SPACE CHARGE NUMERICAL SIMULATION EXPERIMENTS

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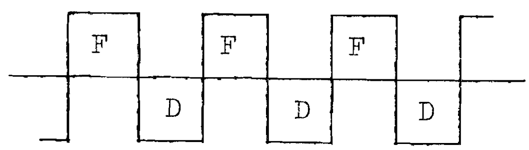
Comments are presented here on the work contained in a CERN report.[a] These comments are mainly oriented toward to comparison with the work of I Haber.[b]

The CERN work was performed about 10 years ago, at the same time as both numerical computations and experimental measurements of emittance growth in a linac were being carried out, in the hope of contributing to the understanding of the phenomena or of being the start of some theory.

1. Focusing System Considered

FD Focusing:



is defined by the betatron phase shift per focusing period at zero beam intensity (represented by and expressed in radians in the report; μ(radians) = σ_o (degrees).

The intensity of space charge is expressed by the dimensionless parameter:

$$\delta \cong \frac{q}{mv^2} \frac{\lambda_o I}{\varepsilon\beta\gamma}$$

where q is the charge, m is the mass, v is the velocity of the particle, λ_o is the betatron wavelength and I is the current (in coulombs per unit time).

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a) Etude numerique d'effets de charge d'espace en focalisation periodique, by P. M. Lapostolle, CERN report ISR/78-13.

b) Presented at HIF Workshop, Oct. 29-Nov. 9, 1979, Claremont Hotel, Oakland, CA (See present proceedings).

There is a direct connection between δ and σ/σ_0:

for $\delta = 1.5$ $\quad \sigma/\sigma_0 \cong 0.5$,

for $\delta = 2.0$ $\quad \sigma/\sigma_0 \cong 0.4$

(The latter value is the threshold of the first Gluckstern mode for a round beam, K-V distribution.)

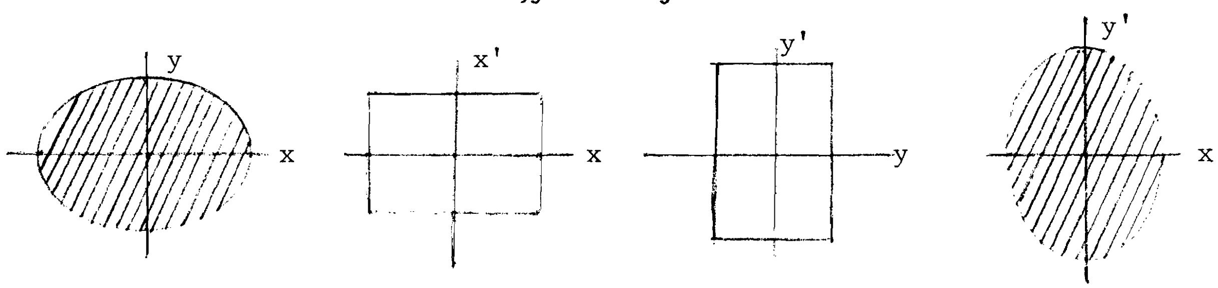
2. Particle Filling at the Input

In most of the runs, 2,500 macroparticles were used with a random uniform distribution inside a fixed boundary.

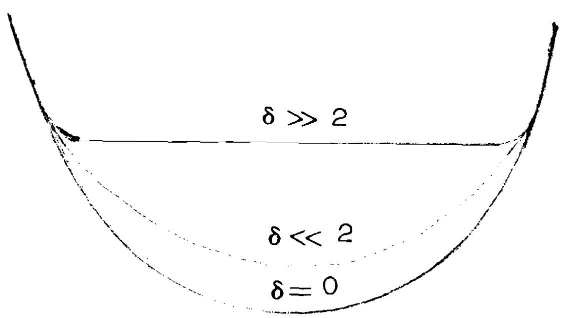
Such a fixed boundary only exists for continuous focusing and is then defined through a Bessel function. The boundary was approximated in actual cases by a properly sized (final adjustment empirical):

hyperellipsoid for $\delta < 2$,

uniform distribution in x,y and x'y' for $\delta > 2$.



In the case, $\delta > 2$, the potential well in which particles move is no longer paraboloidal, but tends to present a flat bottom where no restoring



force exists, except on the edges where the force extends over a thickness equal to the Debye length.

