EMITTANCE GROWTH IN RF LINACS*

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ABSTRACT

As the space-charge limit is approached, the current that can be accelerated in an rf linac and the output emittance that can be expected are discussed. The role of the envelope equations to estimate limits is outlined. The results of numerical experiments to explore general properties of emittance growth are given.

In the study of beam motions through accelerator structures, the useful analytical expressions obtained from the transverse and longitudinal envelope equations give information on the effects of parameter changes over wide ranges. The effect of current is included using a linearized treatment of the space-charge forces from an ellipsoidal uniformly charged beam bunch. Steady-state emittance is included, allowing matched parameters to be calculated which provide quite good results in numerical simulations. The current limits found from the envelope equations have been found to agree well with the saturated output current obtained in computer runs, which also show that a great deal of the input current is lost before saturation is reached. As the current saturates, the output emittance also grows to a level defined by the machine acceptance. This emittance limit, found from the computer code at the onset of saturation, is found to agree well with the emittance calculated using the envelope equation at a tune shift of $\sigma/\sigma_0 = 0.4$.

The envelope equations will not, however, account for the emittance growth due to nonlinear forces seen in all real machines, and there is no theory at present for these effects. Emittance growth occurs from nonlinearities in the rf gaps, space-charge forces, and coupling effects, and would have to be analyzed as a transient problem. A detailed theory is probably impossible, and one in convenient form even less probable, but it would be very useful to have

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a crude theory which would allow prediction of the size and rate of emittance growth under different parameter conditions.

Lacking any theoretical base for calculating emittance growth, except at the saturated limit, our understanding at this point comes from numerical experiments, and to a much smaller extent, from actual machines. Emittance growth is observed in all operating linacs, but production demands generally preclude machine development. It is clear that future high-performance accelerator development, such as for HIF, will require an understanding of these effects and therefore will also require accelerator experiments to be conducted--an exciting prospect.

Computer codes for linear accelerator beam dynamics have been extensively developed for over twenty years. The PARMILA code, used at LASL, LBL, and numerous other places, is the most complete and versatile code in the U. S. It can treat any type of particle in several kinds of accelerator structure, including the Alvarez, Wideröe, and RFQ. Input and output beam transport lines, including bunchers, can be handled. The code is fully six-dimensional, including nonlinear effects. Space charge can also be handled in 3-D, but present techniques are expensive, and most computations are done using a ring model on an r-z area-weighted mesh. Over the years, comparison of the code models to actual machine performance has been made whenever possible, including detailed analyses of measurement techniques. Several examples of exacting modeling studies have resulted in agreement with experimental results to a few percent. This work lends confidence to the use of the codes for detailed design work and for exploration of the causes and effects of beam characteristics such as emittance growth. The disadvantage of the numerical approach to basic studies is that the parameter space is large, making it difficult to infer general results.

Simulations (or numerical experiments) exploring the nature of emittance growth in rf linacs have been in progress at LASL for a number of years; results of some of the latest efforts were reported at the 1979 Linac Conference[1] and will be outlined below. The list of references from that paper is appended.[2-21]

The envelope equations can be expressed very compactly in terms of the phase advance per focusing period of the structure:

$$a^2 = \frac{(2n\beta\lambda)\varepsilon_t}{\sigma^t} \quad , \text{ and } \quad b^2 = \frac{(2n\beta\lambda)\varepsilon_\ell}{\sigma^\ell}$$

where a is the average beam radius over the focusing period, b is the beam bunch half-length, $(2n\beta\lambda)$ is the transverse focusing period length, ε_t and ε_ℓ are unnormalized transverse and longitudinal emittances, and σ^t, σ^ℓ are the transverse and longitudinal phase advances per transverse focusing period. We therefore decided to study emittance growth as a function of phase advance. We wanted to be able to measure the average phase advance of the particles in arbitrarily shaped bunches, as well as individual particle phase advances in the frame of the average. We also wanted to generate linacs having prescribed phase advances in both transverse and longitudinal, for arbitrary bunches. This is done using iterative, nonlinear least-squares techniques.

