THE SUPERACCELERATOR CONCEPT

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ABSTRACT

In the superaccelerator concept a high density electron gas is confined and accelerated by a travelling magnetic wave. The electron gas is produced by inductive charge injection and thereafter compressed to high densities by the travelling magnetic wave. The compression of the electron gas by the travelling magnetic wave results in a collisionless shock transition with a large electric field. The γ-value of the collisionless shock transition can become arbitrarily large even though the γ-value of the flowing electron gas remains finite. Particles placed and trapped in the deep electrostatic potential well of the collisionless shock transition can therefore be accelerated to very high energies. Typically, the concept may permit to accelerate as many as 10^{14} particles to more than 10^{13} eV over a length of 10 km. Through magnetic selffocussing of the flowing electron gas, by projecting it into a tenuous plasma, still larger electric fields and hence even higher particle energies can be reached.

The travelling magnetic wave can draw its energy from inexpensive inductive energy storage devices and ultimately from thermonuclear microexlposion reactors. Because of the large input energy, very high beam power levels seem possible, resulting in large beam luminosities as they will be required for high energy physics experiments.

I. INTRODUCTION

If high energy physics is going to have any future, particle energies up to 10^{15} eV and beam luminosities up to 10^{32} cm^{-2}s^{-1} must be attained. The beam energy and power requirements to reach this goal even dwarf the biggest accelerators presently planned.

The beam power requirements can be estimated from the luminosity formula

$$L = N^2 f/4\pi r^2 \quad , \qquad (1.1)$$

where N is the number of colliding particles, f the frequency of such collisions and r the beam radius at the position where the collisions take place. In case of the single pass collider concept we may put $f \simeq 1\ s^{-1}$. If one then chooses $N = 10^{14}$, one would obtain $L = 10^{31}\ cm^{-2}s^{-1}$ for $r = 10^{-2}$ cm. To accelerate $N = 10^{14}$ particles to 10^{14} eV requires an energy of $\simeq 1.6$ Gigajoule, and with $f = 1\ s^{-1}$ the average beam power would be 1.6 Gigawatt. Assuming optimistically a 10% efficiency for the conversion of electric input energy into beam energy, an accelerator of this kind would therefore require more than 10 Gigawatt in electric power. These numbers show that several concepts proposed to reach ultrahigh particle energies, like laser or beam front accelerators, will fail on the basis of insufficient power and energy, and therefore can hardly be taken seriously.

In the superaccelerator concept(1,2,3) a powerful travelling magnetic wave drives an electron gas and which in turn accelerates particles to ultrahigh energies. In this concept a beam continuously draws energy from a travelling magnetic wave, with the travelling magnetic wave driven by cheap inductive energy storage devices, having energy densities larger by several orders of magnitude than those which can be reached in microwave cavities or laser media. Eventually, the travelling magnetic wave can be driven by thermonuclear microexplosion reactors, if and when they become available, because these reactors can deliver gigajoule energy pulses through magnetohydrodynamic energy conversion of the expanding fireball from the microexplosion.

In the original superaccelerator concept it was proposed to compress and push an electron gas by a travelling magnetic wave, with fresh electrons continuously added by inductive charge injection. The large electric field at the head of the electron gas is then used to accelerate particles to ultrahigh energies. In a more refined version of this concept presented here, we show that the acceleration of the electron gas by the travelling magnetic wave leads to a collisionless shock transition possessing a large electric field and which can be used to accelerate both electrons or ions to high energies.

II. BRIEF DESCRIPTION OF THE ACCELERATOR CONCEPT

The idea is explained in Fig. 1, showing a radial (Fig. 1a), and axial (Fig. 1b) cross section, through the accelerator tube. The accelerator consists of many one-turn magnetic solenoids MS positioned in series

