



Deeply Virtual Neutrino Production of π^0 from Nucleon & Nuclear Targets: Part II

Spin Dependence, Cross Sections and Asymmetries

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and Tracy McAskill¹

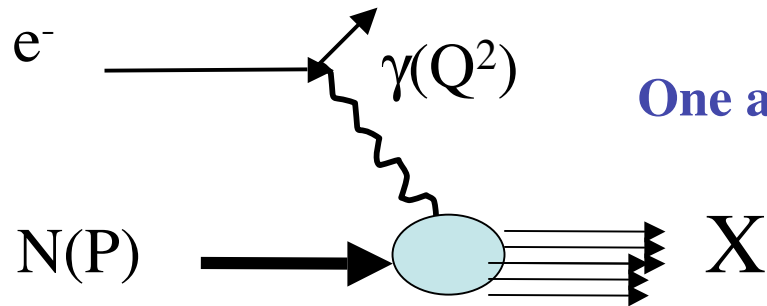
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Generalized Parton Distributions (GPDs) spin & transversity distributions

See Ahmad, Goldstein, Liuti
PRD 49, 054014 (2009)



One amp for Deep *InElastic* Scattering via **X**

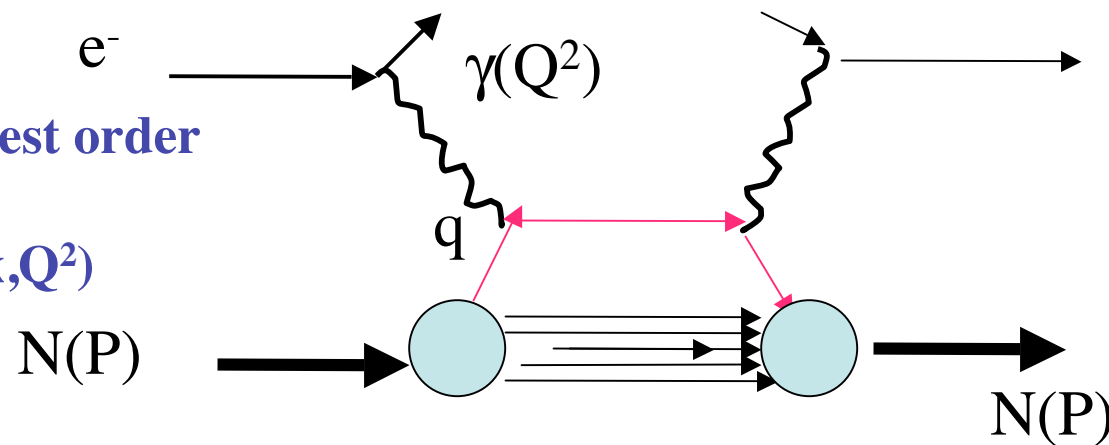
$\sigma(x, Q^2)$ for DIS from squaring amp
& summing over **X**'s

$$\propto \text{Im}(f(\gamma N \rightarrow \gamma N))_{t=0}$$

“Handbag” model is lowest order

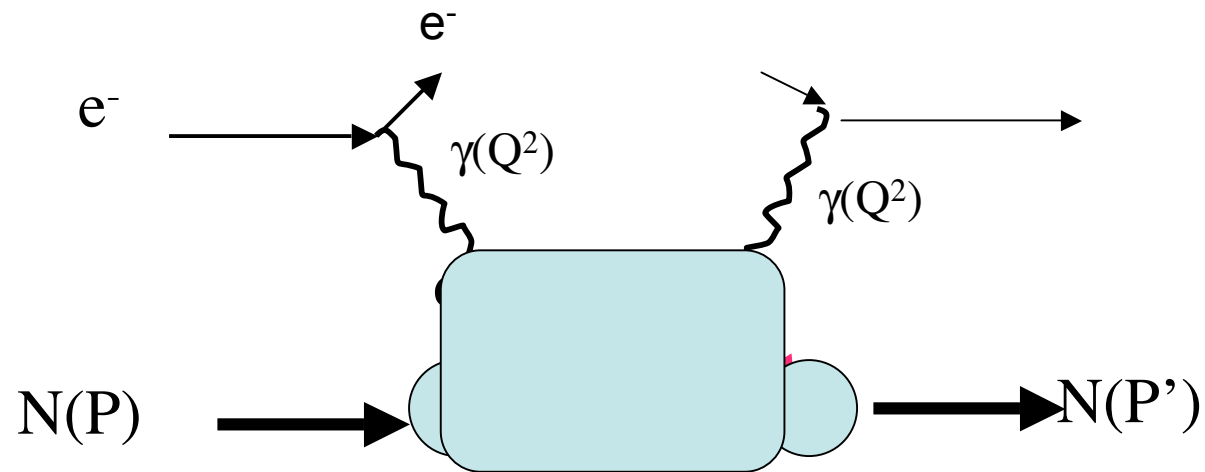
Perturbative QCD

→ Structure functions (x, Q^2)





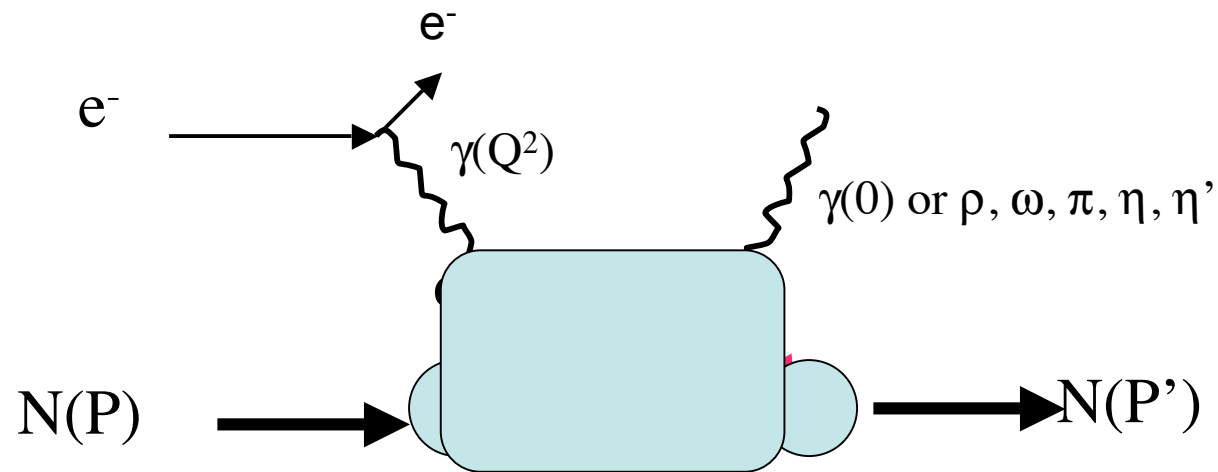
From DIS to exclusive electroproduction



$$T_{EM}^{\mu\nu} = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N(p_2, s_2) | T \{ J_{EM}^\mu(y) J_{EM}^\nu(x) \} | N(p_1, s_1) \rangle$$



