Overview of Neutrino Mixing Models and Ways to Differentiate Among Them

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Present Oscillation Data and Unknowns

- Present data within 3 σ accuracy

Fogli et al.

$$\begin{aligned}
\Delta m^2_{32} &= 2.39^{+0.42}_{-0.33} \times 10^{-3}\ \mathrm{eV}^2, \\
\Delta m^2_{21} &= 7.67^{+0.52}_{-0.53} \times 10^{-5}\ \mathrm{eV}^2, \\
\sin^2\theta_{23} &= 0.466^{+0.178}_{-0.135}, \\
\sin^2\theta_{12} &= 0.312^{+0.063}_{-0.049}, \\
\sin^2\theta_{13} &\le 0.046, \quad (0.016 \pm 0.010).
\end{aligned}$$

- Data suggests the approximate tri-bimaximal mixing texture of Harrison, Perkins and Scott:

$$U_{PMNS} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}$$

with $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{12} = 0.33$ and $\sin^2\theta_{13} = 0.$

- Present unknowns

Hierarchy and absolute mass scales

Whether neutrinos are Dirac or Majorana

CP-violating phases of mixing matrix

How close to zero is the reactor angle θ_{13} ?

How near maximal is the atmospheric mixing?

Is the approximate tri-bimaximal symmetry a softly-broken or accidental symmetry?

Will neutrino-less double beta decay be observable?

How large is charged lepton flavor violation?

Does leptogenesis play a major role in creating the baryon asymmetry in the universe?

Outline

- Present Oscillation Data and Unknowns
- Theoretical Framework
  - Top - Down Model Approach
  - Bottom - Up Mixing Matrix Approach
- Models and Mixing Angle Predictions
  - Discrete Horizontal Flavor Symmetry Groups
  - Grand Unification Models (with Flavor Symmetry)
- Other Tests
  - TBM mixing: softly-broken or accidental symmetry?
  - Neutrino-less Double Beta Decay
  - Charged Lepton Flavor Violation
- Conclusions

Theoretical Framework

- Neutrino oscillations require "massive" neutrinos

$$\Delta m^2_{21} \simeq 7.9 \times 10^{-5}, \quad |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$

$\sum_i m_i \leq 0.17\ \mathrm{eV} \rightarrow 2\ \mathrm{eV}$ (WMAP, SDSS, Lyman alpha)

- Possible extensions of the Standard Model
  - Introduce dim-5 effective non-renormalizable operator
  - Add RH neutrinos with Yukawa interactions
  - Add direct mass terms with RH Majorana couplings
  - Add Higgs triplet with LH Majorana couplings
  - Add fermion triplet with Higgs doublet couplings

- General 6 x 6 neutrino mass matrix in flavor basis $\mathcal{B}(\nu_{\alpha L}, N^c_{\alpha L})$ of the 6 LH fields:

$$\mathcal{M} = \begin{pmatrix} M_L & M_N^T \\ M_N & M_R \end{pmatrix}$$

where M_N is the Dirac neutrino mass matrix, M_L the LH and M_R the RH Majorana neutrino mass matrix.

- M_L only with Higgs triplets/or higher dimensional effective interactions and no RH neutrinos
- M_N only with Higgs doublets and Dirac Yukawa couplings
- Type I seesaw with $M_L = 0, \ M_N << M_R$:

$$M_\nu = -M_N^T M_R^{-1} M_N$$

- Type II seesaw with $M_L \neq 0$ resulting from Higgs triplet; type I + II implies $M_\nu = M_L - M_N^T M_R^{-1} M_N$

- Top - Down Approach
  - Models differ due to horizontal flavor symmetry chosen, vertical family symmetry (if any) selected, fermion and Higgs representation assignments made.
  - M_ν constructed directly or with seesaw formula once $M_N,\ M_R$ (and M_L) are specified.
  - Neutrino and charged lepton matrices diagonalized by

$$M_\nu^{diag} = U_{\nu_L}^T M_\nu U_{\nu_L} = \mathrm{diag}(m_1, m_2, m_3)$$

$$(M_{lept}^{diag})^2 = U_{lept_L}^\dagger M_{lept}^\dagger M_{lept} U_{lept_L} = \mathrm{diag}(m_e^2,\ m_\mu^2,\ m_\tau^2)$$

  - Neutrino mixing matrix is then given by

$$\begin{aligned} V_{PMNS} &\equiv U_{lept_L}^\dagger U_{\nu_L} = U_{PMNS}\Phi \\ \Phi &= \mathrm{diag}(1,\ e^{i\alpha},\ e^{i\beta}) \end{aligned}$$

- Bottom - Up Approach
  - In the diagonal lepton flavor basis and with the general PMNS matrix, one can determine the general texture of the neutrino mass matrix to be

$$
\begin{aligned}
M_\nu &= U^*_{PMNS}\Phi^* M_\nu^{\rm diag}\Phi^* U^\dagger_{PMNS} \\
&= U^*_{PMNS}{\rm diag}(m_1,\ m_2 e^{-2i\alpha},\ m_3 e^{-2i\beta})U^\dagger_{PMNS} \\
&\equiv \begin{pmatrix} A & B & B' \\ \cdot & F' & E \\ \cdot & \cdot & F \end{pmatrix}
\end{aligned}
$$

    where the matrix elements are expressed in terms of the unknown neutrino masses, mixing angles and phases.
  - By restricting the mixing matrix, one can learn that some of the matrix elements may not be independent.

Models and Mixing Angle Predictions

- Texture Zeros assigned to M_ν in hopes of identifying some flavor symmetry, but procedure is basis dependent.

- $L_e - L_\mu - L_\tau$ Lepton Flavor Symmetry : leads to inverted hierarchy only
$$M_\nu = \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}$$

- $\mu - \tau$ or 2 - 3 Interchange Symmetry with $\mathbf{B}' = \mathbf{B}, \mathbf{F}' = \mathbf{F}$ leads to $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{13} = 0,\ \sin^2\theta_{12}$ arbitrary.

- Tri-bimaximal mixing with $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{13} = 0,$ $\sin^2\theta_{12} = 0.333$ leads to $\mathbf{B}' = \mathbf{B},\ \mathbf{F}' = \mathbf{F} = \frac{1}{2}(\mathbf{A} + \mathbf{B} + \mathbf{D}),$ $\mathbf{E} = \frac{1}{2}(\mathbf{A} + \mathbf{B} - \mathbf{D})$ in terms of just three unknowns.