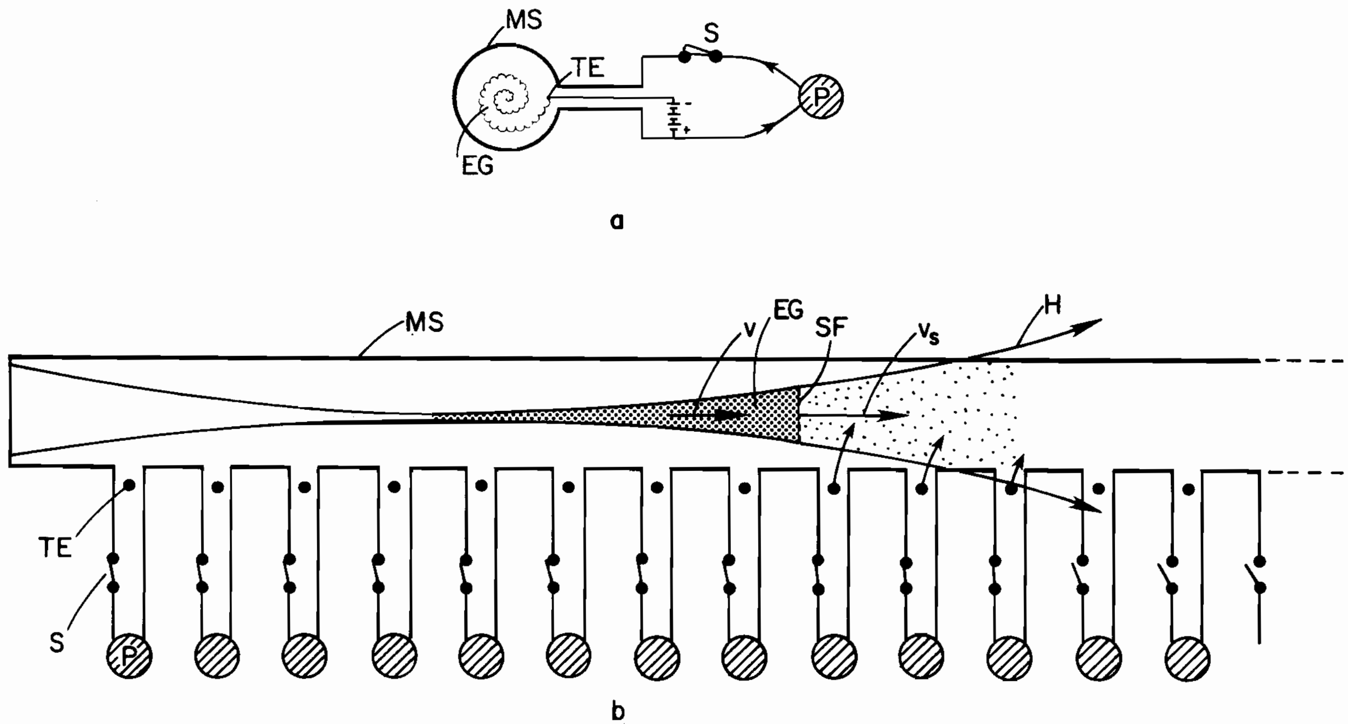

Fig. 1. (a) Radial and (b) axial cross sections through accelerator. MS, one-turn magnetic solenoid; P, power supply; S, switch; TE, thermionic emitter electrode; EG, magnetically confined electron gas; H, magnetic lines of force; SF, collisionless shock front.

along its entire length. Each solenoid has its separate power source P and can be magnetized by closing the switch S. After closing the switch at some solenoid, magnetic flux surfaces created at the inner side of the solenoid move inward. If, as shown in Fig. 1a, a thermionic emitter connected to an auxiliary voltage source, is positioned at the inner side of the solenoid, electrons from the thermionic emitter attach themselves to these flux surfaces and move inward. In this way, known as inductive charge injection[4] (see Appendix I), an electron gas of intiallly low density can be formed and magnetically confined within each solenoid. However, if the solenoids are magnetized in such a programmed way that a travelling magnetic wave of specified velocity moves to the right, the magnetic mirror formed by this travelling magnetic wave can push and accelerate the low density electron gas to the right, and if the pushing of the magnetic travelling wave is strong enough, the electron gas will be axially compressed to much higher densities. This compression is accompanied by a collisionless shock transition SF, separating the low density from the high density gas. The maximum attainable density which in this way can be reached is determined by the magnetic field strength of the travelling magnetic wave and which also radially confines the electron gas. This maximum electron gas density is determined by the condition that

$$E < H \quad , \qquad (2.1)$$

where E is the electric field produced by the electron gas and where H is the confining magnetic field of the moving wave, with E and H both measured in electrostatic units. Inequality (2.1) is just the well known condition for magnetic insulation. It implies, that if for example $H = 10^5$ G, then $E < 10^5$ esu $= 3 \times 10^7$ V/cm. This electric field strenght compares well with the already reached 10^7 V/cm in magnetically selfinsulated pulse power transmission lines. Particles accelerated with 3×10^7 V/cm over 10 km would reach 3×10^{13} eV.

III. THE MAGNETICALLY CONFINED ELECTRON GAS

We are considering an electron gas cylindrical in shape and confined in an axial magnetic field. Assuming that the electron gas has a uniform density n, the radial electric field E_r at the surface of the cylinder of radius r is given by

$$E_r = 2\pi ner \quad . \qquad (3.1)$$

In the presence of an axial magnetic field H_z, the electron gas is confined provided

$$E_r < H_z \quad . \tag{3.2}$$

However, to prevent the diocotron instability it is furthermore required that

$$\omega_p << \omega_c \quad , \tag{3.3}$$

where $\omega_p = (4\pi ne^2/m)^{1/2}$ is the electron plasma frequency and $\omega_c = eH_z/mc$, the electron cyclotron frequency. Condition (3.3) is well satisfied for the electron gas densities of interest. As an example we take $H_z = 10^5$ G and $r = 10$ cm. Inequality (3.2) together with eq. (3.1) then predicts that $n \leq 3 \times 10^{12}$ cm^{-3}, and one computes that $\omega_p = 9 \times 10^{10}$ s^{-1}. Since furthermore $\omega_c = 2 \times 10^{12}$ s^{-1} it therefore follows that condition (3.3) is well satisfied.