From DIS to exclusive electroproduction of γ or mesons

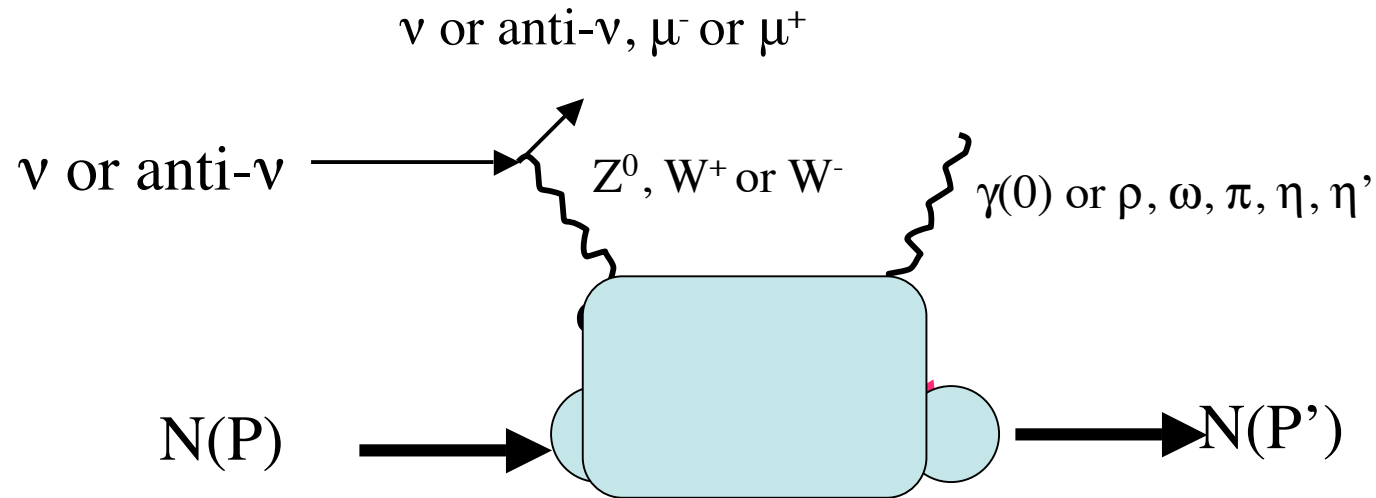


$$T_{EM}^{\mu\nu} = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N(p_2, s_2) | T \{ J_{EM}^\mu(y) J_{EM}^\nu(x) \} | N(p_1, s_1) \rangle$$

Deeply Virtual Compton Scattering or
Deeply Virtual Meson Production



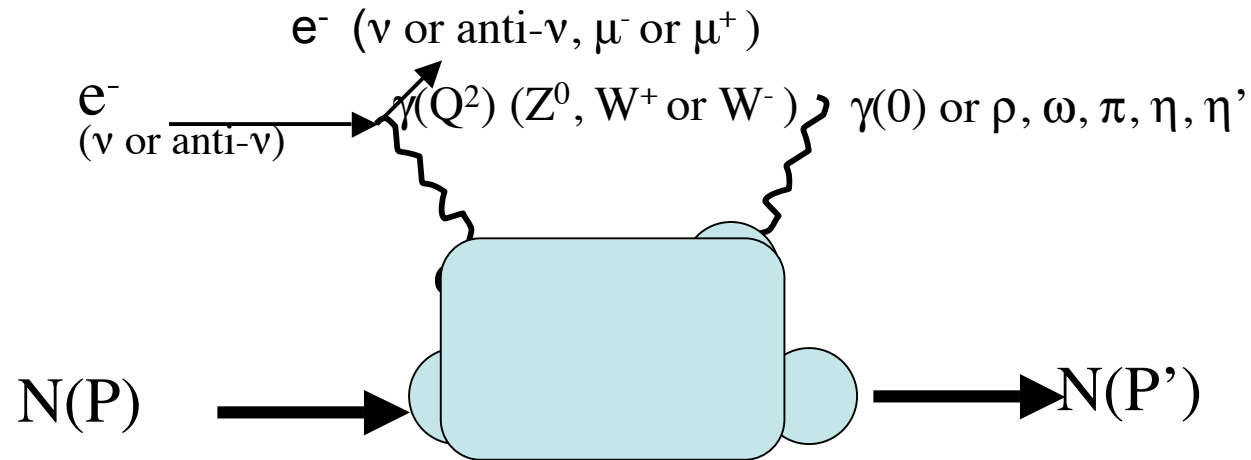
From exclusive electroproduction to neutrino-production



$$T_{EM}^{\mu\nu} = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N(p_2, s_2) | T \{ J_{EM}^\mu(y) J_{EM}^\nu(x) \} | N(p_1, s_1) \rangle$$



From exclusive electroproduction to neutrino-production



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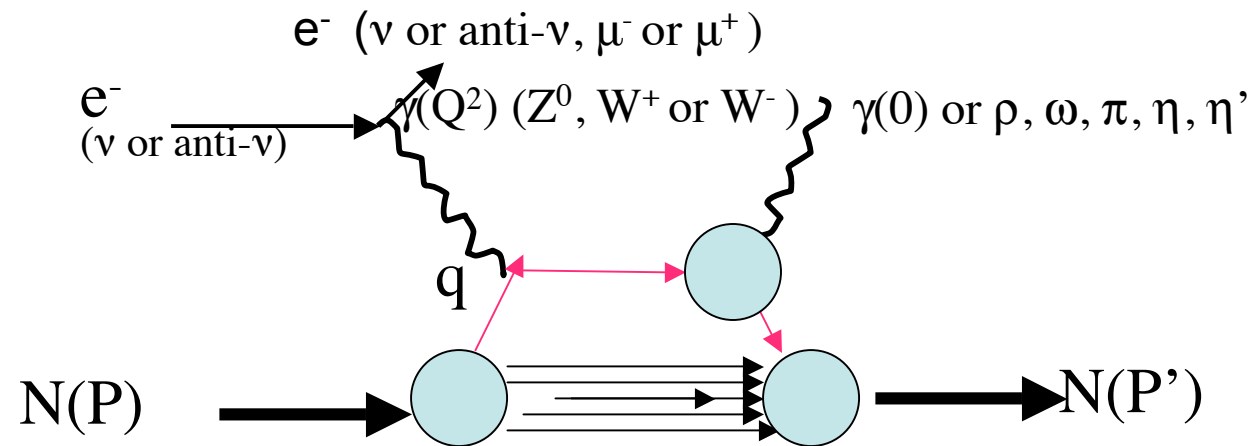
Replace EM current entering with Weak current.

Replace outgoing photon with meson e.g. π^0 .

$$T_W^\nu = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N'(p_2, s_2) | T \{ J_{\pi^0}(y) J_W^\nu(x) \} | N(p_1, s_1) \rangle$$



From hadrons to partons



$$T_{EM}^{\mu\nu} = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N(p_2, s_2) | T \{ J_{EM}^\mu(y) J_{EM}^\nu(x) \} | N(p_1, s_1) \rangle$$

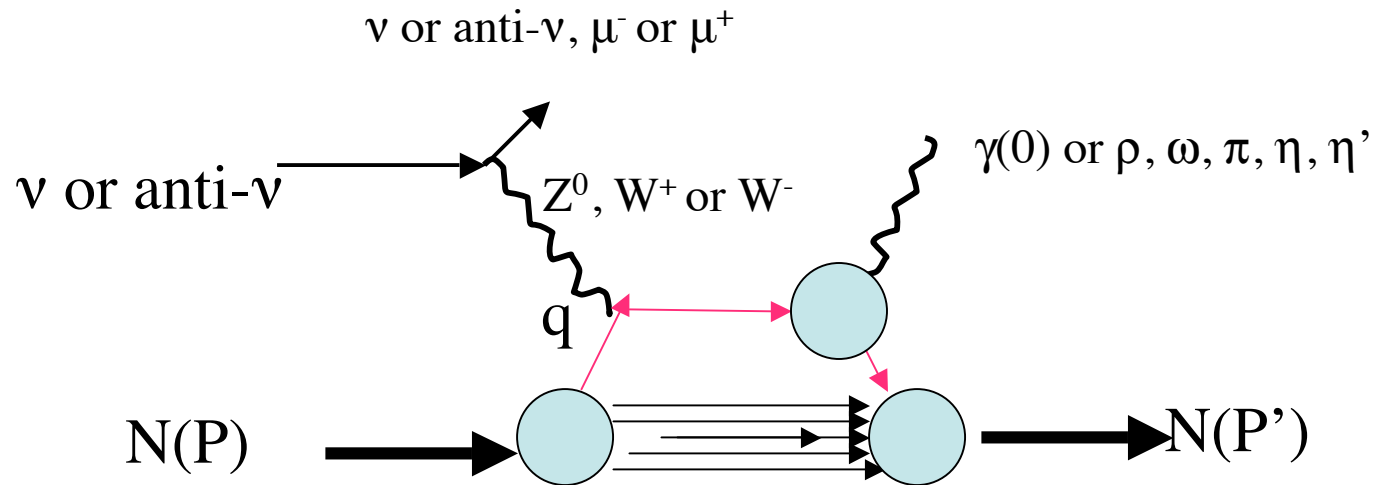
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From hadrons to partons