- Discrete Horizontal Flavor Symmetry Groups

used as starting points and then generally broken

- $\mathbf{S_3}$: permutation group of 3 objects
  6 elements, IR's: $\mathbf{1, 1', 2}$
  same eigenstates as TBM, but 2-fold mass degeneracy
- $\mathbf{A_4}$: even permutation group of 4 objects
  12 elements, IR's: $\mathbf{1, 1', 1'', 3}$
  $U(1)_F$ flavon group imposed to fix mass scale
  Can not get non-diagonal CKM matrix for quarks
- $\mathbf{T'}$: covering group of $\mathbf{A_4}$ but $\mathbf{A_4} \not\subset \mathbf{T'}$
  24 elements, IR's: $\mathbf{1, 1', 1'', 3, 2, 2', 2''}$
  TBM obtained for leptons, satisfactory CKM matrix
- $\mathbf{S_4}$: smallest symmetry group naturally related to TBM   Lam
  24 elements, IR's: $\mathbf{1, 1', 2, 3, 3'}$
  Can avoid vacuum alignment problems and obtain CKM

- Examples involving GUT Models

  - "Minimal" SO(10) Models with Higgs in 10, 126, (120, 45, 54) result in symmetric and antisymmetric contributions to quark and lepton mass matrices

  - SO(10) Models with Higgs in 10, 16, 16bar, 45 result in in "lopsided" down quark and charged lepton mass matrices due to the SU(5) structure of the EW VEVs appearing in the 16 and 16bar representations

  - Type I seesaws only lead to a stable Normal Hierarchy, while type I + II seesaws can also result in an Inverted Hierarchy

  - GUT models based on SU(5), SO(10) or E_6 may or may not also have a direct product horizontal flavor group such as U(1), SU(2), SU(3), etc. or one of earlier discrete groups.

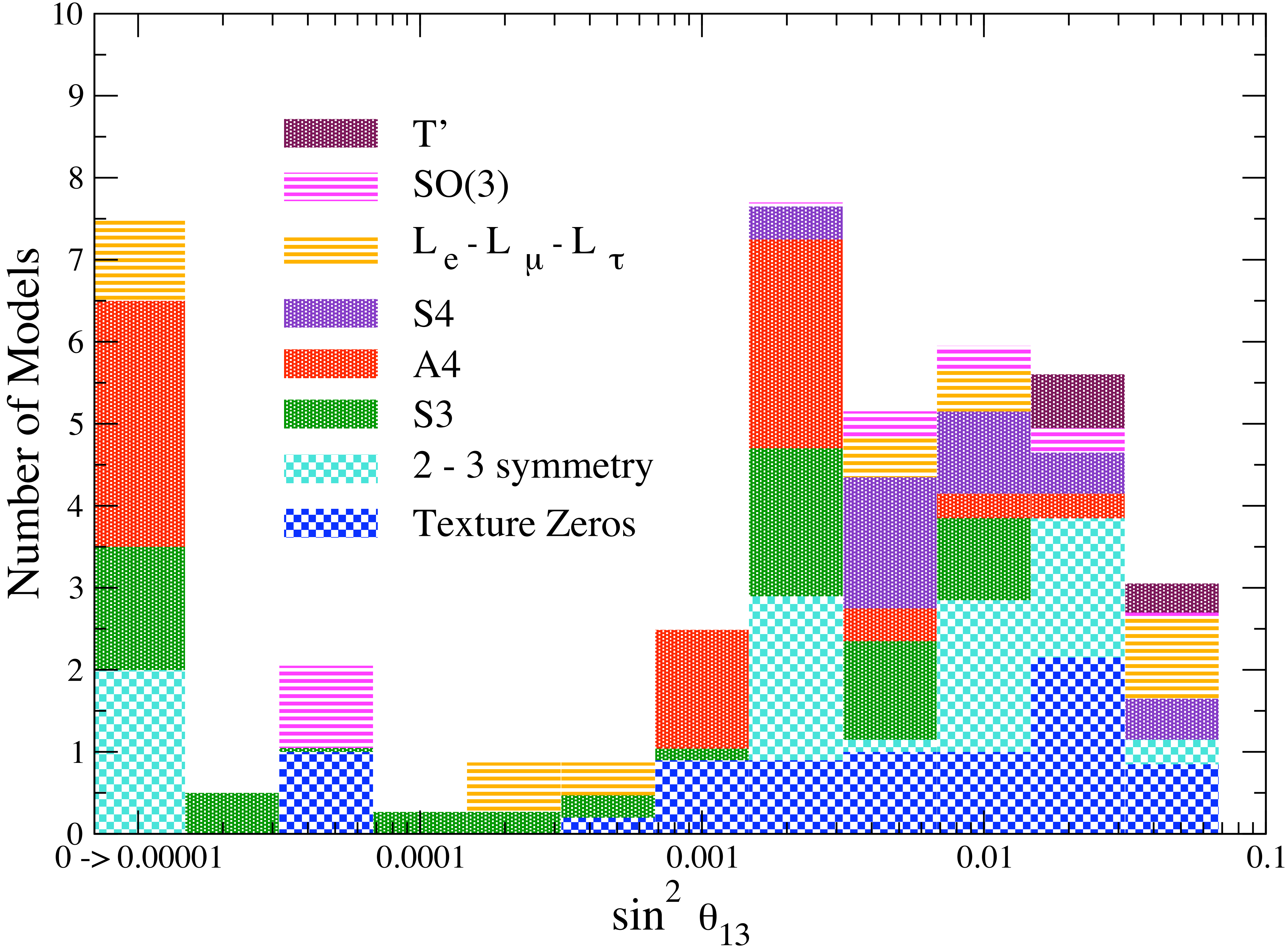
Survey of Mixing Angle Predictions

- Survey made of 80+ models in literature which satisfy the the 3   bounds and give reasonably restrictive predictions for the reactor neutrino angle. (Cutoff date: 1/09)

