FOCAL SPOT SIZE PREDICTIONS FOR BEAM TRANSPORT THROUGH A GAS-FILLED REACTOR

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ABSTRACT

Results from calculations of focal spot size for beam transport through a gas-filled reactor are summarized. In the converging beam mode, we find an enlargement of the focal spot due to multiple scattering and zeroth order self-field effects. This enlargement can be minimized by maintaining small reactors together with a careful choice of the gaseous medium. The self-focused mode, on the other hand, is relatively insensitive to the reactor environment, but is critically dependent upon initial beam quality. This requirement on beam quality can be significantly eased by the injection of an electron beam of modest current from the opposite wall.

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I. SUMMARY OF RESULTS

Results from calculations of focal spot size for heavy ion beams transported through a gas-filled reactor are summarized below. Three transport modes have been studied. The first is the ballistic mode where beams are injected through large portholes ($\sim$15 cm radius) to converge at the target. The second is the self-focused mode where beams are injected through small portholes (sub-cm), and propagate in their self-generated pinch field. The third mode is a variant of the second: an electron beam of modest current is injected from the opposite wall to aid the ion beam pinching.

The basic tools for our analyses are a set of 1-D codes which calculate the r.m.s. radius of a beam during its passage through the reactor. Analytic formulae for the r.m.s. radius of the beam in the target region were derived on the basis of several approximations which were shown by the 1-D codes to be reasonable in the range of parameters of interest for HIF. We have also checked 1-D code results against simulations and 2-D codes. These codes include only classical effects (no instabilities), and the predicted focal spot size represent the minimum disruption the beam is expected to experience.

Ballistic Mode

We find an enlargement of the focal spot size due to multiple scattering and classical self-field effects. The magnitude of these effects seem large enough to be of concern for target requirements. However, there is still sufficient uncertainty in the theory, particularly in connection with the calculation of beam-generated electrical conductivity, that firm predictions are not yet possible. Detailed calculations of the electrical conductivity with Boltzmann codes are in progress. Calculations up to this point indicate the sensitivity of final focal spot size to various beam and reactor parameters, and suggest several ways for minimizing the spot size enlargement:

1. Small reactors. The neck radius of the beam is a very sensitive function of the reactor size. To keep the neck radius small, the

total path of propagation in gas should be kept to a minimum ($z_o \leq 5$ meters). In addition, the neck radius is reduced by maintaining large portholes. The geometric factor R_w/z_o should be maximized, where R_w is the beam radius at reactor wall.

2. Proper choice of gas medium. To minimize multiple scattering effects, one is restricted to a low-Z material below a few torr. Self-field effects could be significantly reduced by choosing a good conducting gas, e.g., Ne and N_2 are better conductors than He or Li. One might also consider mixtures involving more complex gases, e.g., H_2O. Self-field effects are relatively insensitive to gas pressure.

3. Some pulse shaping in the radial direction would be helpful. Emittance growth is minimized by having a nearly square current profile. The magnitude of the current in a given beamlet has relatively little effect on the emittance increase in the classical calculations. Since the change in beam emittance in the reactor is independent of its initial value, little is gained by improving beam quality at entrance. Overall, the effects of initial beam parameters on focal spot size enlargement due to classical phenomena are secondary. One should note, however, that the beam energy cannot be lowered much below 10 Gev, since multiple scattering, which is proportional to β^{-4}, would become intolerable.

Pinched Modes

Whereas the reactor geometry and reactor environment are constrained in the ballistic mode of propagation, the self-focused mode is insensitive to the reactor size, and can accommodate a large range of gas types and pressure ($\sim$1 to 50 torr). In addition, the small portholes associated with the self-focused mode ease pumping requirements. The constraint for this mode is on beam quality. The self-focused beam characteristically expands at the beam head, and is pinched at the tail. The amount of beam head lost, and the final pinch radius at the tail are both sensitive to the initial beam emittance. Our code indicates that with initial emittance of 1 mrad-cm, the beam is pinched to a radius of $\sim$1 mm with only a small portion of the beam head lost. But the ion beam pinching rapidly degrades as the initial emittance is increased. Beam radius at injection is also an important parameter. The optimal beam size at entrance is a few millimeters. Generally, the more current there is in a given beamlet, the tighter is the final pinch. However, beam current is not as critical a parameter as the beam quality in determining the final beam radius.

The requirement on beam quality could be significantly eased by the injection of an electron beam from the opposite wall simultaneously with or prior to the injection of the heavy ion beam. Our code indicates that vast improvements in the ion beam pinching is achieved with a modest electron beam current. With a 3 kA/cm^2 e-beam, a 3 mrad-cm ion beam is pinched to slightly larger than 1 mm at the beam tail with little loss of the beam head. The stable propagation of such a e-beam has been demonstrated experimentally.[1]

II. THEORY

The basic ansatz for our beam transport studies is the envelope equation[2] which describes the evolution of the r.m.s. radius R of a beam segment (characterized by $x = \beta c\tau$, the distance from the beam head) as a function of z, the distance from the reactor wall:

$$\frac{\partial^2 R}{\partial z^2} = \frac{E^2}{R^3} - \frac{\overline{k_\beta^2 r^2}}{R} \tag{1}$$

The envelope equation is quite general, and is derived by taking appropriate moments of the particle equations of motion. The complications lie in evaluating the evolving emittance E, and the pinch force, characterized by the average betatron oscillation $\overline{k_\beta^2 r^2} = I_{net}/I_A$, where I_{net} and I_A are the net current and Alfven current, respectively. Since the physics of the ballistic mode and the pinched modes are quite different, they will be discussed separately.