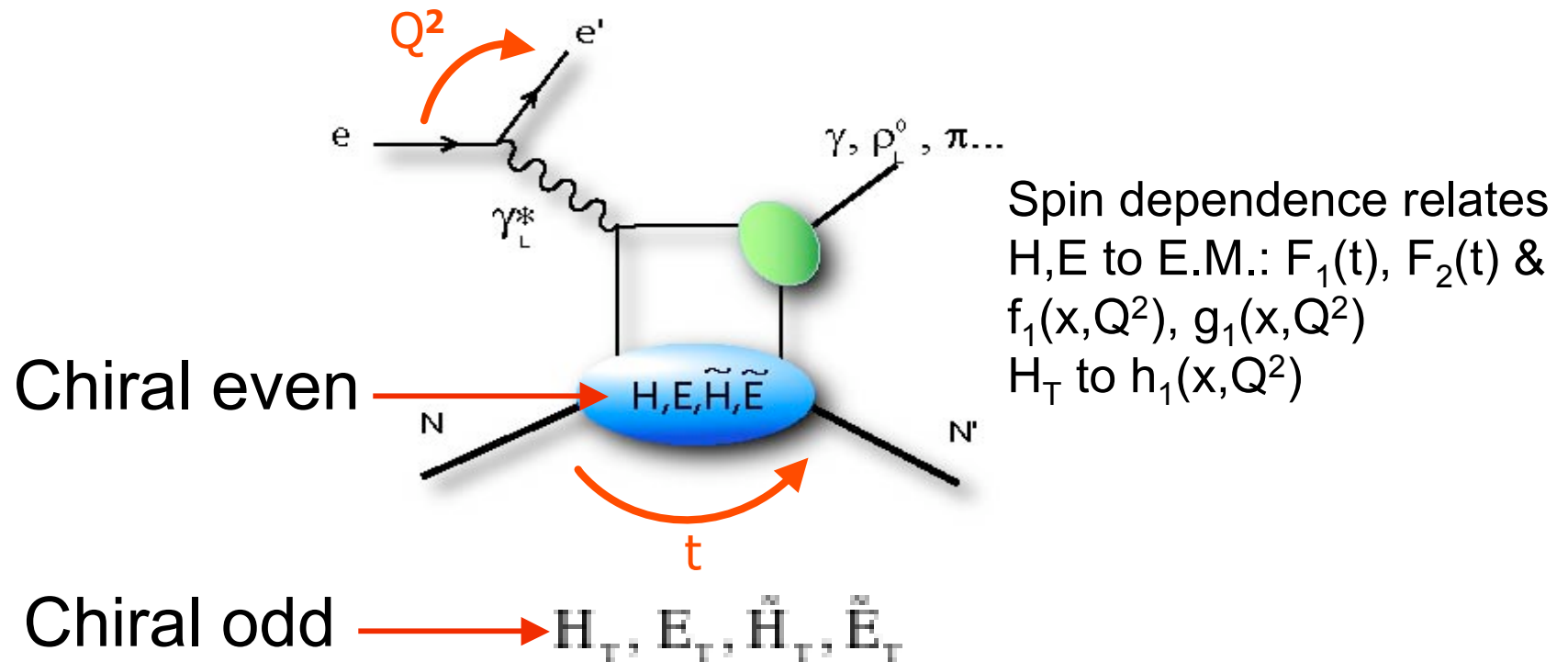
$$T_{W}^{\nu} = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N'(p_2, s_2) | T \{ J_{\pi^0}(y) J_W^{\nu}(x) \} | N(p_1, s_1) \rangle$$

Product of current operators has decomposition into quark field operators (OPE)

Leading twist involves free quark spinors. Now consider π production (outgoing $\gamma \rightarrow \pi$)

$$i T \{ J_{\pi^0}(z/2) J_{WN}^{\nu}(-z/2) \} = i \sum_f \left[\bar{\psi}_f(z/2) \gamma^5 i \not{\beta}(z) \gamma^{\nu} \frac{1}{2} (c_V^f - \gamma_5 c_A^f) \psi_f(-z/2) \right. \\ \left. + \bar{\psi}_f(-z/2) \gamma^{\nu} \frac{1}{2} (c_V^f - \gamma_5 c_A^f) i \not{\beta}(-z) \gamma^5 \psi_f(z/2) \right] .$$

Spin dependent GPDs upon insertion of T product into nucleon matrix elements



Parity violating V-A coupling doubles the number of helicity amps from 6 to 12.

Procedure: GPDs to $d\sigma^n/d\Omega_{\text{lepton}}d\Omega_{\text{meson}}dt$ & asymmetries, especially ϕ for $\rightarrow\gamma$ or $\rho,\omega,\pi,\eta,,K$

Exclusive spin dependent e or ν amps

Factorized into handbag with quarks (2 scales)

PQCD for Virtual boson+quark $\rightarrow\gamma$ (or ρ,ω,π,η,K)+quark'

Soft quark/nucleon amp \rightarrow GPD

4 chiral even + 4 chiral odd GPDs
Connected to 8 helicity amps

Combine soft & hard. Integrate over p_{quark} \rightarrow T-matrix for each helicity combination \rightarrow form $d\sigma/d\Omega dt\dots$

Models: diquark or exchanges \rightarrow predictions

Defining GPDs via quark correlation on light cone

$$\begin{aligned}
 F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^+ q(\frac{1}{2}z) | p \rangle \Big|_{z^+=0, \mathbf{z}=0} \\
 &= \frac{1}{2P^+} \left[H^q(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(p) \right] \\
 \tilde{F}^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^+ \gamma_5 q(\frac{1}{2}z) | p \rangle \Big|_{z^+=0, \mathbf{z}=0} \\
 &= \frac{1}{2P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2m} u(p) \right]
 \end{aligned}$$

These are chiral even – no quark helicity flip

4 other chiral odd GPDs with $\gamma^5 \sigma^{+j}$

Collinear factorization applied (k_τ quark momenta integrated over)



GPD is Real function of x (unobservable longitudinal momentum of parton),

ξ or ζ (fraction of long. Mom. lost by target), t, Q^2 .

> Observables cannot depend on ξ or ζ .

> Integrated GPD is like a Compton Form Factor - complex

> phases enable interferences & asymmetries

$$T^{\mu\nu}(v, Q^2, t) = \frac{1}{2} g^{\mu\nu} \bar{u}(p') \hat{n} u(p) \sum_{\text{flavors}} e_f^2 \mathcal{H}_f(\xi, t)$$

$$\mathcal{H}_f(\xi, t) = \int_{-1}^{+1} dx \frac{H_f(x, \xi, t)}{x - \xi + i\epsilon}$$

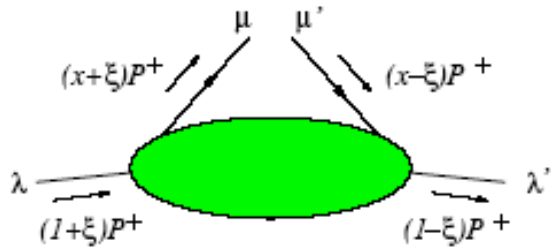
$$\text{Im } \mathcal{H}_f(\xi, t) = H_f(\xi, \xi, t) \quad \text{Re } \mathcal{H}_f(\xi, t) = \frac{1}{\pi} PV \int_{-1}^{+1} dx \frac{H_f(x, \xi, t)}{x - \xi}$$

$$\text{Dispersion Relation: } \text{Re } \mathcal{H}_f(\xi, t) = \frac{1}{\pi} PV \int_{-1}^{\xi(\text{Max})?} dx \frac{H_f(x, x, t)}{x - \xi}$$