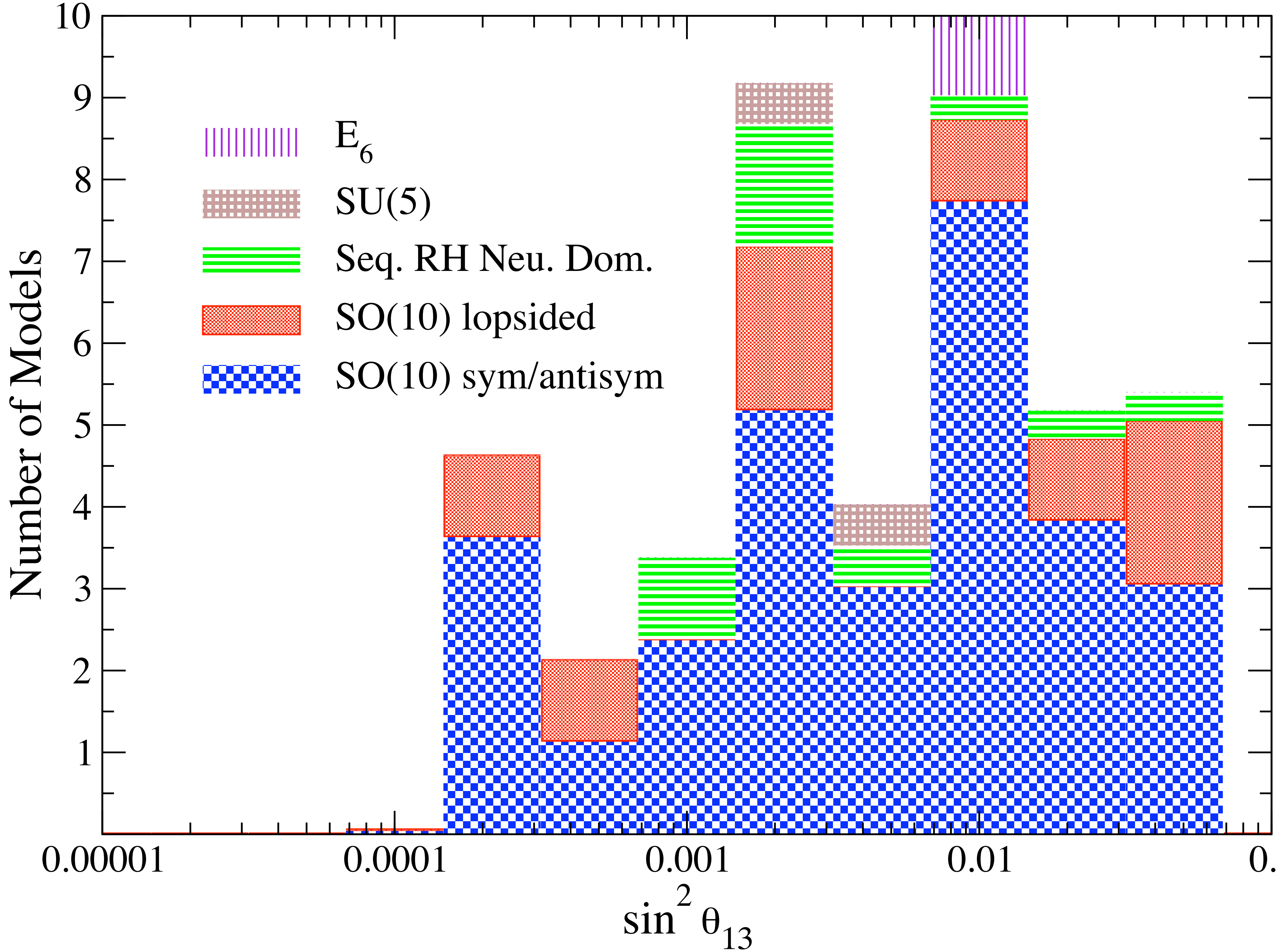
  Update of survey from 2006 made with M.-C. Chen.

- All models considered given same area on histograms.
- Many of these models do not predict other mixing angles.
- Several hundred models exist in the literature, but many lack firm predictions for any of the mixing angles.