Ballistic Mode

Only the effects of multiple scattering and classical pinch fields are included in the present study. The classical field effects arise from incomplete current neutralization, which results in self-magnetic fields that deflect beam particles.[3] Degradation of the focal spot size happens in two ways. First, when particles at different radial positions of a given beam segment experience different focusing forces (due to anharmonic pinch fields), there is a net increase in the emittance of that disc, which leads in turn to an enlarged neck radius. Secondly, the average pinch force experienced by various beam segments are different. The resulting variable focal lengths imply that as the beam impinges upon a target placed at any point in the reactor, most of the beam discs will be slightly out of focus.

For the converging beam, it is possible to derive an approximate solution for the neck radius of a beam segment from Eq. (1). It is given by

$$R_n = E_f \left(\frac{z_o}{R_w}\right) / \left[1+\delta\right]^{1/2} \tag{2}$$

where the final emittance is the initial emittance enhanced by multiple scattering and self-field effects

$$E_f^2 = E_0^2 + (\Delta E^2)_{scatt} + (\Delta E^2)_{field} \tag{3}$$

The geometric factor (z_o/R_w) in Eq. (2) determines the amount of emittance growth that can be tolerated to achieve a given neck radius. The parameter δ represents a correction due to gross pinching effects, given by

$$\delta = 2\left(\frac{z_o}{R_w}\right)^2 \overline{k_\beta^2 r^2}\, \ell n\left(\frac{Rw^2}{E_f\, z_o}\right) \tag{4}$$

Emittance increase due to multiple scattering is well known, and is given by

$$(\Delta E^2)_{scatt} = \sigma_{scatt}\, n_g\, R_w^2\, z_o/3 \tag{5}$$

where n_g is the gas density, and the factor of 1/3 results from the converging beam geometry. σ_{scatt} is an effective cross section for multiple scattering, given by

$$\sigma_{scatt} = \frac{8\pi\, e^4\, Z_i^2\, Z_g^2}{M^2\, c^4\, \beta^4\, \gamma^2}\, \ell n \tau \tag{6}$$

where Z_i and Z_g are the atomic number of the heavy ion and the gas, respectively, and M is the mass of the ion. $\ell n\tau$ provides the cutoff in momentum transfer.

To calculate the emittance growth due to self-fields requires detailed knowledge of the evolving velocity distribution of the beam particles. Explicit evaluation of this effect is not known in general. However, for the converging beam, it is possible to employ a perturbative approach upon the assumption that deviations from ballistic trajectories are slight. A simple formula then emerges:[4]

$$(\Delta E^2)_{field} = s\ (\overline{k_\beta^2 r^2})\ z_o^{\,2} \tag{7}$$

where s is a radial shape factor, given by

$$s = \left(\frac{\overline{k_\beta^4 r^2}\ R^2}{(\overline{k_\beta^2 r^2})^2} - 1 \right) \tag{8}$$

s = 0 if the net current profile is square, and is 1/45 for a parabolic profile and 0.15 for a Gaussian profile.

To complete the derivation of the neck radius, $\overline{k_\beta^2 r^2}$ must be evaluated. The pinch force results from the aggregate effect of the beam current and the plasma current. Initially, one might expect $\overline{k_\beta^2 r^2}$ to be a very sensitive function of the beam current and the degree of stripping of the heavy ion. However, in the highly current neutralized regime relevant for heavy ion fusion, the plasma current varies in such a way as to cancel both the beam current and the effective ion charge state dependence to leading order. The resultant $\overline{k_\beta^2 r^2}$ is sensitive only to atomic properties of the gas. It is given approximately by

$$\overline{k_\beta^2 r^2} = \frac{1}{4}\left(\frac{m}{M}\right)\left(\frac{\beta}{\gamma}\right)\left(\frac{\nu m}{c\,\sigma_{ie}}\right) \ell n\, \kappa \tag{9}$$

where νm is the momentum transfer rate, and σ_{ie} is the relativistic limit of the non-relativistic ionization cross section for the gas by electron impact. The dependence on beam particle current I_{bo}/e, the effective charge state of the ion Z, and the beam segment position x are contained in the single parameter κ. $\kappa = 1$ at beam head, and away from the beam head, ($x \gg x_r = \beta c \tau_r$, where τ_r is the rise time), we have

$$\kappa = \left(\frac{2eZ^2\ \sigma_{ie}\ I_{bo}}{m\ \nu_m\ \beta^2\ c^2} \right)^2 \left(\frac{x}{x_r} - \frac{1}{2} \right) \qquad x \gg x_r \tag{10}$$

The size of the neck radius of a beam segment is given by Eqs. (2) to (10). The position of the neck z_n can also be obtained from Eq. (1):

$$z_n = z_o - \overline{k_\beta^2 r^2}\left(\frac{z_o^3}{R_w^2}\right) \tag{11}$$

where z_o is the geometric focal point for a ballistic beam. Variations of the focal length with x comes through the factor $\overline{k_\beta^2 r^2}$.