GPDs & helicity

Functions of x, ξ, Δ



$$\begin{aligned}
 A_{\lambda'\mu',\lambda\mu} &= \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \mathcal{O}_{\mu',\mu}(z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0} \\
 &= \int \frac{d^2k_T}{(2\pi)^3} \left[\int dz^- d^2z_T e^{ik \cdot z} \langle p', \lambda' | \mathcal{O}_{\mu',\mu}(z) | p, \lambda \rangle \right]_{z^+=0, k^+=xP^+}
 \end{aligned}$$

Quarks
do **not**
flip helicity
for these
amps
 \Rightarrow **not** quark
transversity

$$A_{++,+} = \sqrt{1-\xi^2} \left(\frac{H^q + \tilde{H}^q}{2} - \frac{\xi^2}{1-\xi^2} \frac{E^q + \tilde{E}^q}{2} \right),$$

$$A_{-+,-} = \sqrt{1-\xi^2} \left(\frac{H^q - \tilde{H}^q}{2} - \frac{\xi^2}{1-\xi^2} \frac{E^q - \tilde{E}^q}{2} \right),$$

$$A_{++,-} = -\epsilon \frac{\sqrt{t_0-t}}{2m} \frac{E^q - \xi \tilde{E}^q}{2},$$

$$A_{-+,+} = \epsilon \frac{\sqrt{t_0-t}}{2m} \frac{E^q + \xi \tilde{E}^q}{2},$$

$H(x, 0, 0) = f_1(x)$
& $H^{-q}(x, 0, 0) = g_1(x)$

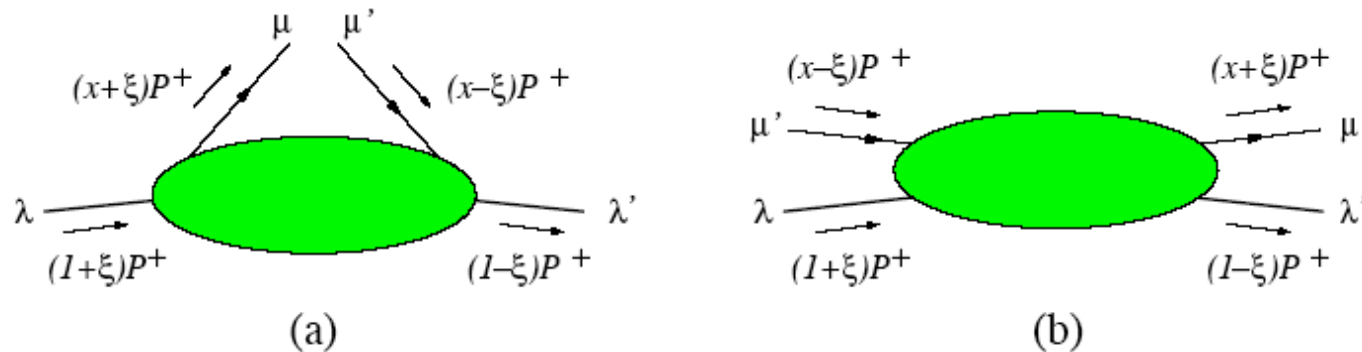
M. Diehl; Boglione & Mulders



Chiral odd GPDs

$$\begin{aligned} & \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0} \\ &= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[H_T^q i\sigma^{+i} + \tilde{H}_T^q \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right. \\ & \quad \left. + E_T^q \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda). \end{aligned}$$

Eqns connecting GPD & helicity amps - M. Diehl, Eur.Phys.J.C19 (2001) 485;
Boglionne & Mulders, Phys.Rev.D 60 (1999) 054007.

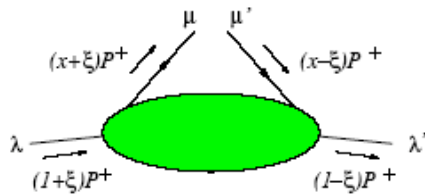


- Exploit these relations to evaluate H_T^q with diquark spectator (scalar & axial vector \rightarrow u & d distributions) with constraints from form factors & **lattice calculations**. (Hägler, Schierholtz, et al. See especially [S.Liuti, et al. DIS 2008 tomorrow.](#))



How does transversity enter?

- Quark helicity flip amps \Rightarrow quark transversity



$$A_{++,+-} = \epsilon \frac{\sqrt{t_0 - t}}{2m} \left(\tilde{H}_T^q + (1 - \xi) \frac{E_T^q + \tilde{E}_T^q}{2} \right),$$

$$A_{-+,-} = \epsilon \frac{\sqrt{t_0 - t}}{2m} \left(\tilde{H}_T^q + (1 + \xi) \frac{E_T^q - \tilde{E}_T^q}{2} \right),$$

$$A_{++,--} = \sqrt{1 - \xi^2} \left(H_T^q + \frac{t_0 - t}{4m^2} \tilde{H}_T^q - \frac{\xi^2}{1 - \xi^2} E_T^q + \frac{\xi}{1 - \xi^2} \tilde{E}_T^q \right)$$

