Studies of a Beam Line for Transport to a Target

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Introduction

An example beam line for transport of a 10 GeV U^{+4} beam from a periodic lattice to a 4 mm target has been designed by K. Brown and J. Peterson for the zero space charge case.[1] Two variants of this line exist: a straight 60 m four quadrupole module without chromatic corrections, and a line with three of these modules including dipoles and sextupoles for chromatic correction.

As an approach to some of the problems of final vacuum transport of intense beams, this note describes calculations to modify the beam line of Brown and Peterson to take account of space charge, and to assess its performance with respect to momentum spread and intensity variation along the bunch.

Secondly a computational program is outlined for design of the sextupole chromatic correction system of beams with space charge, including a way to obtain their dispersion.

Beam Line Without Sextupole Corrections

The beam line to be considered is exhibited in Fig. 1 and Table I. On the left is a periodic FODO cell lattice, to the right a transport channel that focusses the beam onto the target. The cells are 4 m long, with 1 m long quadrupoles and 1 m long drifts. The gradients are such as to produce μ_0 = 60 degrees betatron phase advance at zero current.

The system is intended for the 10 MJ case: U^{+4} K.E. = 10 GeV, target radius a = 4 mm. The current I is 780 A (electrical), which depresses the phase advance to μ = 24°. (Many such beams are required to deliver the needed total energy).

Table 1 Lengths and quadrupole gradients of the last half cell and final focus transport line for 10 GeV, U^{+4} beams at three current levels.

		Gradient (T/m)		
Element	Length (m)	I = 0A	390A	780
		Q = 0	3.194×10^{-4}	6.387×10^{-4}
QD/2	0.5	-34.44	-34.44	-34.44
L	1.0			
QF/2	0.5	34.44	34.44	34.44
L1	2.0			
Q1	1.0	1.84	-5.55	-14.86
L2	13.0			
Q2	1.0	0.0	5.33	9.53
L3	11.5			
Q3	1.0	-3.64	-5.50	-7.16
L4	10.0			
Q4	1.0	0.0	7.39	4.95
L5	8.5			
Q5	2.0	5.91	5.96	5.14
L6	2.0			
Q6	2.0	-7.88	-7.58	-8.53
L7	5.0			

The following calculations were performed, using the SYNCH computer program:

1. Periodic solutions to the equations for the beam envelope radii a_x, a_y

$$\frac{d^2 a_{x,y}}{ds^2} + K\, a_{x,y} - \frac{E^2_{x,y}}{a^3_{x,y}} - \frac{Q}{a_x + a_y} = 0 \quad (1)$$

were obtained for the cells. Here $K = (dB/dx)/B\rho$, $E_{x,y}$ are the emittance areas divided by π,

$$Q = \frac{4r_p q^2 N}{A\beta^2\gamma^3} = 1.288 \times 10^{-7} \frac{qI}{A(\beta\gamma)^3} \quad (2)$$

where N is the number of ions per unit length and I is the current in amperes.

2. The tune depression is calculated as follows. Single particle rays are integrated using

$$\frac{d^2(x,y)}{ds^2} + \left(\pm K - \frac{Q}{A_{x,y}(A_x + A_y)}\right)(x,y) = 0,$$

cell transfer matrices are obtained from the final values of rays with initial values (1,0) or (0,1), and then the tunes are obtained from the matrix traces.

3. The envelope equations (1) are tracked through the channel to the target, using the cell periodic solutions as starting conditions. An optimizing routine adjusts the gradients to obtain the desired conditions at the target:

$$a_x = a_y = 4 \text{ mm}, \quad a_x' = a_y' = 0$$

Additional constraints were added to limit the maximum beam radii. Two quadrupoles were added, so that the maximum radius along the channel increased only from 25 cm in the zero-current line to 35 cm in the modified line with 780 A.

The principal results are shown in Table II.

Chromatic Behavior of the Example System

An approximate estimate of the increase in spot size to be expected from the final doublet

Table II Properties of 10 GeV, U^{+4} beams in FODO cells and final transport channel at three current levels