If the cylindrical electron gas has a finite axial extension, the axial electric field at the end of the cylinder is given by (see Appendix II):

$$E_z \simeq E_r \quad . \tag{3.4}$$

One therefore also always has

$$E_z < H_z \quad . \tag{3.5}$$

IV. THE ACCELERATION OF THE ELECTRON GAS BY THE TRAVELLING MAGNETIC WAVE

We now depart slightly from the previous assumption of a cylindrical electron gas and rather assume that the gas is confined by a slender magnetic mirror field, produced by the travelling magnetic wave as shown in Fig. 1. In this case the nonstationary magnetic mirror field induces an axial electric field and which accelerates the electron gas to the right. To keep the electron gas inside the mirror field of the travelling magnetic wave, the wave must likewise be accelerated by the same amount.

A simplified form of the magnetic travelling wave is shown in Fig. 2. The length of the mirror, that is the length over which the magnetic field appreciably changes, shall be called the length λ_H of the travelling wave. The magnetic wave acts like a piston pushing the electron gas, and because

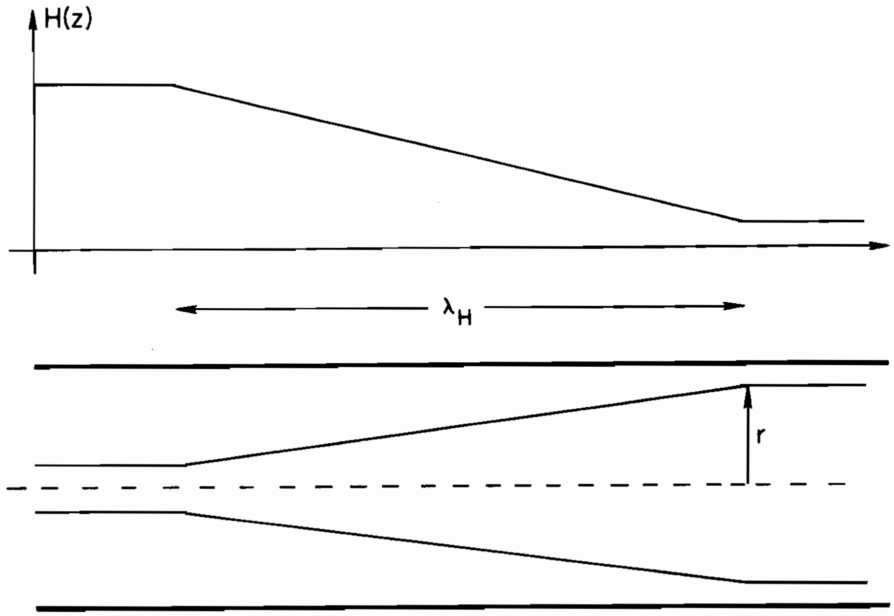

Fig. 2. Schematic display of field lines of magnetic travelling wave.

the electron gas is very light, this piston must move close to the velocity of light. The minimum length of the wave is then determined by the technical ability to make a fast rising magnetic field. If we call the rise time for the magnetic field τ_H, we have

$$\lambda_H = c\tau_H \quad . \qquad (4.1)$$

In practice $\tau_H \simeq 10^{-6}$ sec is possible and with considerable difficulty $\tau_H \simeq 10^{-7}$ sec. In the first case $\lambda_H \simeq 300$ meters, and in the second case $\lambda_H \simeq 30$ meters. The travelling wave appears rather long, but if the accelerator is $\simeq$ 10 km long, the length of the travelling wave is small by comparison.

The acceleration of the electron gas can be most easily understood by the magnetic pressure concept. The magnetic pressure force acting on the gas of cross section πr^2 is $(H_z^2/8\pi)\pi r^2$. This force can be interpreted as the result of an effective induced electric field E_{in}, acting on all the electrons confined in the wave. The number of these electrons is of the order $n\pi r^2\lambda_H$, and one therefore has

$$(H_z^2/8\pi)\pi r^2 \sim n\pi r^2 \lambda_H e E_{in} \quad . \tag{4.2}$$

With $H_z \sim E_r = 2\pi n e r$, one finds that

$$E_{in} \sim (r/\lambda_H) E_r/4 \quad . \tag{4.3}$$

If, for example, $E_r \sim 3 \times 10^7$ Volt/cm, $r \sim 10$ cm and $\lambda_H \sim 10^4$ cm, one has $E_{in} \sim 0.7 \times 10^4$ Volt/cm. Through the action of the travelling wave, the electron gas assumes a γ-value ($\gamma = (1 - v^2/c^2)^{-1/2}$, v velocity of electron gas) which is approximately given by

$$\gamma \simeq E_{in} z \simeq (r/\lambda_H) E_r z/2 \quad . \tag{4.4}$$

In eq. (4.4) both E_{in} and E_r are measured in MV/cm.