$H_T^q(x, t=0, \xi=0)$
 $= h_1(x)$ Norm δq

Also $H(x, 0, 0) = f_1(x)$

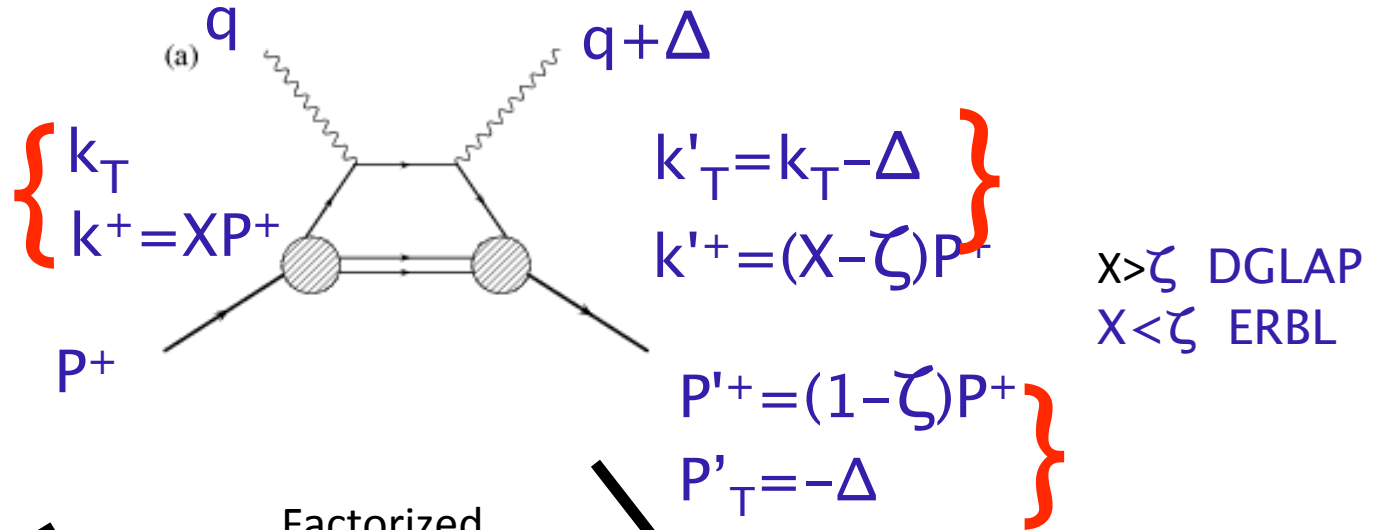
& $H^{-q}(x, 0, 0) = g_1(x)$ Norm Δq

$$A_{-+,+} = -\sqrt{1 - \xi^2} \frac{t_0 - t}{4m^2} \tilde{H}_T^q$$

M. Diehl; Boglione & Mulders



DVCS $\gamma^* + P \rightarrow (\gamma \text{ or meson}) + P'$
partonic picture



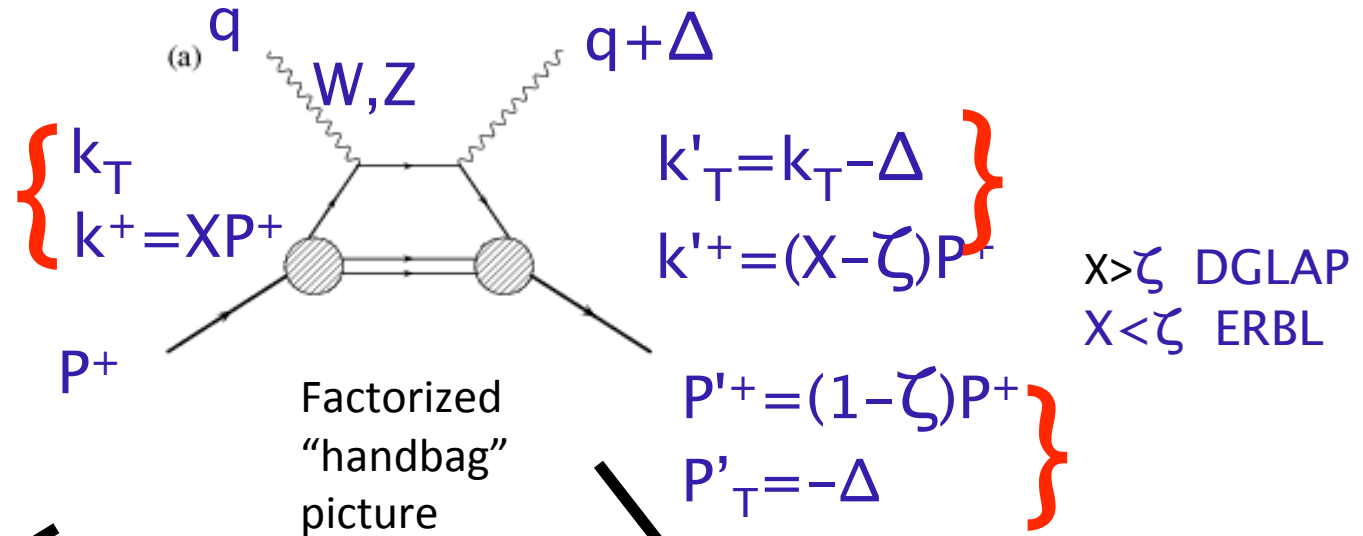
Factorized
"handbag"
picture

$\zeta \rightarrow 0$
High E

Quark-spectator
quark + diquark



DVCS $\gamma^* + P \rightarrow (\gamma \text{ or meson}) + P'$
partonic picture



$$f_{\Lambda_\gamma, \Lambda; 0, \Lambda'} = \sum_{\lambda, \lambda'} g_{\Lambda_\gamma, \lambda; 0, \lambda'}(X, \zeta, t, Q^2) \otimes A_{\Lambda', \lambda'; \Lambda, \lambda}(X, \zeta, t).$$

12 g amps in V and A combinations

For vDVCS get 4 chiral even GPDs (see Psakar, et al., PRD 75,054001 (2007))

For vDVπ get 4 chiral odd GPDs **Pion selects chiral odd, C-parity odd in electro**

$$i \int \frac{d^4 l}{(2\pi)^4} e^{-il \cdot z} \frac{l_\mu}{l^2 + i0} \cdot \sum_f \left[g^{\mu\nu} \left(c_V^f \mathcal{O}_5^+ - c_A^f \mathcal{O}_I^- \right) + ic_A^f \mathcal{O}_{\mu\nu}^+ - ic_V^f \mathcal{O}_{5\mu\nu}^- \right]. \quad (13)$$

$$\langle N_2 | \mathcal{O}_\Gamma | N_1 \rangle = \bar{u}_2 \left[\sum_A (\text{Dirac} \times \text{external kinematic factors})_A \int_{-1}^{+1} dx e^{ixp \cdot z} GPD_A(x, \xi, t) \right] u_1. \quad (14)$$

Insert this into Eqn. 4 and use the integral over l of the propagator, Eqn. 5, to obtain for each \mathcal{O}_Γ

$$-\frac{1}{2} \int_{-1}^{+1} dx \int \frac{d^4 l}{(2\pi)^4} \int d^4 z e^{i(xp+q-l) \cdot z} \frac{l_\mu}{l^2 + i0} \bar{u}_2 \left[\sum_A (\text{Dirac} \times \text{kinematic})_A^{\mu\nu} GPD_A(x, \xi, t) \right] u_1 \quad (15)$$

$$= -\frac{1}{2} \int_{-1}^{+1} dx \frac{l_\mu}{l^2 + i0} \bar{u}_2 \left[\sum_A (\text{Dirac} \times \text{kinematic})_A^{\mu\nu} GPD_A(x, \xi, t) \right] u_1 |_{l=xp+q} \quad (16)$$

For $\mathcal{O}_{\mu\nu}^+$ we have

$$-\frac{i}{2} \int_{-1}^{+1} dx \frac{l_\mu}{l^2 + i0} \bar{u}_2 \left[H_T^+ i\sigma^{\mu\nu} + \tilde{H}_T^+ \frac{P^\mu \Delta^\nu - \Delta^\mu P^\nu}{M^2} + E_T^+ \frac{\gamma^\mu \Delta^\nu - \Delta^\mu \gamma^\nu}{2M} + \tilde{E}_T^+ \frac{\gamma^\mu P^\nu - P^\mu \gamma^\nu}{M} \right] u_1 |_{l=xp+q}. \quad (19)$$

For $\mathcal{O}_{5\mu\nu}^-$ we have

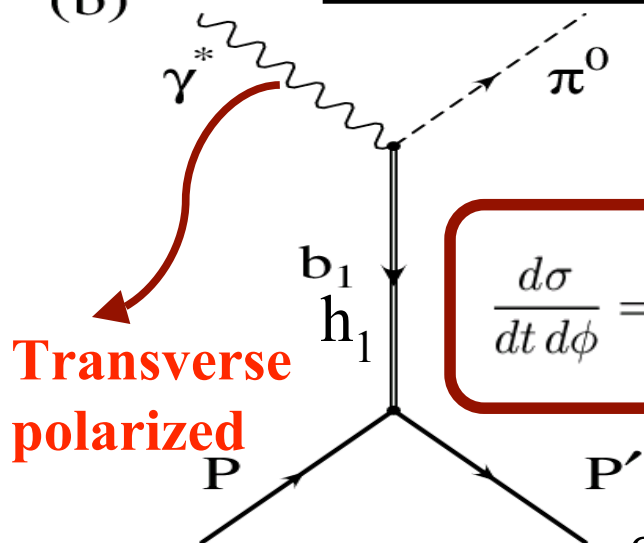
$$-\frac{1}{2} \int_{-1}^{+1} dx \frac{l_\mu}{l^2 + i0} \bar{u}_2 \left[H_T^- \gamma^5 \sigma^{\mu\nu} + \tilde{H}_T^- \frac{\epsilon^{\mu\nu\rho\sigma} \Delta_\rho P_\sigma}{M^2} + E_T^- \frac{\epsilon^{\mu\nu\rho\sigma} \Delta_\rho \gamma_\sigma}{M^2} + \tilde{E}_T^- \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho \gamma_\sigma}{M^2} \right] u_1 |_{l=xp+q}. \quad (20)$$



(b)

Exclusive π^0 -electroproduction

$$ep \rightarrow e'p'\pi^0$$



Transverse polarized

$$\frac{d\sigma}{dt d\phi} = \left(\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} \right) + \epsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi + \sqrt{2\epsilon(\epsilon + 1)} \frac{d\sigma_{LT}}{dt} \cos \phi$$