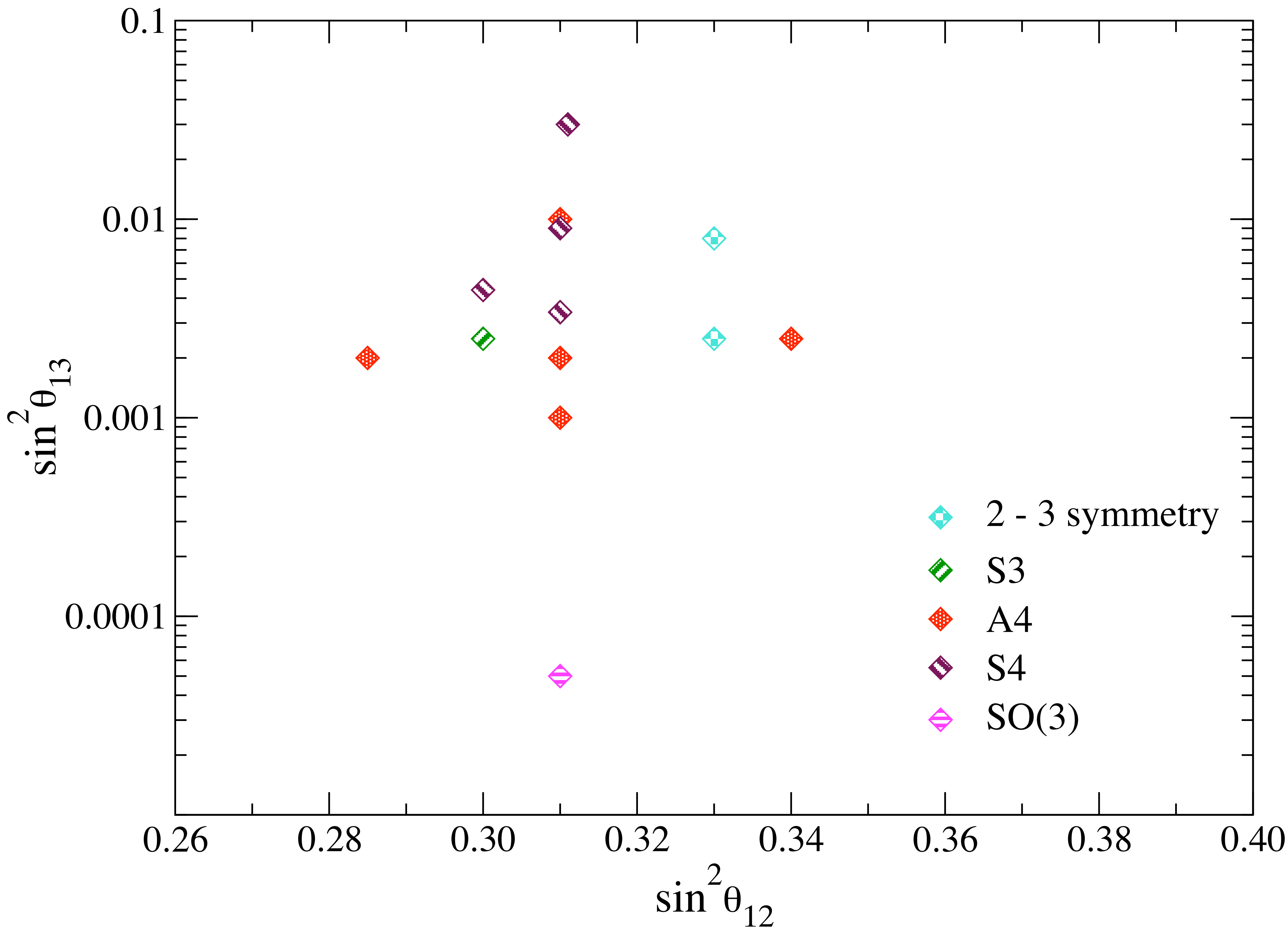
Predictions of Lepton Flavor Models



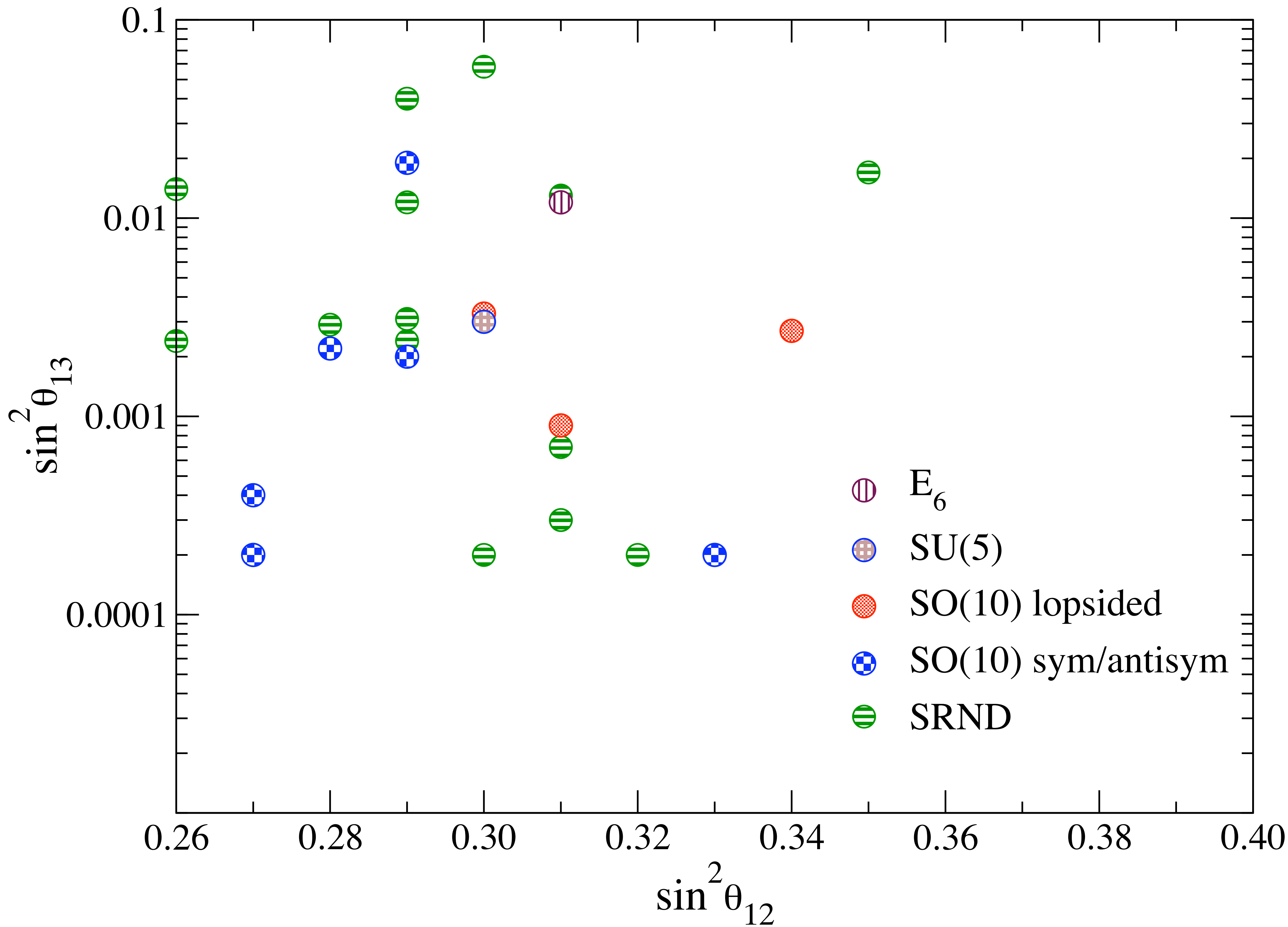
Predictions of Grand Unified Models



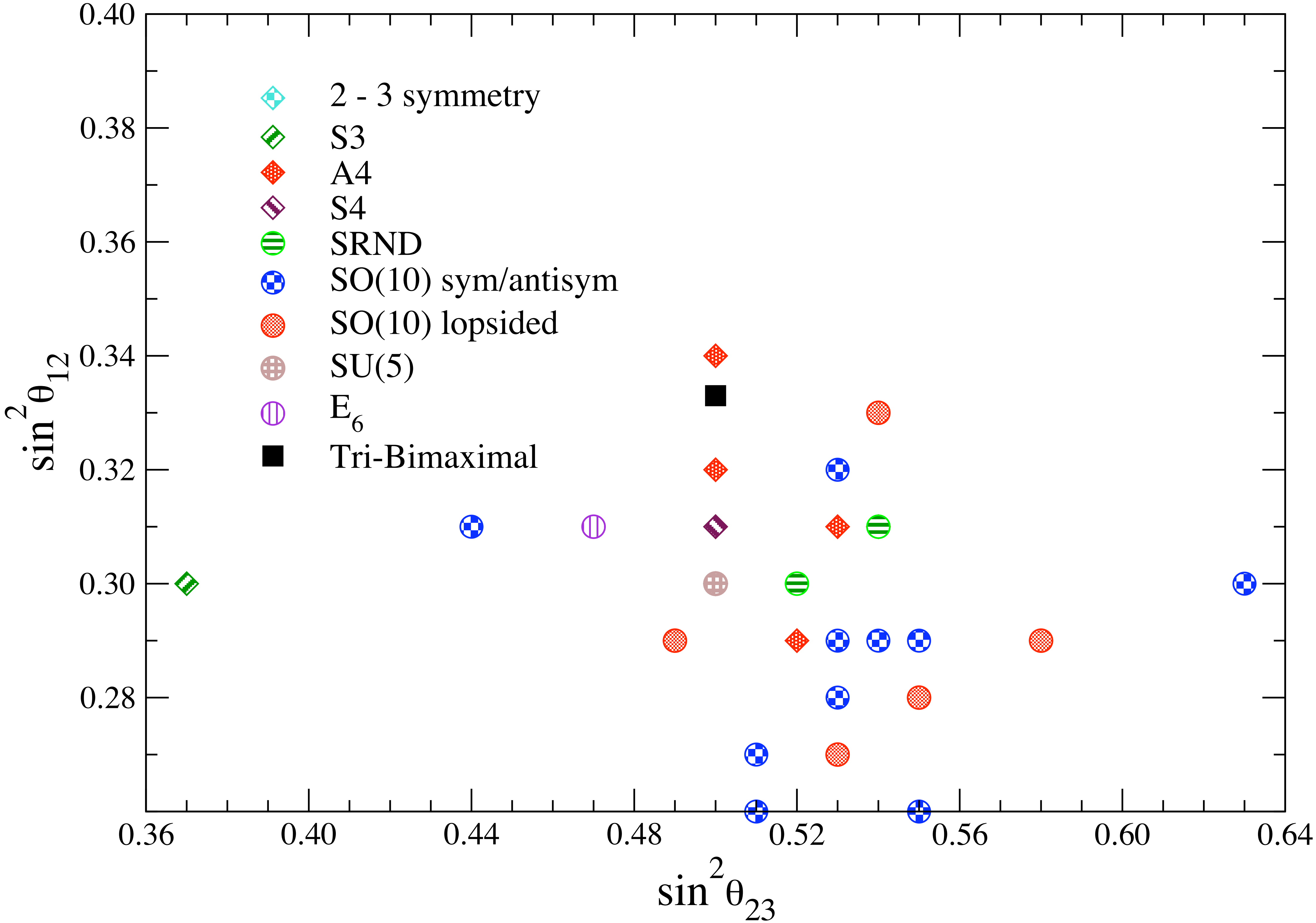
Scatterplot for Lepton Flavor Models



Scatterplot for Grand Unified Models



Scatterplot for Both Types of Models



Survey of Mixing Angle Predictions

- Survey made of 80+ models in literature which satisfy the the 3σ bounds and give reasonably restrictive predictions for the reactor neutrino angle. (Cutoff date: December 2008)