We have made two major sets of runs so far--both with $n = 1$ and $\varepsilon_t \sim \varepsilon_\ell/5$, and differing in that one set kept the accelerating gradient and synchronous phase constant at the value required to give the desired σ_0^ℓ at the first cell of each case, while the other set required the accelerating gradient to rise along the machine so that σ^ℓ was constant. The results were very similar in all qualitative aspects, the only real difference being more longitudinal emittance growth for the constant σ^ℓ case. We generated 7 linacs with zero-current phase advances of $\sigma_0^\ell \cong 42°$, and $\sigma_0^t = 50,70,90,100,110,120$, and $130°$. The tune of each linac was depressed by adding current, maintaining matched conditions, and the emittance growth* observed, with the results shown in Fig. 1. For each initial condition (points on the abscissa in Figs. 1a and 1b), current was added until the longitudinal stability limit was approached. The resulting traces in Fig. 1 show the emittance growth as the tune was depressed For the $\sigma_0^t = 50°$ case, we then raised the electric field to keep some longitudinal focusing, and raised the current further (open circles, Fig. 1). The longitudinal emittance growth is shown in Figs. 1c and 1d for all the transverse cases. These studies were done with short 20-cell FDFD Alvarez linacs. No particles were lost on any of the runs. Future work will address asymptotic behavior and other aspects.

We found a violent effect on the transverse (but not longitudinal) emittance in the linacs with $\sigma_0^t > 90°$, which appears to be analogous to the envelope instabilities studied in detail by Smith, Laslett and others for K-V beams in transport systems.[4-7] This is discussed further in the Conference paper--zero-current tunes above 90° should clearly be avoided.

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*Total effective emittance is found by fitting ellipses with the rms emittance parameters through each particle and taking the largest.

Below $\sigma_0^t = 90°$, the transverse emittance growth behavior indicates a preference for smaller beam radius, as would be expected to minimize the longitudinal-transverse coupling effect in the rf gaps. The other main feature is that the growth begins to increase rapidly, in both transverse and longitudinal dimensions, as the tune is depressed to about $\sigma/\sigma_0 = 0.4$ and below. This is also evidenced in the numerical runs by increasing difficulty in finding the best