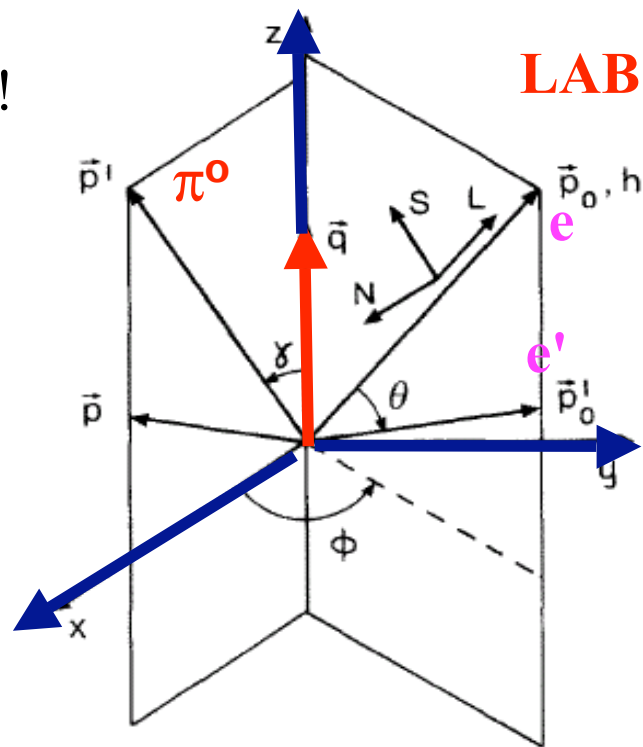
Sensitive to tensor charge!

$$d\sigma \propto L_{\mu\nu}^{h=\pi^0} W_{\mu\nu}$$

$L_{\mu\nu}^{h=\pi^0} \approx \gamma^*$ polarization density matrix

$W_{\mu\nu} = \sum_f J_\mu J_\nu^* \delta(E_i - E_f) =$ hadronic tensor

$$\frac{d\sigma_{TT}}{dt} = W_{yy} - W_{xx} \equiv 2\Re e(J_1 J_{-1}^*)$$



Neutrino analog

Electroproduction differential cross section

$$\frac{d^4\sigma}{d\Omega d\varepsilon_2 d\phi dt} = \Gamma \left\{ \frac{d\sigma_T}{dt} + \varepsilon_L \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon_L(\varepsilon+1)} \cos\phi \frac{d\sigma_{LT}}{dt} \right\}$$

where subscript T = transversely polarized virtual γ

L =longitudinal γ , TT = \perp - \parallel to hadron scattering plane,

LT =long. \times transverse interference

Neutrino (antineutrino) cross section

$$\frac{d^4\sigma}{d\Omega d\varepsilon_2 d\phi dt} = \Gamma \left\{ \frac{d\sigma_T}{dt} + \varepsilon_L \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon_L(\varepsilon+1)} \cos\phi \frac{d\sigma_{LT}}{dt} \right\}$$

$$\pm(\varepsilon \text{ factor}) \sin\phi \frac{d\sigma_{L'T}}{dt} + (\varepsilon \text{ factor}) \sin 2\phi \frac{d\sigma_{TT}}{dt} \left. \right\}$$

Neutrino (antineutrino) cross section

$$\frac{d^4\sigma}{d\Omega d\varepsilon_2 d\phi dt} = \Gamma \left\{ \frac{d\sigma_T}{dt} + \varepsilon_L \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon_L(\varepsilon+1)} \cos\phi \frac{d\sigma_{LT}}{dt} \right. \\ \left. \pm (\varepsilon \text{ factor}) \sin\phi \frac{d\sigma_{L'T}}{dt} + (\varepsilon \text{ factor}) \sin 2\phi \frac{d\sigma_{TT'}}{dt} \right\}$$

- Extra terms from parity violation - V and A interference -
- **sin ϕ** term would be obtained from polarized electron beam spin asymmetry
- Neutrino beam is 100% polarized left \Rightarrow separates parity odd & even combinations of helicity amps \Rightarrow spin dependent GPDs
- **sin 2ϕ** term is transverse in-plane V interference (Im) with transverse perp-plane A
- Would have no phases w/o rescattering- GPDs involve removing quark from target at origin of light front & returning quark further up the light front.
- How to see ϕ dependence? CC like $\nu N \rightarrow \mu^- \pi^+ N$ allows 2 planes to be seen
- What to expect? From $eN \rightarrow e \pi^0 N$ see $d\sigma_L/dt$, showing importance of long. Photon.

Procedure: GPDs to $d\sigma^n/d\Omega_{\text{lepton}}d\Omega_{\text{meson}}dt$ & asymmetries, especially ϕ for $\rightarrow\gamma$ or $\rho,\omega,\pi,\eta,,K$

Exclusive spin dependent e or ν amps

Factorized into handbag with quarks (2 scales)

PQCD for Virtual boson+quark $\rightarrow\gamma$ (or ρ,ω,π,η,K)+quark'

Soft quark/nucleon amp \rightarrow GPD

4 chiral even + 4 chiral odd GPDs
Connected to 8 helicity amps

Combine soft & hard. Integrate over p_{quark} \rightarrow T-matrix for each helicity combination \rightarrow form $d\sigma/d\Omega dt\dots$

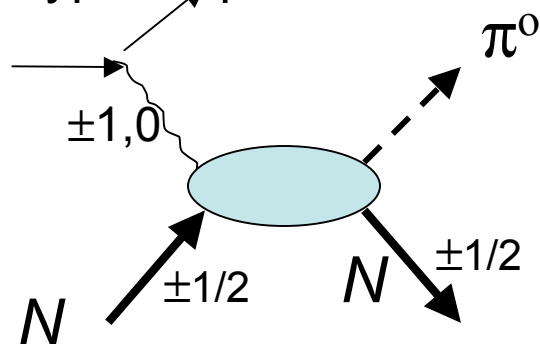
Models: diquark or exchanges \rightarrow predictions



Exclusive π^0 electroproduction and Transversity

$$\frac{d\sigma_{TT}}{dt} = W_{yy} - W_{xx} \equiv 2\Re(J_1 J_{-1}^*)$$

Connect to helicity
amps- make spin
behavior explicit
In each observable
there will be both
V and A type amps



$$2 \operatorname{Re}(f_{+1+,0+}^* f_{+1-,0-} - f_{+1+,0-}^* f_{+1-,0,+})$$

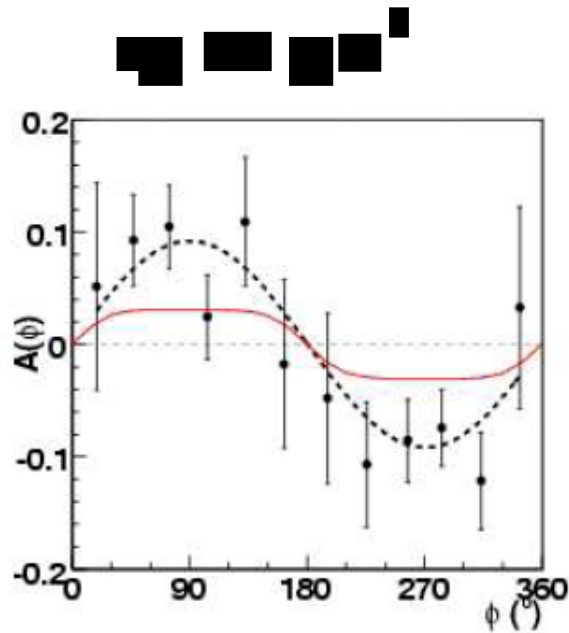
only $f_{+1+,0-}(s,t,Q^2) = f_2$
survives at $t \rightarrow 0$

Target asymmetry for γ_T
 $A_{UT} \propto 2\operatorname{Im}(f_{+1+,0+}^* f_{+1-,0,+} - f_{+1-,0-}^* f_{+1+,0-})$

$$d\sigma_T \propto |f_{+1+,0+}|^2 + |f_{+1+,0-}|^2 + |f_{+1-,0+}|^2 + |f_{+1-,0-}|^2$$