- Two thirds of both the lepton flavor and GUT models predict $0.001 < \sin^2\theta_{13} < 0.05$ while the lepton flavor models have a much longer tail extending to very small reactor angles.
- Planned reactor experiments will reach $\sin^2 2\theta_{13} \sim 0.01$, so roughly two thirds of models will be eliminated if no $\bar{\nu}_e$ depletion is observed.
- Most models prefer $\sin^2\theta_{12} \leq 0.31$ rather than 0.333 for TBM
- Most models prefer $\sin^2\theta_{23} \geq 0.50$ compared with the best fit value of 0.44
- Normal hierarchy is preferred 3 : 1

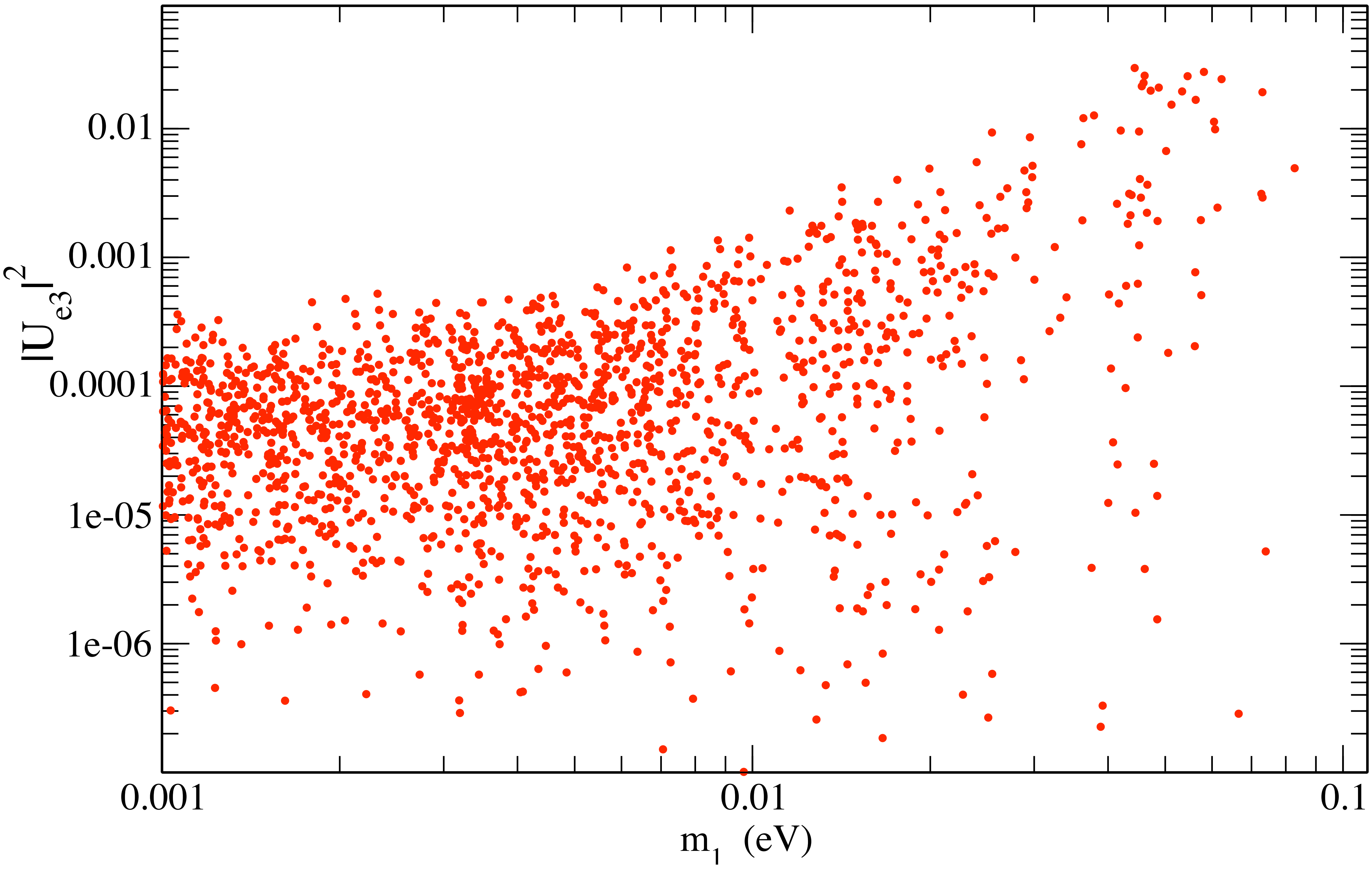
Tri-bimaximal Mixing: Hidden or Accidental Symmetry?