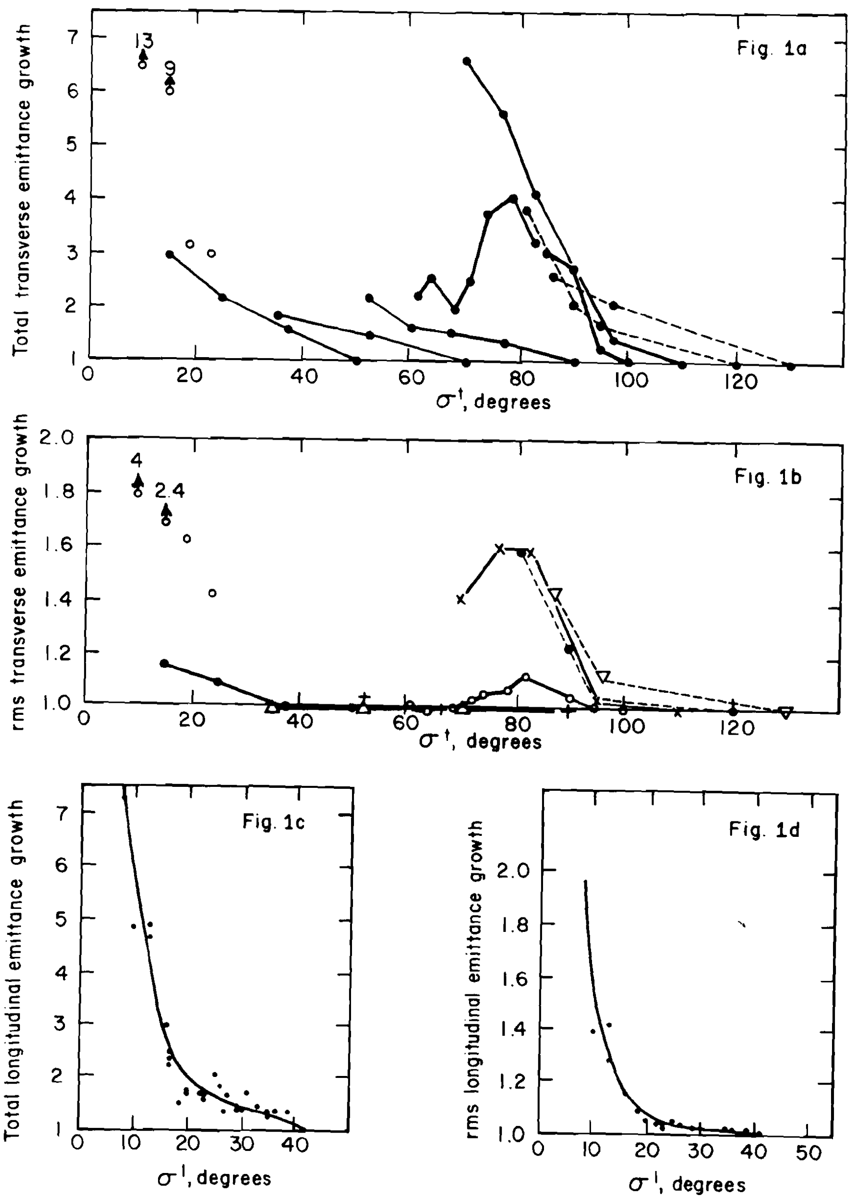

Fig. 1.
Emittance growth after 20 cells, as a function of tune shift from various initial zero-intensity phase advance per transverse focusing period.

input matching conditions to keep the beam matched in the machine. The effect of mismatching is discussed further below.

As the space-charge limit is approached, ($\sigma^t \rightarrow 0°$), we would start losing beam, as well as have the output emittance grow to the geometrically defined output limit. The growing sensitivity to matching will compound the problem. This evidence causes us to be suspicious of designs which claim operation "near the space-charge limit." This may be a matter of clarifying the definitions being used in each case - some are apparently unconcerned with beam loss and may be able to tolerate the resulting saturated output emittance. Others may use $\sigma/\sigma_0 = 0.4$ as their "space-charge limit." It does appear that by backing off on the current per channel, and/or by control of the frequency transition points in a funneled design for HIF, emittance can be kept in bounds. We need, however, to explore asymptotic behavior and the effects of frequency transitions in detail.

Since practical parameter choices for applications commonly result in $\sigma^\ell < \sigma^t$, we made a preliminary search for resonances of the $2\sigma^t = n\sigma^\ell$ type. Keeping $\sigma^t = 50°$, E_0/β was adjusted for constant σ^ℓ with n from 2 to 8. No differences in emittance growth were seen out to 60 cells, which is beyond the point to which the E_0 ramp could practically be sustained.

In considering other preliminary slices of the parameter space, we looked at some $\sigma^t/\sigma_0{}^t = 0.75$ cases in which the transverse emittance was reduced by another factor of 6 ($\varepsilon_t \sim \varepsilon_\ell/30$). Somewhat more transverse and less longitudinal growth was observed. Such transfers are commonly observed. In this case, far from the space-charge limit, the added growth was not large. The ratio of emittances is undoubtedly an important parameter, and may suggest multidimensional matching with equal emittances, especially if the parameters change along the machine.

We changed the frequency by a factor of five in each direction, keeping $a/\beta\lambda$, $b/\beta\lambda$, ϕ_s and injection energy constant, changing the accelerating gradient to keep $\sigma_o{}^\ell$ at 42° and changing σ_t to keep $a/\beta\lambda$ constant. This scaling reproduced Fig. 1 very closely.

We reran the $\sigma_o{}^t = 100°$ cases (which exhibited the instability in Fig. 1a) for input distributions uniform in 6-D, and Gaussian in 6-D (truncated at 3σ), keeping the rms emittances constant. The quadrupole strengths were those used to achieve a constant phase advance for the original distribution, approximately uniform in real space. The 6-D distributions grew more rapidly in the first two to three cells. From Cells 3-20 the growth in total emittance was very similar, but the rms growth for the 6-D cases was about double that of the 3-D case. The 6-D cases became somewhat mismatched as the beam progressed through the cells. We could reset the quads for each particular distribution; we expect that this would smooth but not necessarily reduce the growth--it may in fact increase (see below). The unstable mode evidenced in Fig. 1 is thus not the result of a particular particle distribution. Similar general influences of the distribution have also been observed for other choices of parameters. Figure 2 shows a typical redistribution of emittance. We conclude that the shape of the distribution does influence emittance growth, with greater effect as the beam brightness is increased, and with greater growth as the central density is increased.