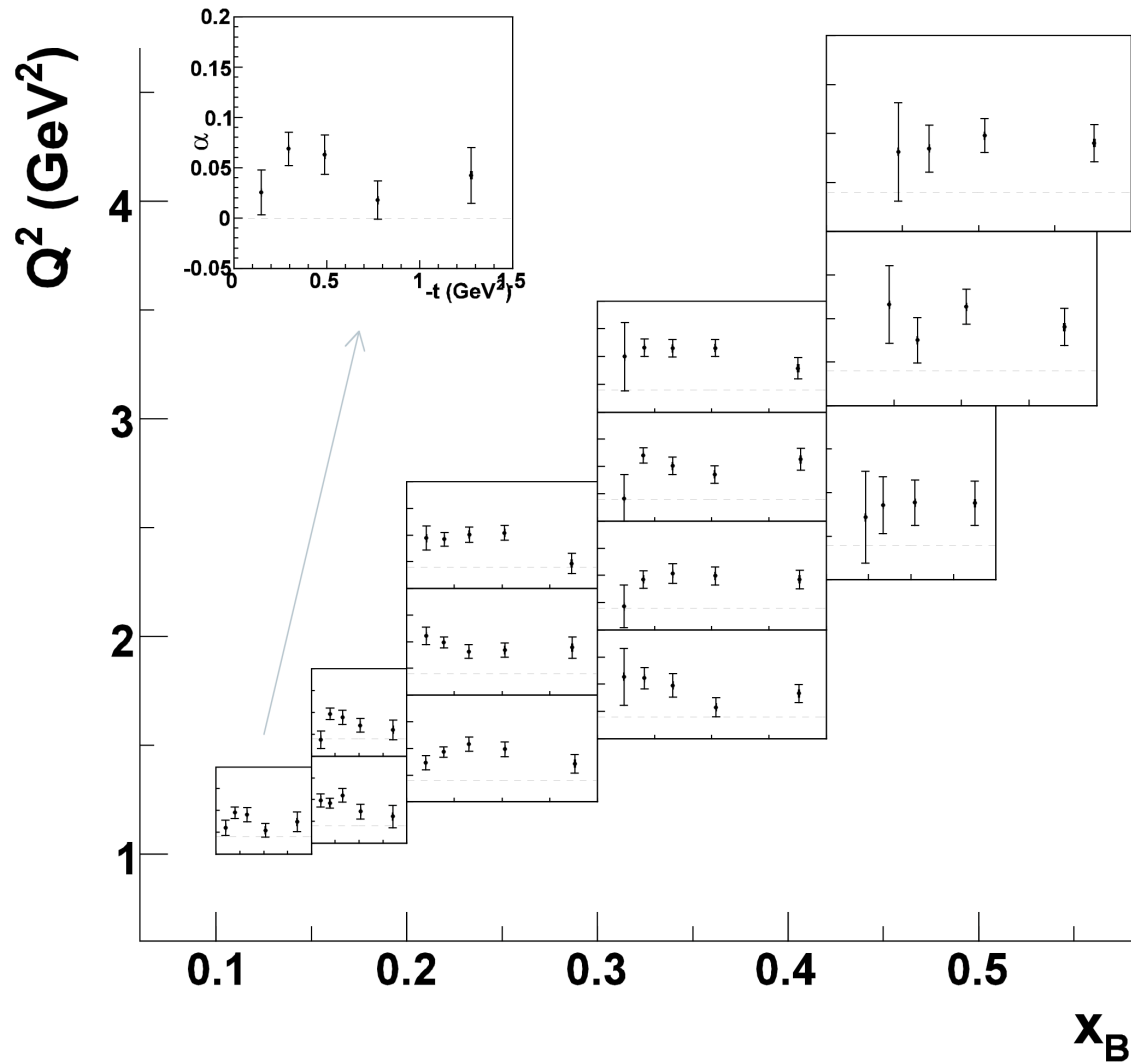
Beam asymmetry measures L-T interference

α vs t Preliminary Hall B data



----- $A = \alpha \sin \phi$

→ any non-zero BSA indicates L-T interference



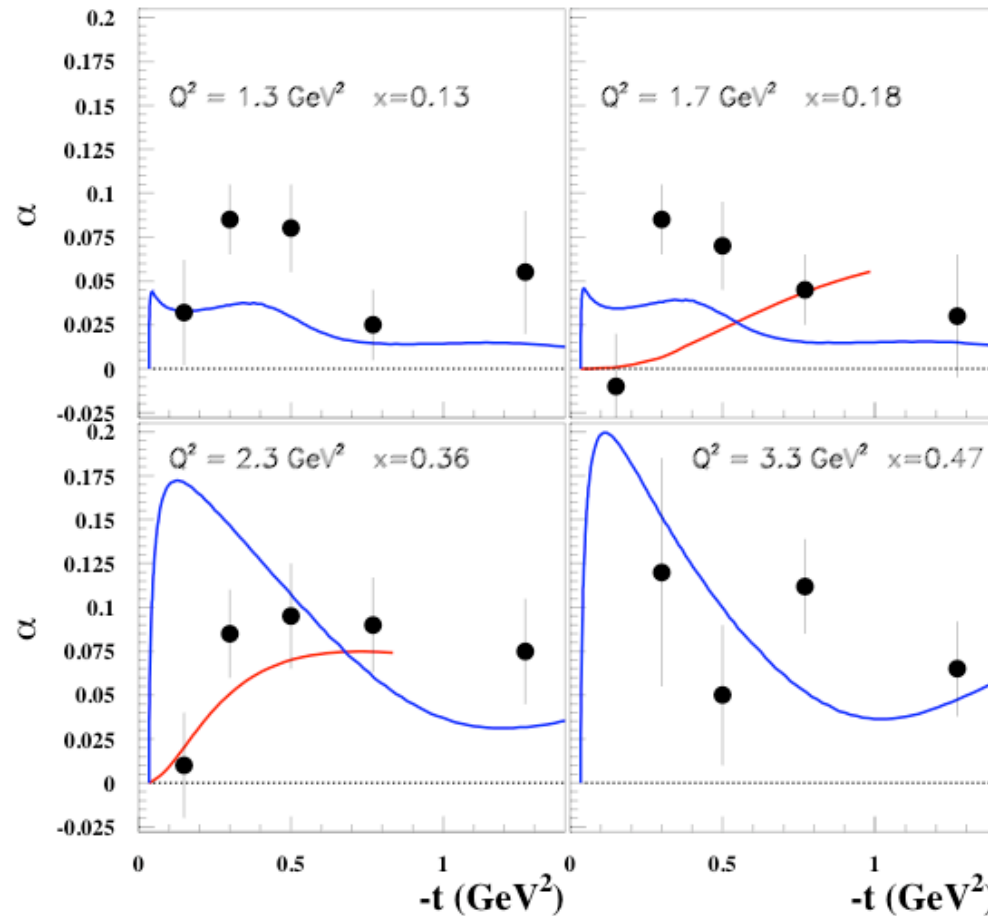
Models Beam-spin asymmetry α

data R. De Masi et al., Phys.Rev.C77, 042201 (2008).