V. LORENTZ-TRANSFORMATION OF THE MAGNETICALLY CONFINED ELECTRON GAS

For an electron gas moving with a velocity v relative to an observer frame S into the positive z-direction, one has the following relations between the observer frame S and electron gas frame S':

$$\left.\begin{aligned} n &= \gamma n' && \text{(a)} \\ E_r &= \gamma E_r' && \text{(b)} \\ E_z &= E_z' && \text{(c)} \\ H_z &= H_z' && \text{(d)} \\ H_\phi &= \beta\gamma E_r' = \beta E_r && \text{(e)} \end{aligned}\right\} \quad . \tag{5.1}$$

These transformations change (3.2) into

$$E_r < \gamma H_z \quad . \tag{5.2}$$

Inequality (5.2) can be understood as follows: In the observer frame S, the gas appears as an intense electron beam into the z-direction, and in this frame pressure equilibrium at the beam surface therefore requires that

$$E_r^2 < H_z^2 + H_\phi^2 \quad . \tag{5.3}$$

By virtue of (5.1e), (5.3) then yields (5.2). Inequality (5.2) shows, at least in principle, that for an electron gas moving at relativistic velocities, much larger electric fields can be reached than even with the simple magnetic insulation criterion (3.2). However, if the axial field E_z is used for acceleration, the transformation equations, as before, only give $E_z < H_z$. Therefore, unless a way can be found to use E_r rather than E_z for acceleration, the relativistic magnetic insulation effect cannot be applied.

For sufficiently large γ-values the moving electron gas is strongly Lorentz-contracted and which therefore can lead to large beam currents. From eq. (5.1e) we can obtain a value for the beam current given by

$$I_B = 5r\beta E_r \lesssim 5\gamma r H_z \quad . \qquad (5.4)$$

VI. THE COLLISIONLESS SHOCK TRANSITION

Initially, the electron gas, produced by inductive charge injection at the head of the travelling magnetic wave, has a comparatively low density. As the electron gas, pushed by the travelling magnetic wave, moves down the accelerator tube it snowplows the low density gas positioned in front of it. The dense electron gas thus grows, with the result that the region separating the low density from the high density gas can move faster than the electron gas itself. This behavior is reminiscent of a piston pushed into a gas-filled cylinder, which leads to a shock wave in the gas moving ahead of the piston and with a velocity larger than the piston velocity. The difference between this situation, occuring in classical gas dynamics, and the situation occuring here is twofold: First, the gas flow is relativistic, and second, because the mean free path of the low density electron gas is very large, the shock is collisionless. A collisionless shock is here made possible by the large magnetic and electric forces in the transition region.

To describe the shock transition we go into a coordinate system at rest with the electron gas behind the shock, that is a coordinate system which relative to the accelerator rest frame moves with the velocity $v = \beta c$. In this coordinate system, the electron number density is n', and in front of it shall be n_1'. Relative to this system the shock front moves with the velocity $v_s' = \beta_s' c$, which, of course, is not a material velocity. In relativistic gas dynamics v_s' cannot exceed 1/3 the velocity of light. In collisionless electrostatic shocks v_s' can come arbitrarily close to the velocity of light, because the large electric field having its source at the shock discontinuity can move ahead of the shock with the velocity of light.

First, we note that as long as inequality (5.2) is satisfied, the electron gas, both in front of the shock and behind, is radially confined by the external magnetic field of the travelling wave. Inequality (5.2) permits a γ-fold Lorentz-contraction of the electron gas, if seen in frame S. As a radial static equilibrium condition however, it is separated from the dynamic behavior of the shock along the axial direction.