- Model-independent approach adopted to test whether TBM is an accidental or softly-broken hidden symmetry. with W. Rodejohann
- In the lepton flavor basis, deviations from TBM were considered by perturbing each element of neutrino mass matrix by up to 20%:

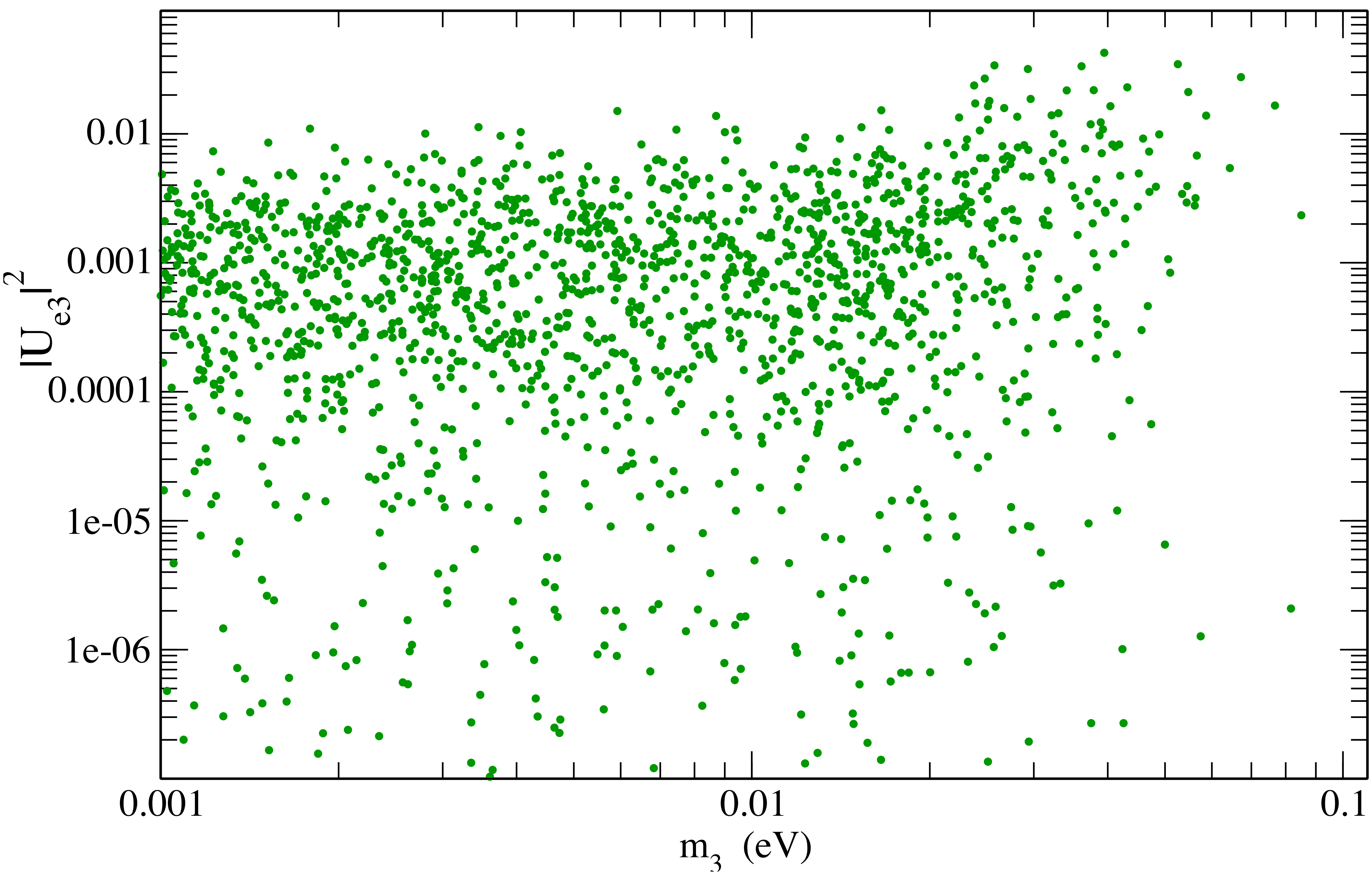
$$m_\nu = \begin{pmatrix} A(1+\epsilon_1) & B(1+\epsilon_2) & B(1+\epsilon_3) \\ \cdot & \frac{1}{2}(A+B+D)(1+\epsilon_4) & \frac{1}{2}(A+B-D)(1+\epsilon_5) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)(1+\epsilon_6) \end{pmatrix}$$

- Scatter points allowed according to following prescription:
  - For TBM, $A = (2m_1 + m_2 e^{-2i\alpha})/3$, $B = (m_2 e^{-2i\alpha} - m_1)/3$, $D = m_3 e^{-2i\beta}$.
  - Start with central best values for the masses, hold m_3 (NH) or m_2 (IH) fixed and let other masses vary by up to 20%.
  - Vary Majorana phases in their full ranges.
  - Vary each ϵ_i within $|\epsilon_i| \leq 0.2$ for its full phase range.