Regge-cut predictions - comparisons involve $\varepsilon_L, \varepsilon$

Ahmad, GRG, Liuti, PRD79,054014 (2009)

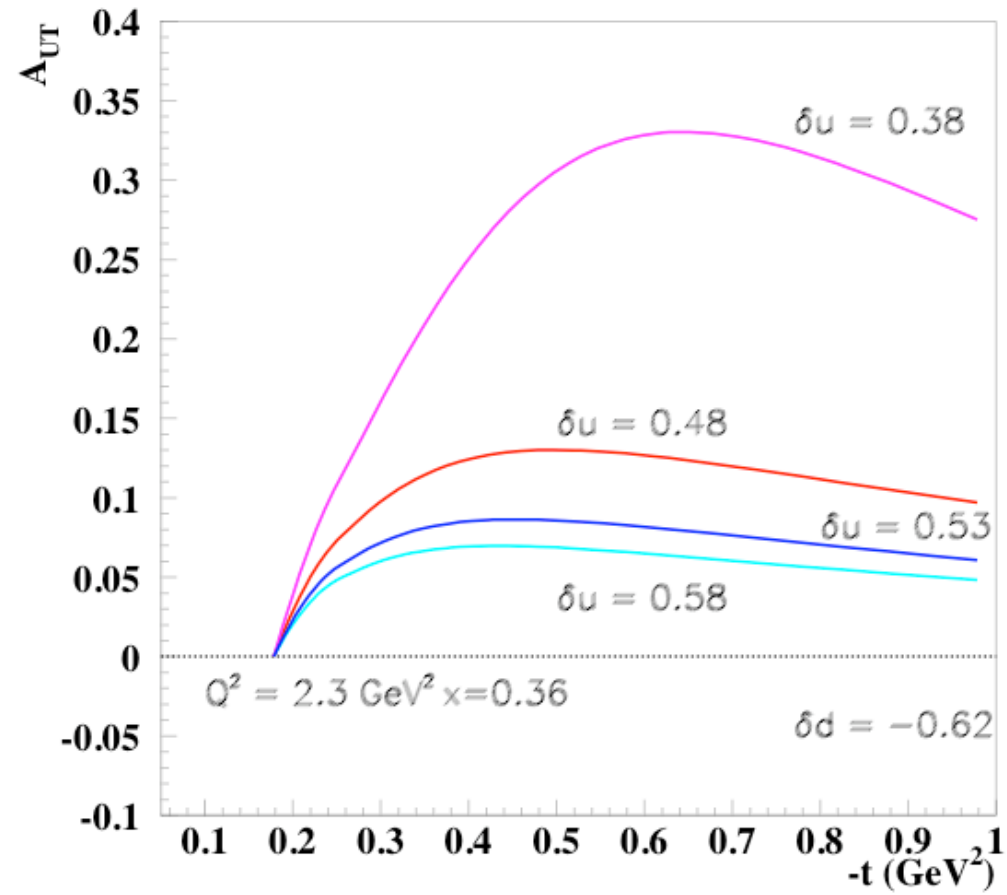
blue



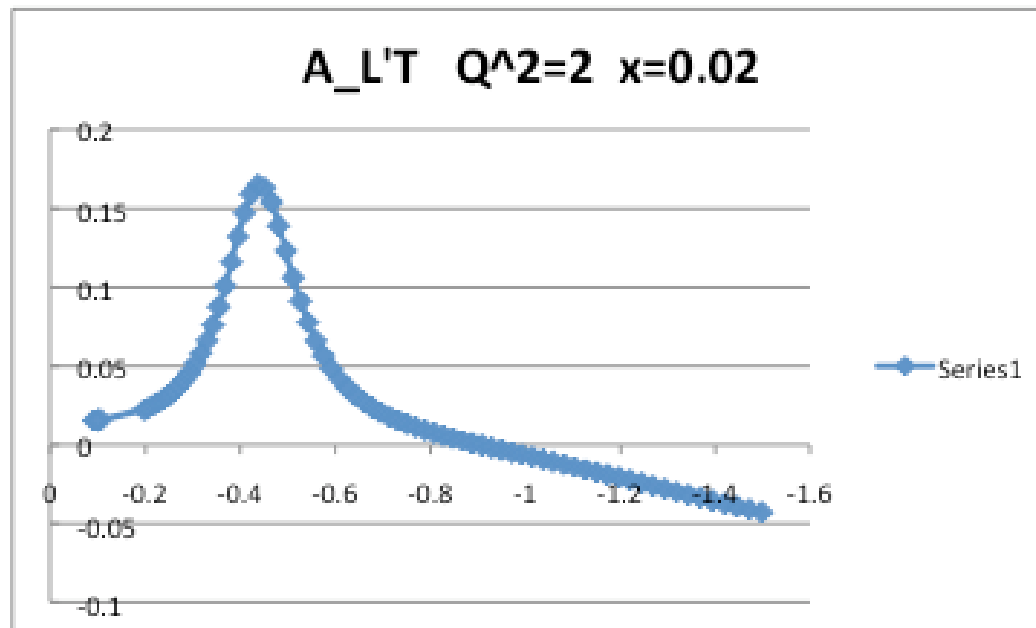
GPD predictions
(preliminary) red

Variation of asymmetries with tensor charge All GPDs

Ahmad, GRG, Liuti, PRD79,054014 (2009)



$\sin\phi$ term \Rightarrow parity violating Longitudinal x Transverse
Interference \Rightarrow V & A phase difference
example from exchange model

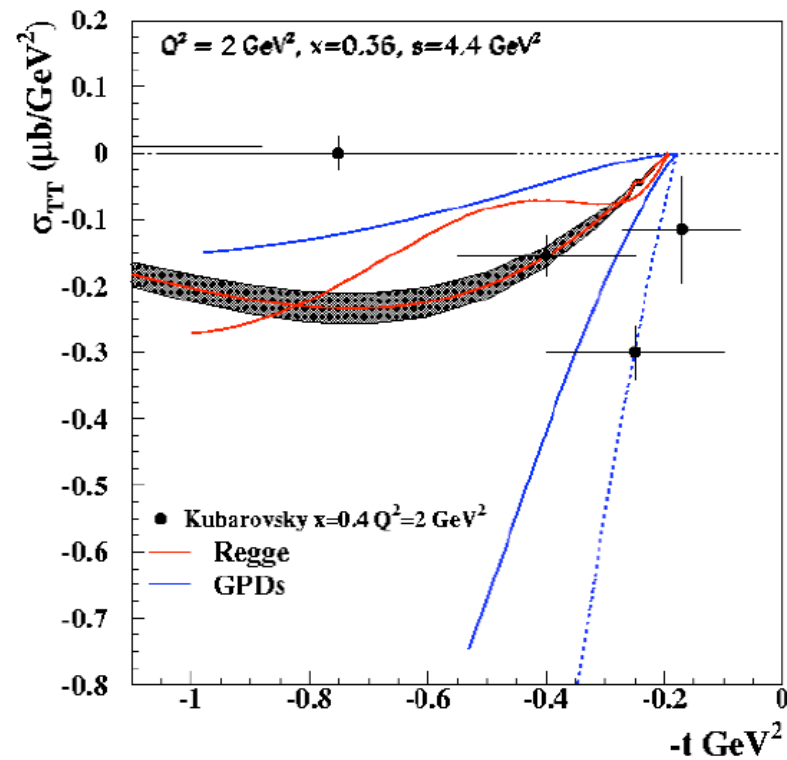




How to compare σ_{TT} with data for $Q^2 > 0$?

Different choices of κ_T^q from small to large u&d

SAID curve





Conclusions

- ☯ electroproduction of γ or ρ, ω, π, η at $Q^2 \approx 2$ to 5 , $W \approx 6$ GeV is described by GPDs
 - Coherent neutrino-production of γ or ρ, ω, π, η in similar energy regime should be amenable to similar description of dynamics
 - Exclusive π^0 electroproduction observables depend on $p(\pi)$ & *azimuthal* orientation of hadron vs. lepton plane
 - $d\sigma_T/dt$, $d\sigma_{TT}/dt$, A_{UT} , beam asymmetry, beam-target correlations, $d\sigma_L/dt$, $d\sigma_{LT}/dt$ contribute to full $d\sigma^5/dtd$. . . along with parity-violating observables $d\sigma_{L,T}/dt$ and $d\sigma_{T,T}/dt$
- ☯ Charge Current interactions \Rightarrow azimuthal asymmetry measurements & T-odd being related to GPD loop integral
- ☯ electroproduction, GPDs in overlapping kinematic region can bring enlightenment to neutrino scattering & transversity of nucleon within nucleus.