The equation describing the conservation of electrons across the shock front is given by

$$n'\beta_s' = n_1'(\beta + \beta_s') \quad , \qquad (6.1)$$

where $\beta_s' = v_s'/c$, as seen from the frame at rest with the electron gas behind the shock. The ℓ.h.s. of this equation is the flux of electrons (divided by c) entering into the front, whereas the r.h.s. side is the flux of electrons (divided by c) leaving the electron gas positioned in front of the shock. In the accelerator rest frame $n' = n/\gamma$ and $n_1' = n_1\gamma$, hence

$$\beta_s' = \frac{n_1\gamma^2\beta}{n - n_1\gamma^2} \quad . \qquad (6.2)$$

Using the velocity addition theorem one finds in the accelerator rest frame:

$$\beta_s = \frac{\beta_s' - \beta}{1 - \beta_s'\beta} = \frac{(2n_1\gamma^2 - n)\beta}{n(1 - \beta^2) - n_1\gamma^2} \simeq 2 - \frac{n}{n_1\gamma^2} \quad . \qquad (6.3)$$

In the limit $\beta_s = 1$ one has

$$n_1/n = 1/\gamma^2 \quad , \qquad (6.4)$$

or

$$n_1/n' = 1/\gamma \quad . \qquad (6.5)$$

The value $\gamma_s = (1 - \beta_s^2)^{-1/2}$ valid for the shock front, and which tells us what energies the particles trapped in the front can reach, follows from eq. (6.3). One finds

$$\gamma_s \simeq \frac{1}{\sqrt{2}} \left[\frac{n}{n_1\gamma^2} - 1\right]^{-1/2} \quad . \qquad (6.6)$$

Therefore, as n_1/n approaches the value given by (6.4), $\gamma_s \to \infty$, even though

γ can be much smaller.

In a collisionless electrostatic shock the conservation of momentum simply means that the stagnation pressure of the particles hitting the shock must counterbalance the electric pressure resulting from the axial field E_z at the shock discontinuity. As seen from the frame at rest with the shocked material one has

$$E_z'^2/8\pi = E_z^2/8\pi = \gamma m\beta c n_1'(\beta + \beta_s')c$$

$$= mc^2\gamma^2 n_1\beta(\beta + \beta_s')$$

$$\simeq 2mc^2\gamma^2 n_1 \quad . \qquad (6.7)$$

In the limit $\beta_s = 1$ one has according to eq. (6.5) $n_1 = n'/\gamma$, hence

$$E_z^2/8\pi = 2mc^2\gamma n' \quad . \qquad (6.8)$$

In the rest frame of the electron gas one has

$$\left(\frac{\omega_p}{\omega_c}\right)^2 = \frac{4\pi n' mc^2}{H_z^2} \simeq \frac{4\pi n' mc^2}{E_z^2} \quad . \qquad (6.9)$$

Because n' is the number density in the rest frame of the electron gas, it must be computed from eqs. (3.1) and (3.2) by substituting n' for n. For the above given example we thus have $n' \leq 3 \times 10^{12}$ cm^{-3}.

Eliminating E_z^2 from eqs. (6.8) and (6.9) one obtains a condition for the axial confinement of the shock discontinuity and which can be expressed as a condition for the γ-value of the electron gas. One finds that

$$\gamma \simeq (1/4)(\omega_c/\omega_p)^2 \quad . \qquad (6.10)$$

Taking the above given example, $\omega_p = 9 \times 10^{10}$ s^{-1} and $\omega_c = 2 \times 10^{12}$ s^{-1}, one finds $\gamma \simeq 100$. It is therefore sufficient to accelerate the electron gas to about 50 MeV. For $E_r \simeq 30$ MV/cm, $r \simeq 10$ cm and $\lambda_H = 3 \times 10^4$ cm, one computes from eq. (4.4) that the electrons reach this energy after being accelerated by the travelling over a distance of $\sim$ 70 meters. For $\gamma \simeq 100$ and $n' \simeq 3 \times 10^{12}$ cm^{-3}, eq. (6.5) then gives $n_1 \simeq 3 \times 10^{10}$ cm^{-3}. This comparatively low value of n_1 is well within the densities reached in inductive charge injection experiments[(5)].

The electric current for $\gamma = 100$, $r = 10$ cm and $H_z = 10^5$ G is computed from eq. (5.4) and one finds that $I_B \leq 5 \times 10^8$ A.

After the electron gas is accelerated to $\gamma \simeq 100$, the energy supplied by the travelling magnetic wave goes into accelerating the snow-plowed low density electron gas to the same γ-value, but also into the fewer particles which are trapped in the deep electrostatic potential well of the collisionless shock discontinuity, and which are the particles accelerated to ultrahigh energies.

