Longrightarrow q^2 \approx 0$.
- Eikonal approximation is not appropriate to evaluate pion distortion at low energies.
  - It should become better at higher neutrino energies that imply higher pion energies.
- PCAC-based model predictions for angular distributions with respect to the incoming neutrino direction might not be reliable.