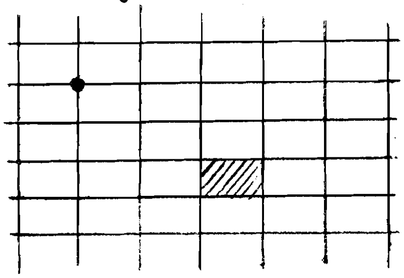
For $\delta = 2$, the Debye length is about equal to the beam radius.

For large δ, individual particles do not perform sinusoidal oscillations and their frequency spectrum may become wide. (This was one reason not to take, during this work, σ/σ_0 as a parameter but, rather, δ, along with the fact that it was intended to keep the possibility of considering unequal sizes in x and y).

Some other distributions have been tested: a) KV, to check the stability limit (when stable, the 4-D surface immediately become slightly wrinkled due to statistical fluctuations, but with a constant thickness). b) Gaussian and uniform-gaussian have also been tested to represent the output of a source.

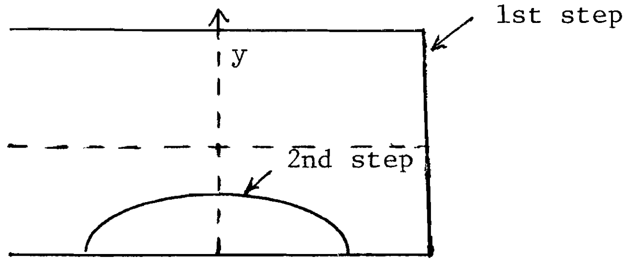
3. Methods of Computation

Space charge is computed in free space, but assuming a 4-fold symmetry (2-fold only in certain cases).



direct 10 x 10 box-to-nodes computation
or
FFT (16 x 16 or 32 x 11)
} + interpolation

The rectangular mesh is variable (sized to 1.1 outer particles) in order to keep maximum accuracy. (The potential on the boundary is computed from a series expansion of the outer potential: with a rectangular mesh of aspect ratio larger than 1.1, a 2-step computation is performed, the first is on a square mesh to compute the potential on a close boundary which would enter the circle of convergence of the expansion. For example, consider the 2-fold symmetry case illustrated here:



4. Results

Detailed results appear in the report quoted (a). Only a summary and some remarks are given here:

• For $\sigma_0 > 90^o$, even for $0.4 < \sigma/\sigma_0 < 0.6$, large emittance increases appear (as for KV). (The emittance level reached indicates an overshoot effect in this case).

• For $\sigma_0 < 90^o$ and large enough tune depression ($\sigma/\sigma_0 < 0.4$) slower increases are seen: the smaller the tune depression, the slower the increase, the smaller the σ_0, the slower the increase (at least in the cases computed and with 4-fold symmetry assumed in the computations; on the CERN linac, there was some experimental evidence of this effect).

• For continuous focusing, even with tune depressions as large as 95%, no deviation from the original distribution was observed (apart from the immediate statistical effect). However, no way was found to specify the absence of oscillations.

• It is mainly the outer part of the beam which is affected by higher order space charge effects; the central core almost keeps its 2-D phase space density. (The 4-D is obviously conserved.)

• It is not obvious how to determine from the simulation experiments whether or not a more stable distribution exists.

A few additional tests were performed with the simulation program:

1. Filamentation due to mismatch in a continuous focusing system, even

for cases with very large space charge,

$$x^2_{min} + x^2_{max} = ct$$

$$y^2_{min} + y^2_{max} = ct$$

The above leads, after damping has taken place, to an emittance increase. (Similar behavior was observed with a non-linear third-order field and no space charge.)

2. Some early simulation work with non-circular beams which was not followed up:

Starting with $\sigma_{ox} = \sigma_{oy}$ and unequal ε_{ox}, ε_{oy}, it was found that ε_x and ε_y become equal with their sum remaining constant (constant energy). Starting with $\sigma_{ox} \neq \sigma_{oy}$ (by a factor of $\cong 2$) nothing was seen, neither with equal initial energies in x and y, nor with equal emittances. (These runs were made when the program was not optimized, interpolations were crude and the accuracy was such that small variations were considered irrelevant.