Finally, the r.m.s. radius of the beam in the region around the neck is given by

$$R^2 = R_n^2 + \frac{E_f^2}{R_n^2}(z_n - z)^2 \tag{12}$$

Pinched Modes

The physics of the pinched mode is more complicated, and we do not have simple analytic solutions in this case. Detailed 1-D and 2-D codes are required to calculate the beam envelope and results of these calculations have been reported previously.[5] However, we will attempt to give a heuristic description of the underlying physics.

As a self-pinched beam moves towards the target away from the porthole, the main body of the pulse eventually settles into an equilibrium. The equilibrium radius is obtained by setting $\partial^2 R/\partial z^2 = 0$ in Eq. (1):

$$R_{eq} = \frac{E_f}{\sqrt{\overline{k_\beta^2 r^2}}} \tag{13}$$

The final emittance E_f is again the initial emittance enhanced by multiple scattering and self-field effects. However, emittance growth due to multiple scattering for a self-focused beam is small because $(\Delta E^2)_{scatt}$ is proportional to R^2. Hence, the self-focused mode is not confined to low-Z material and/or low pressure as is the converging beam mode.

Emittance growth due to self-fields can be minimized by injecting the beam at nearly the matched radius. Emittance growth due to mismatch has been studied with simulation codes,[6] and a phenomenological model which reproduces the observed effects was constructed. It is given by[4]

$$\frac{\partial E^2}{\partial z} = -\left[\frac{\overline{k_\beta^2 r^2}\; R^2}{\frac{1}{A}\left(\frac{E}{R}\right) + \frac{1}{B}\left(\frac{\overline{k_\beta^2 r^2}\;.\; R}{E}\right)}\right]\frac{\partial^2 R}{\partial z^2} \tag{14}$$

where A and B are numerical constants. For a slightly mismatched beam, the net emittance gain is predicted to be[4]

$$(\Delta E^2)_{field} = 2\; \overline{k_\beta^2 r^2}\left(R_{eq} - R_w\right)^2 \tag{15}$$

The calculation of $\overline{k_\beta^2 r^2}$ has been performed with detailed numerical codes.[5] To obtain a physical feel for the requirements for pinching, one could invert Eq. (13) to derive the required net current to attain a given target size

$$I_{net} = \frac{\beta\gamma Mc^3}{e\ Z}\ \overline{k_\beta^2 r^2} = \frac{\beta\gamma Mc^3}{e\ Z}\left(\frac{E_f}{R_{eq}}\right)^2 \tag{16}$$

As a numerical example, we consider a 10 Gev uranium beam with 1 kA particle current. Assuming Z = 80, the beam current is 80 kA, while the net current required to pinch a 1 mrad-cm beam to 1 mm [Eq. (16)] is only 3 kA. Hence, more than 95% current neutralization can be tolerated.

The amount of current neutralization is controlled by the magnetic decay time τ_m

$$\frac{I_{net}}{I_{beam}} \sim \frac{\tau}{\tau_m} \tag{17}$$

where τ_m is proportional to the electrical conductivity

$$\tau_m = \frac{4\ \pi\sigma\ R^2}{c^2} \tag{18}$$

The electrical conductivity σ is proportional to $Te^{3/2}/Z_g^{eff}$ where Z_g^{eff} is the effective charge state of the gas, while Te is the electron temperature. The electron temperature is determined by direct heating by ions and Joule heating, combined with thermal cooling in the radial direction. The general trend is for the conductivity, and therefore the magnetic decay time to increase with beam current. However, the increase is less than linear. Eq. (17) therefore predicts an increase in I_{net} with I_{beam} which is also less than linear. Hence, from Eq. (13), we have a decrease in R_{eq} with increased beam current at a rate which is slower than a square root. Thus, independent of the details of the fields and plasma channel, the strong dependence of the pinch radius on beam quality and its weak dependence on beam current can be qualitatively understood.

The theory of electron beam aided pinched propagation, together with numerical results from a parameter search, have been reported.[7] Calculations involving an e-beam current are quite similar to the pure self-focused mode calculations except that there is an additional contribution to $\overline{k_\beta^2 r^2}$ due to the external current. We would stress that the external current of the order of 3 kA/cm^2 is in itself far too weak to pinch the beam. The external current merely serves to hold together the ion beam head loosely. This leads in turn to rapid generation of self-fields.

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