The thickness of the collisionless shock discontinuity is (by order of magnitude) an electron Larmor radius. Near the shock discontinuity there is a large azimuthal magnetic field, as seen from an electron in front of the discontinuity and at rest with the accelerator. This strong azimuthal magnetic field H_ϕ, according to eq. (5.1e), is given by

$$H_\phi = \beta E_r \simeq \gamma H_z \quad . \tag{6.10}$$

The Larmor radius of an electron colliding with the shock front therefore is

$$r_L = \frac{\gamma mc^2}{eH_\phi} \simeq \frac{mc^2}{eH_z} \quad . \tag{6.11}$$

For $H_z \simeq 10^5$ G, one finds $r_L \simeq 2 \times 10^{-2}$ cm. The other characteristic length, which might also play a role in the shock thickness, is the Debye-length

$$\delta = c/\omega_p \quad . \tag{6.12}$$

For $\omega_p = 9 \times 10^{10}\ s^{-1}$ one finds $\delta \simeq 0.3$ cm. The values, both of r_L and δ suggest a shock transition sufficiently sharp to ensure in its vicinity a large axial electric field. If the γ-value of the moving electron gas is sufficiently large, the contribution to the electric field resulting from the tenuous electron gas at rest in front of the shock transition, and acting in the opposite direction, can be neglected. The contribution of the dense electron gas behind the shock transition is proportional to $n' = n/\gamma$, whereas the contribution of the low density electron gas in front of the transition is proportional to n_1. The electric field produced by the denser gas is therefore γ times stronger.

In a collisionless shock the electron density in the transition region has the form of a soliton, qualitatively shown in Fig. 3. The shown periodic density profile leads to a sequence of deep potential wells, and which can be used to accelerate negative and positive charged particles.

The maximum number N_{max} of particles which can be trapped and thereby accelerated is of the order

$$N_{max} \sim \pi r^3 n \quad , \tag{6.13}$$

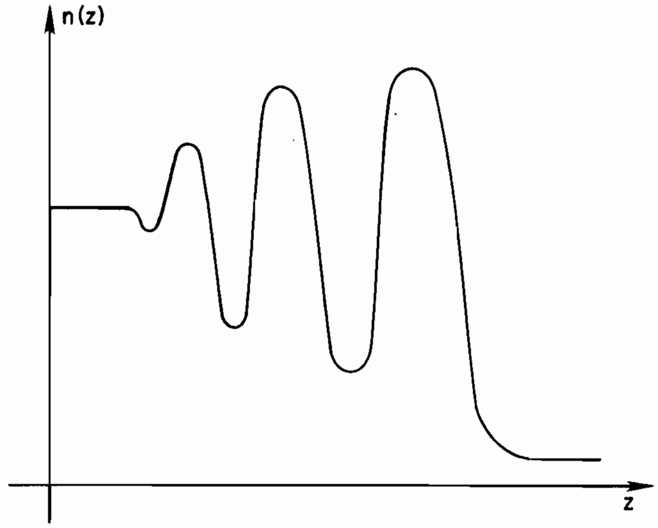

Fig. 3. Schematic display of soliton density distribution in collisionless shock discontinuity.

where $n \simeq E_r/2\pi er$. The axial electric field within the shock, as seen from the rest frame S, extends over a distance $\sim r/\gamma_s$, and the maximum density of the trapped particles is $n\gamma_s$, such that $(r/\gamma_s)n\gamma_s = nr$ is independent of γ_s. For $r = 10$ cm and $n = 3 \times 10^{12}$ cm^{-3} one finds that $N_{max} \simeq 10^{16}$. A 1% trapping would therefore be sufficient to accelerate 10^{14} particles.

VII. PROJECTION OF THE BEAM INTO A TENUOUS BACKGROUND PLASMA

Both, for the single pass collider concept, but also for increasing the accelerating field above the value given by (3.5) one may project the intense electron beam, produced by the travelling magnetic wave pushing the electron gas down the accelerator tube, into a tenuous background plasma (see Fig. 4). Moreover, the beam can be projected into a plasma where the externally applied magnetic field is weak and therefore could propagate into the plasma without possessing a trapped axial field. Under this condition the beam can shrink down to a much smaller diameter, provided the confining magnetic pinch-force becomes larger than the repulsive electric force. If the fraction of charge neutralization by the background plasma is f, the radial force on an electron in the beam is given by

$$F = e[(1 - f)E_r - \beta H_\phi] \quad , \qquad (7.1)$$

or because $H_\phi = \beta E_r$:

$$F = [1/\gamma^2 - f]eE_r \quad . \qquad (7.2)$$

The beam shrinks if F becomes negative, that is if

$$f > 1/\gamma^2 \quad . \qquad (7.3)$$

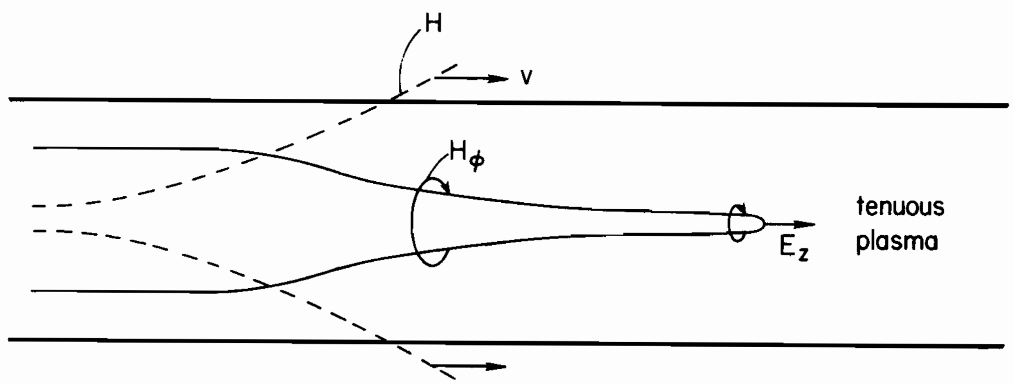

Fig. 4. Partial charge neutralization and beam focussing by projecting electron gas into tenuous plasma.

Therefore, if f is chosen to be just slightly larger than $1/\gamma^2$, the beam not only shrinks but still can produce a collisionless shock discontinuity at its head. In this case though, the beam snowplows the electrons from the tenuous background plasma and which thereafter become part of the beam. In the presence of the background charges, the radial and axial electric beam field is only reduced by the insignificant factor $1 - f = 1 - 1/\gamma^2$. This means that almost the full electric field still acts at the beam head. The electric field of the shrinking beam is proportional to nr and because nr^2 = const., scales as $1/r$. Therefore, if the initial beam diameter was $\sim$ 10 cm and if the beam would shrink down to $\sim$ 0.1 cm, which appears a quite reasonable value, the electric field near the beam head would rise to $E_z \sim 3 \times 10^9$ Volt/cm, assuming that initially $E_z \sim 3 \times 10^7$ Volt/cm. If furthermore, the thusly selfconfined beam can be transported by the travelling magnetic wave over a distance of $\sim$ 10 km, the particles trapped in the collisionless shock wave at the beam head would reach $\sim 3 \times 10^{15}$ eV. Because of the large azimuthal magnetic beam field, the accelerated particles would be radially confined and thus be in a stable configuration.

The very high field at the beam head is sustained by a constant force along the entire beam. Since $E_r \propto 1/r$, the force for each beam cross section is $(\pi r^2)(E_r^2/8\pi)$ = const.. The radial electric pressure must then be only balanced by the pressure of the magnetic wave excerting its force on the rear part of the beam where its diameter is at its initial value.

Under normal conditions, if a beam shrinks to a smaller diameter, the IdL/dt voltage pulse (I beam current, L beam selfinductance), would slow down the beam. This however, appears to be avoided here, because the magnetic travelling wave which is pushing the beam, has plenty of magnetic energy and which it supplies to the beam.

VIII. STABILITY

As far as can be said, the electron gas is in a stable configuration. First, the curvature of the field lines with regard to the electron gas is convex. This prevents Taylor instabilities. Second, inequality (3.3) is always well satified. This prevents the diocotron instability. Furthermore, both inductive charge injection experiments[5,6], and experiments with trapped electron rings[7], have demonstrated stable electron gas configurations with number densities $n \simeq 10^{10}$ cm^{-3}, that is with number densities which are of the same order of magnitude as they must be established in the superaccelerator concept.

IV. CONCLUSION

Many proposals for the attainment of ultrahigh energies, up to 10^3 TeV, have been made over the years. However, only a very few of them may hold the promise to reach the luminosities required for the experiments to be done. The superaccelerator concept, drawing its energy from a travelling magnetic wave, operates with input energies larger by many orders of magnitude than any other concept hitherto proposed. Furthermore, all the physical principles underlying the superaccelerator concept, like the inductive charge injection concept and the stability of the thusly produced electron gas, have been established experimentally.

APPENDIX I

To describe the concept of inductive charge injection, a cylindrical coordinate system is introduced along the solenoid axis. From Maxwell's equation

$$\text{curl}\ \underline{E} = -\frac{1}{c}\frac{\partial \underline{H}}{\partial t} \quad , \qquad \text{(A.I.1)}$$

one obtains for a uniform, but time dependent, field $H_z = H_z(t)$:

$$E_\phi = -\frac{r}{2c}\dot{H}_z \quad . \qquad \text{(A.I.2)}$$

The azimuthal electric field E_ϕ together with the axial magnetic field H_z leads to the radial drift motion, of electrons injected from the thermionic emitter, given by

$$v_r = c\frac{E_\phi}{H_z} = -\frac{r}{2}\frac{\dot{H}_z}{H_z} \quad . \qquad \text{(A.I.3)}$$

The thusly produced buildup of electric charge leads to a radial electric field

$$E_r = 2\pi ner \quad , \qquad \text{(A.I.4)}$$

which in combination with H_z leads to the additional azimuthal drift motion

$$v_\phi = c\frac{E_r}{H_z} \quad . \qquad \text{(A.I.5)}$$

The superposition of both drift motions. (A.I.3) and (A.I.5), results in the spiraling trochoid shown in Fig. 1a.