Mismatched beams will be smeared by the action of nonlinear space-charge forces and eventually will assume an emittance congruent with the machine acceptance. Figure 3 demonstrates how emittance growth is affected by mismatching the input beam size up to a factor of $\sqrt{2}$ at injection, for the range of linac parameters we have been discussing. (Note that these cases have constant accelerating voltage gradient, E_0, rather than constant σ_0^ℓ as in Figs. 1 and 5.) At a given σ_0^t, the sensitivity to matching becomes more pronounced as the tune is depressed. As σ_0^t increases, the sensitivity for a given tune depression increases, an effect of the alternating gradient. The smaller absolute size of the beam (in one dimension) also becomes more important in terms of the required measurement resolution. For K-V beams, a "mismatching" instability mode has been identified[5] for $\sigma_0^t > 90°$; its analog for these distributions may be a factor here.

In the vicinity of the unstable mode, the behavior becomes somewhat unpredictable. For the parameters in Fig. 3, the betatron oscillations generally subjected the beam to a lower average μ_t (higher σ^t) over the 20 cells, sometimes resulting in less growth. The $\sigma^t/\sigma_0^t = 70°/110°$ case is particularly dramatic in this respect. The changes in transverse emittance growth from mismatching are generally rather uniform with respect to the shape of the distribution function, as shown in Fig. 4, or sometimes show more growth for higher percentages.

We then considered off-axis beams. For a single gap without space charge, Crandall[11] showed that the increase in total emittance is proportional to $(a^2 + d^2)$ if $|d| \leqslant a$, and 2da if $|d| \geqslant a$, where a is the half-width of the beam and d is the displacement of the beam center from the axis. The increase in rms emittance is proportional to $(1 + \overline{d^2}/\overline{a^2})$, where $\overline{a^2}$ denotes the mean square

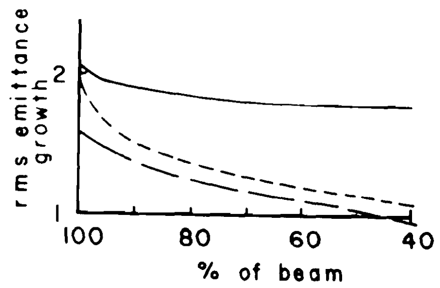

Fig. 2.
Typical variation in rms transverse emittance growth with input distribution.

— — Uniform in 3-D real space

------- Uniform in 4-D transverse space, separate 2-D longitudinal

——— Gaussian in real space, truncated at 3σ

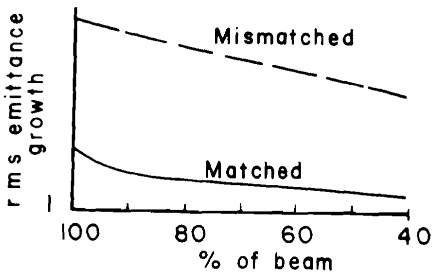

Fig. 4.
Typical redistribution of transverse emittance growth for mismatched beams.