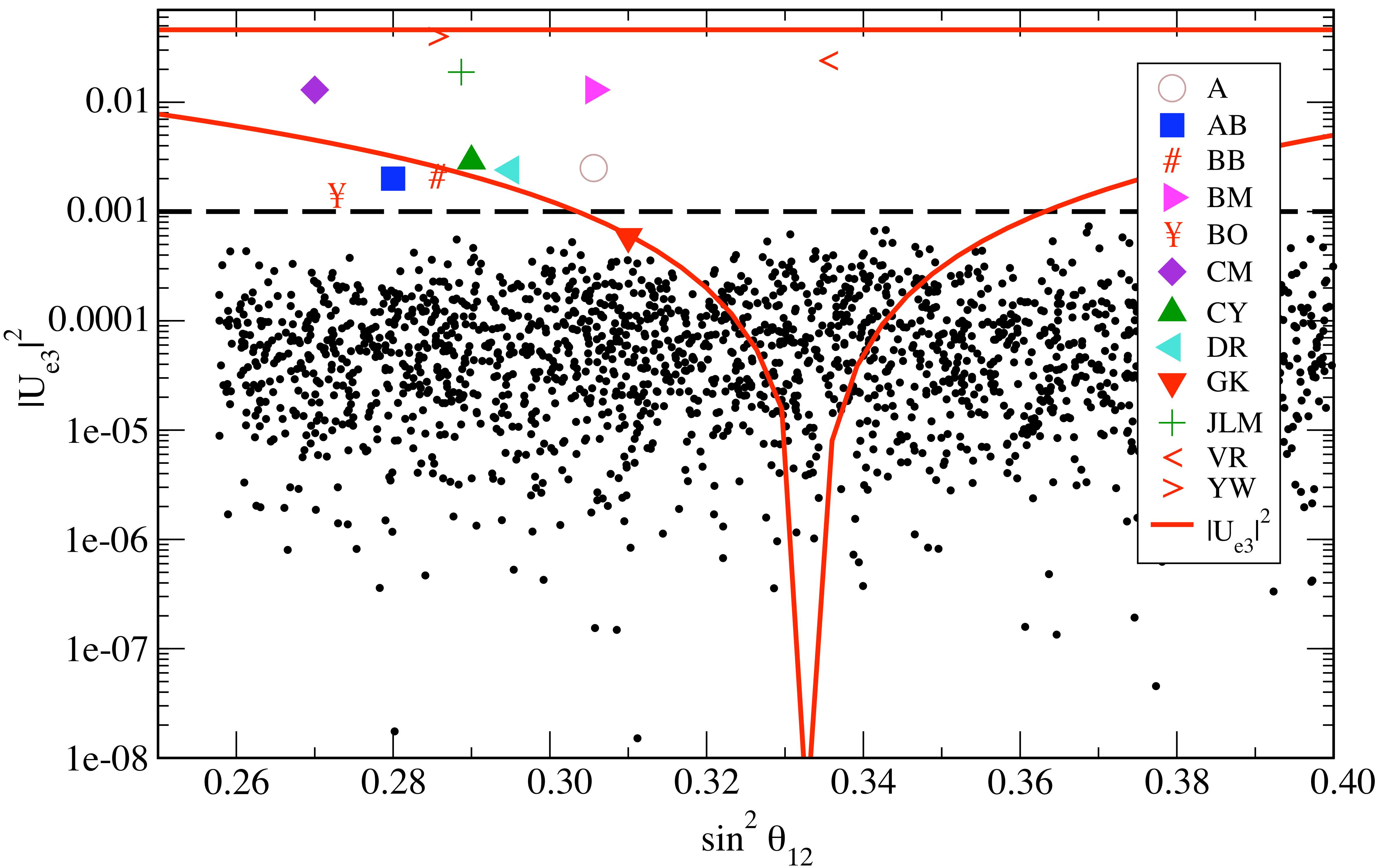
Normal Ordering



Inverted Ordering

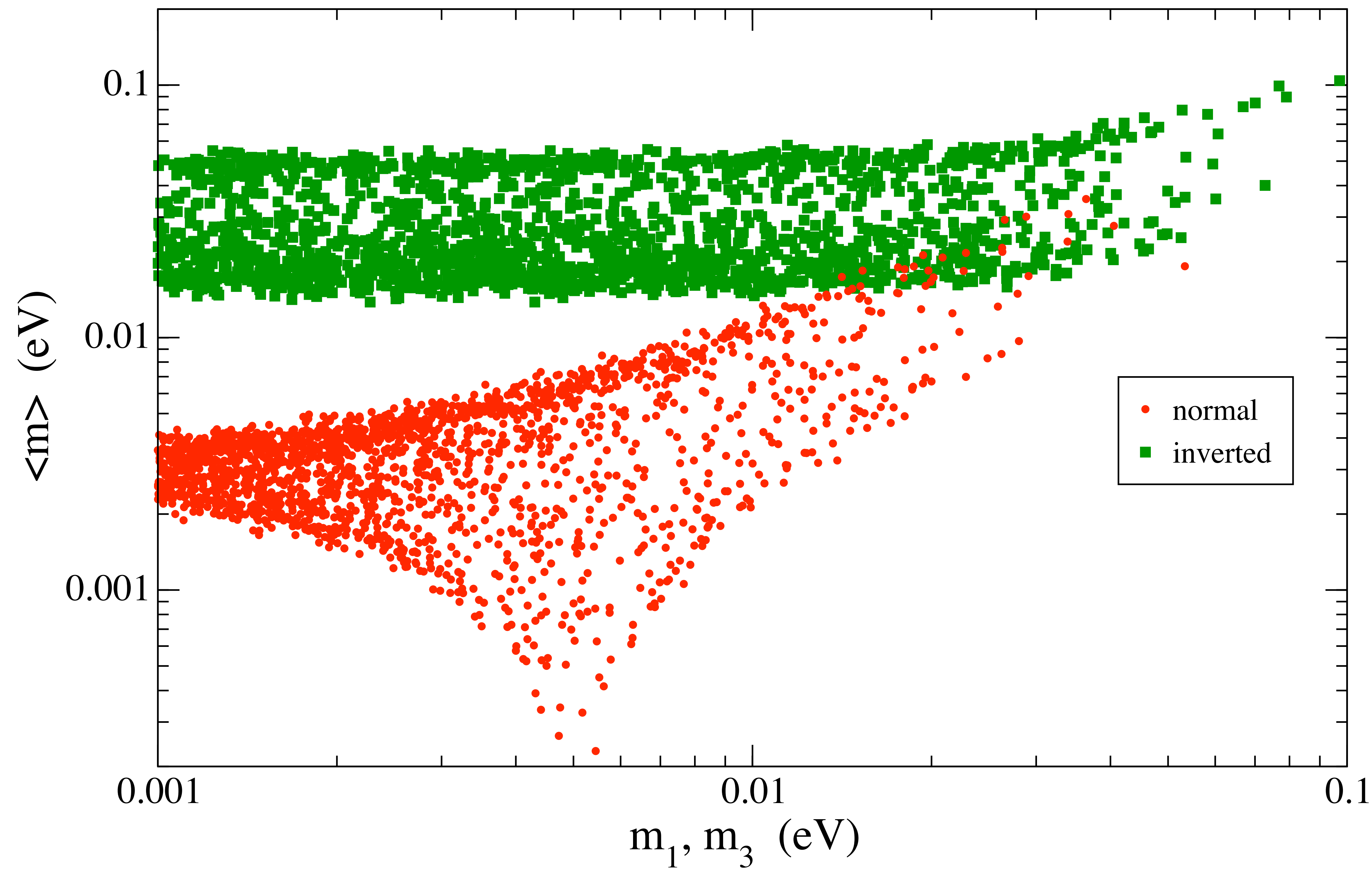


Perturbed TBM Mixing and GUT Model Predictions

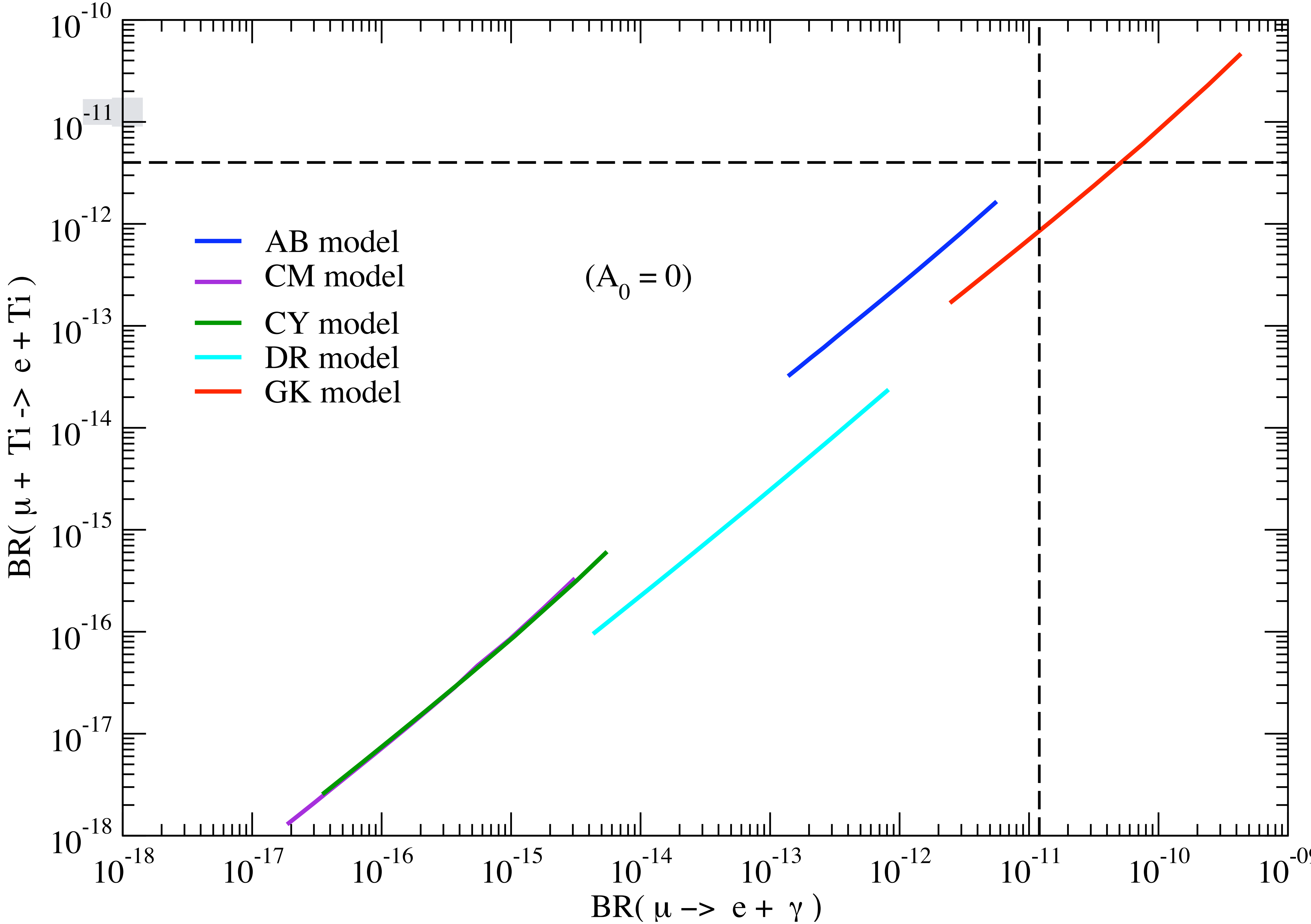


- We find for a normal hierarchy, the maximum TBM mixing deviation from zero for the reactor angle is $\sin^2\theta_{13} \leq 0.001$ for $m_1 < 5$ meV. This maximum deviation rises as the normal ordering approaches near degeneracy.
- This would suggest that for normal ordering the TBM mixing is accidental, if the reactor angle is determined to be larger than the bounded deviation.
- However, if the charged lepton flavor matrix is rotated by $\theta_C/3$ (θ_C) from its diagonal form while the neutrino matrix keeps the TBM form, one finds a larger result is possible independent of m_1: $\sin^2\theta_{13} = 0.0028$ (0.025).
- For an inverted hierarchy the restricted bound on $\sin^2\theta_{13}$ is much weaker for deviations from TBM mixing.

Effective Mass Plot for Perturbed TBM



$\mu - e$ Conversion vs. $\mu \rightarrow e + \gamma$ Decay



Conclusions

Overview based on 80+ models in literature (< 1/09) with discrete horizontal lepton flavor symmetries or vertical Grand Unified family symmetries and firm numerical predictions for $\sin^2\theta_{13}$.