The buildup of the electron gas is derived from the equation of continuity in cylindrical coordinates:

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rnv_r) = 0 \quad . \qquad \text{(A.I.6)}$$

Inserting (A.I.3) in (A.I.6) one finds:

$$\frac{\partial n}{\partial t} - \frac{1}{2r}\frac{\dot{H}_z}{H_z}\frac{\partial}{\partial r}(r^2 n) = 0 \quad . \qquad \text{(A.I.7)}$$

This equation has the general solution

$$n(r,t) = H_z f(r^2 H_z) \quad , \qquad \text{(A.I.8)}$$

where f is an arbitrary function to be determined by the boundary condition of the thermionic charge injectors at the inner wall of the solenoid. If r_0 is the radius of the wall, and ℓ the axial length of the electron gas, the total electron current radially injected is

$$I(t) = 2\pi r_0 \ell e n(r_0,t) v_r = -\pi r_0^2 \ell e f(r^2 H_z)\dot{H}_z \quad . \quad \text{(A.I.9)}$$

If in particular (dropping the index z on H) $I/\dot{H}$ = const., then f = const., and

$$I/I_0 = \dot{H}(t)/\dot{H}(0) \quad . \quad \text{(A.I.10)}$$

The average density of the injection current is

$$j_r = 2\pi e n v_r = 2\pi e n r_0/\tau_H \quad , \quad \text{(A.I.11)}$$

where τ_H is the rise time of the magnetic field. The inductive charge injection permits to make electron gases of different densities, but of course not of a density larger than

$$n = E/2\pi e r \simeq H/2\pi e r \quad . \quad \text{(A.I.12)}$$

From eq. (A.I.3) it follows that

$$-\frac{dr}{r} = \frac{1}{2}\frac{dH}{H} \quad , \quad \text{(A.I.13)}$$

which by integration yields

$$H(t)/H(t_0) = (r_0/r)^2 \quad . \quad \text{(A.I.14)}$$

This last equation determines how much the electron gas has been compressed during the rise of the magnetic field.

APPENDIX II

The axial electric field E_z at the end of an cylindrical electron gas of length ℓ and radius r is obtained by integration of the electric field coming from all cylindrical discs of the gas. One finds for the electric field contribution of one disc positioned at z':

$$dE_z = 2\pi d\sigma \left[1 - \frac{|z|}{(r^2 + z^2)^{1/2}}\right] \quad , \quad \text{(A.II.1)}$$

where $d\sigma = en(z')dz'$. For $n(z) = n$ = const. one has

$$E_z = 2\pi ne\left[\int_{-\ell}^{z}\left(1 - \frac{z - z'}{(r^2 + (z - z')^2)^{1/2}}\right)dz'\right.$$

$$\left. - \int_{z}^{\ell}\left(1 + \frac{z - z'}{(r^2 + (z - z')^2)^{1/2}}\right)dz'\right]$$

$$= 2\pi ne\left[2z + (r^2 + (z - \ell)^2)^{1/2} - (r^2 + (z + \ell)^2)^{1/2}\right] . \quad \text{(A.II.2.)}$$

At $z = \ell$, E_z has a maximum and which for $\ell >> r$ is

$$E_z \simeq 2\pi ner = E_r \quad . \quad \text{(A.II.3)}$$

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DISCUSSION

Reiser. There is a question of physics which I think you have overlooked. This is relevant if you want to inject electrons into a volume, as in HIPAC, or as Rostoker wants to do at Irvine, or as Trivelpiece tried to do a few years ago in electron rings. In these schemes the aim is to add electrons to keep the density of an expanding electron cloud constant. You have not put into your calculations the 'self potential' (which gives rise to the concept of limiting current) and I suggest that this prevents densities as high as 10^{14} or 10^{15} from being accumulated. I think that your entire concept is at fault from this point of view.

My second comment is that the virtual cathode formation which I discussed is likely to prevent your beam from propagating.

Winterberg. I will answer first the second question. There is a magnetic field which presses the whole thing together, your analysis of limiting current is not relevant.

Reiser. I don't agree.

Winterberg. This cloud has a very large γ.

Reiser. This is not relevant, just integrate the electric field from the cloud to the wall, and you will get enormous potential differences which prevent electrons from being injected.

Schumacher. These devices suffer from many instabilities. Flute instabilities, anisotropy, lower hybrid drift and many others. These have to be investigated.

Winterberg. These instabilities must, of course, be investigated. This type of scheme involves relativistic electromagnetic fluid mechanics and is not an ordinary type of beam, so the considerations are different.

Participant. The diocotron instability may be quite severe.

Winterberg. This is prevented because ω_p/ω_c is small.