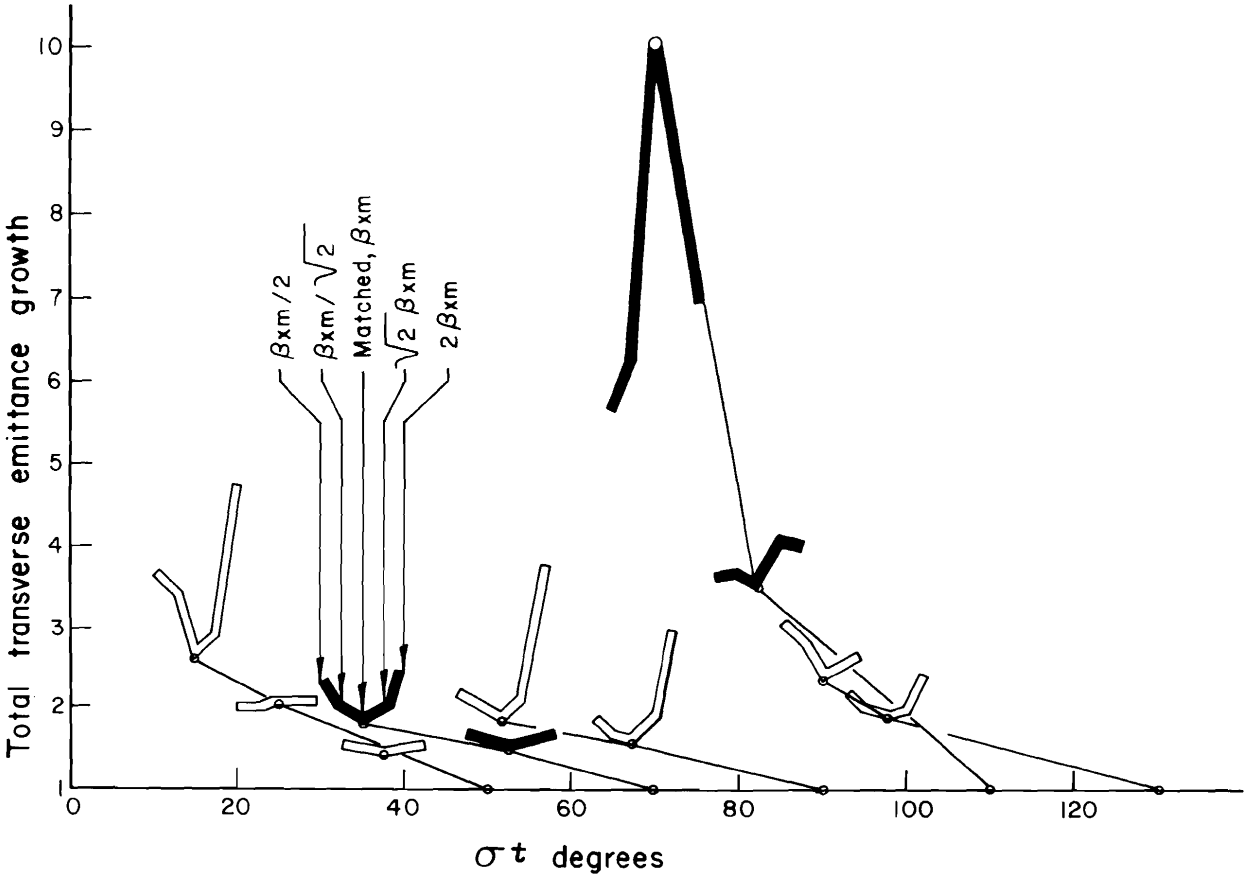

Fig. 3.
Sensitivity of transverse emittance growth to input matching. Linac has constant accelerating gradient E_0. At each tune, the smaller dimension of the input beam is varied by changing the input matching-ellipse parameter β by $\pm\sqrt{2}$ and ±2. The y and z inputs are matched. Growth is shown after 20 cells.

half-width of the beam. It is seen that the rms emittance grows relatively faster than the total emittance. The growth over n gaps will depend on what happens to the relative sizes of a and d. Figure 5 shows the emittance growths for (x-off-set/average input-beam radius) = 1.0 for five of the cases of Fig. 1. Again the sensitivity increases for larger tune depressions and for higher σ_0^t.