- Tried to differentiate models based on neutrino mass hierarchy, mixing angles, neutrino-less double beta decay (and charged lepton flavor violation) predictions.
- Most models allow either hierarchy with exceptions being inverted only for $L_e - L_\mu - L_\tau$ models and normal only for type I seesaw models. Normal hierarchy preferred 3 : 1
- Double CHOOZ and Daya Bay reactors will be able to eliminate roughly two thirds of the neutrino models surveyed, if their planned sensitivity reaches $\sin^2 2\theta_{13} \simeq 0.01$

- No smoking gun apparently exists to rule out any type of model based on accurate data for $\sin^2\theta_{13}$ alone.
  - Of the order of 5 models have similar values for in the interval 0.001 - 0.08.
  - Only lepton flavor models allow $\sin^2\theta_{12} \ll 0.0001$
- Most models prefer $\sin^2\theta_{12} \leq 0.31$ rather than 0.333 for TBM in agreement with present best value of 0.312.
- Most models prefer $\sin^2\theta_{23} \geq 0.50$ compared with the best fit value of 0.44
- Effective mass plots for perturbed TBM mixing show a clear separation of the normal and inverted ordering distributions.
- It is clear that very accurate determination of the three mixing angles and eventually the three CP-violating phases will be required to pin down the most viable model(s).