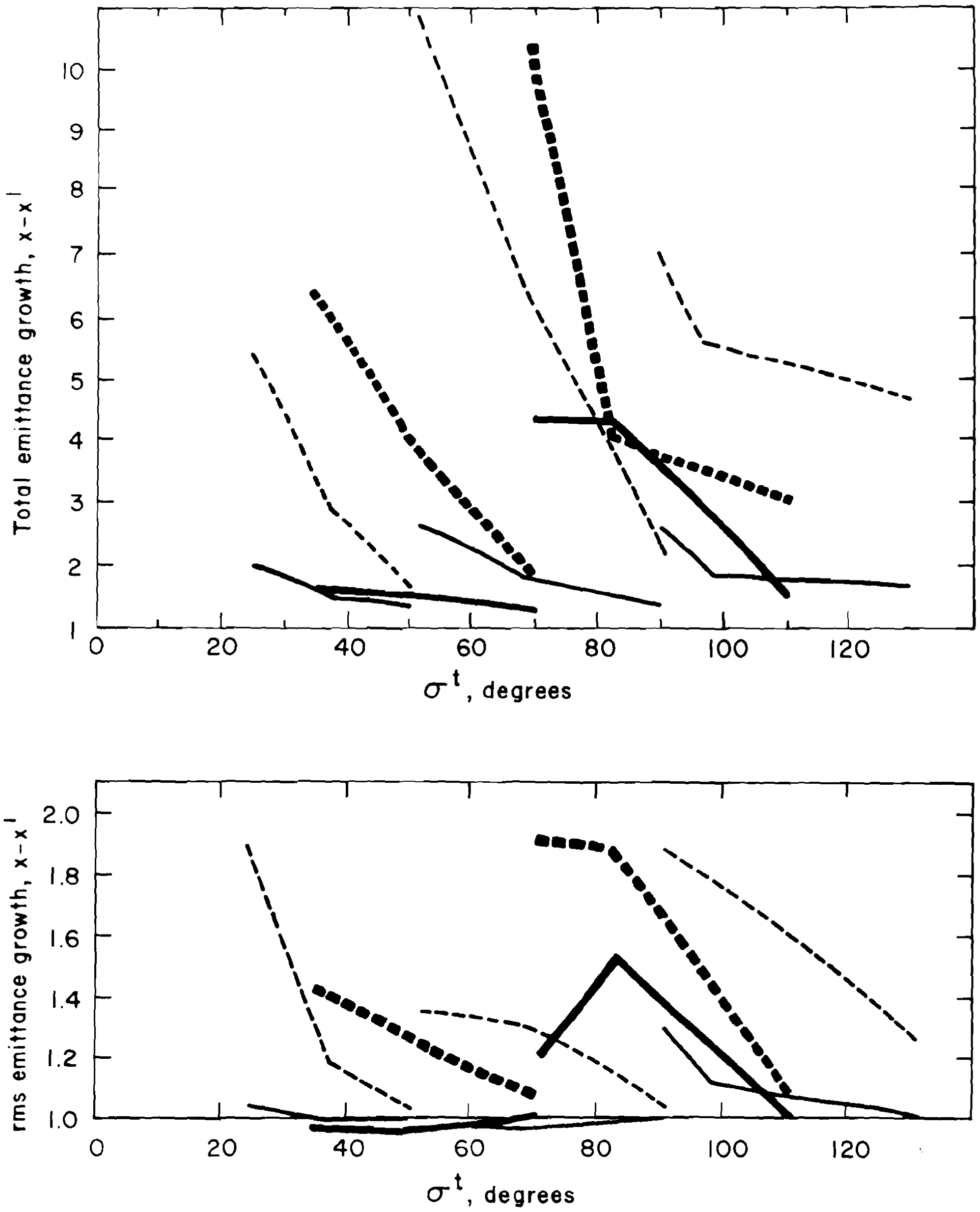

Fig. 5.
Sensitivity of transverse emittance growth, after 20 cells for 100% of beam, to horizontal offsets equal to the average input beam radius. Solid - no offset; Dashed - offset.

The interaction of off-axis beams with the envelope mode is complicated. The longitudinal emittance growth also is increased by the transverse oscillation. Figure 6 shows the typical redistribution that occurs in the transverse-phase space. This feature, and the contrasting signature of the mismatched beam, Fig. 4, can be valuable aids in machine tuning for detecting the presence of a centroid oscillation or mismatch.

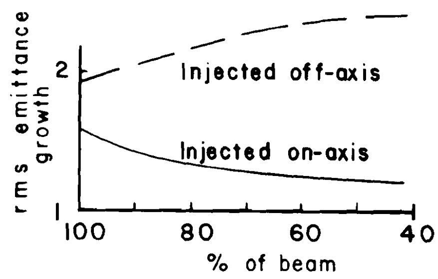

Fig. 6.
Typical redistribution of transverse emittance growth for missteered beams.

There are clearly many more things to be done. This initial work is encouraging in the sense that at least some scalings over wide ranges appear to produce the same emittance growth behavior. The effect of constraints on parameters as scaling is done must be investigated.

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