Current (electrical)	I	0	390	780	Amp.
Space charge parameter	Q	0	3.19×10^{-4}	6.39×10^{-4}	
Cell phase advance	μ	60	36.7	24.4	deg.
Cell beam radius max.	a_{max}	1.99	2.51	3.06	cm.
Cell beam radius min.	a_{min}	1.20	1.55	1.91	cm.
Channel phase hor.	μ_x	180	115	91	deg.
Channel phase vert.	μ_y	180	152	156	deg.
Channel radius max.	a_x	24.8	28.7	35.5	cm.
Channel radius max.	a_y	16.1	30.0	30.4	cm.
Target beam radius	a^*	0.40	0.40	0.40	cm.
Relative velocity	β		0.291		
Relative momentum	$\beta\gamma$		0.304		
Magnetic rigidity	$B\rho$		56		T-m
Emittance/π	E		60		mm-mrad

ignoring space charge may be obtained using the thin lens approximation. If the beam has a waist at the target and is nearly parallel at the doublet entrance, it can then be shown* that the β-function at the target depends on momentum deviation $\delta = \Delta p/p$ as

$$\beta_0(\delta) \simeq \beta_0 \left[1 - 4\delta + (2M_{12}\ \delta/\beta_0)^2\right] \qquad (5)$$

where M is the transfer matrix from doublet entrance to target, $\beta_0 = r_0^2/\varepsilon$, and

$$M_{12} = L + \ell + \sqrt{\ell(L + \ell)} \qquad (6)$$

Here L is the distance from the final lens to the target, and ℓ is the doublet separation. If the maximum δ is taken to be that which doubles β (40% increase in r_0), and we take the mean of the horizontal and vertical values of M_{12}^2 we get

*Specifically one sets $M_{11} = 0$ in both planes. The δ^2 term is larger than the δ term if $\delta > (\beta_0/L)^2 \sim 10^{-3}$ for the example beam line.

$$\left(\frac{\Delta p}{p}\right)_{max} = \pm \frac{r_0^2}{2\varepsilon\sqrt{(L + \ell)(L + 2\ell)}} \qquad (7)$$

For the example beam line L = 6 m, ℓ = 2 m and $(\Delta p/p)_{max} = +0.011\%$

To estimate the chromatic effect of the example beam line more accurately, beam envelopes were tracked through the system for off-momentum particles. In this calculation K in Eq. (1) is made proportional to $(1 + \delta)^{-1}$ and Q to $(1 + \delta)^{-2}$. The calculation is inexact in that the denominator of the space charge term should contain the effective size of the beam due to superposition of the different momentum components. However the space-charge term is more important than the emittance term at places where the beam sizes are large, and there the dependence of a_x and a_y on momentum is relatively small. Hence the results, shown in Table III, may not be too inaccurate.

Table III Effect of Momentum Error on Channel Performance

Momentum deviation	$\Delta p/p$	-0.01	0	0.01	
Q = 0:					
Radii at channel entrance	a_x	.020	.020	.012	m
($a_x'=a_y'=0$)	a_y	.012	.0120	.012	m
Radii at target position:	a_x	.0072	.0040	.0073	m
	a_y	.0044	.0040	.0043	m
Waist position (from target)	S_x	-0.04	0	+0.04	m
	S_y	-0.1	0	+0.1	m
Radii at waist	a_x	.0038	.0040	.0043	
	a_y	.0041	.0040	.0040	
Q = .0000639:					
Radii at entrance	a_x	.0307	.0306	.0306	
	a_y	.0190	.0191	.0192	
Radii at target	a_x	.0235	.0040	.0199	
	a_y	.0054	.0040	.0037	
Waist position	S_x	-1.0	0		
	S_y	-0.1	0	+0.1	
Radii at waist	a_x	.003	.004		
	a_y	.005	.004	.003	

It was assumed that each momentum constituent was matched in the cells -- this accounts for the variation of radii a_x, a_y at the channel entrance. For the zero space charge case the horizontal beam size has increased from 4 to 7 mm at the target for ± 1% momentum error, the vertical size hardly at all. The spot area a_x a_y increases by a factor of 2, while from Eq. (5) one predicts a factor 1.4. For 780A the area at the target is increased by a factor of six.

Thus the theoretical momentum acceptance for zero current is about ± 1% , for the channel example 0.7% at zero current and about 0.2 at 780A. The increased sensitivity at 780A may not be directly due to space charge, but rather to increased chromaticity, which varies as $\sum K_i\beta_i$. It may be possible to reduce this effect by more careful design of the channel.

Sensitivity to Current Level

The change of the beam radii at the target as a function of current was calculated by tracking envelopes through the channel whose quadrupole gradients were fixed at values to focus the 780A beam to 4 mm radii at the target. As with the momentum dependence calculations, matched envelopes in the periodic lattice were taken as initial values in the channel. The results, Table IV, show higher tolerance to current than to momentum variation. For example the current must be depressed about 20% to produce a 50% growth of spot-radius.

Evaluation of Example Beam-Line

The resulting beam-line appears satisfactory in that it produces the desired spot size, the beam dimensions are reasonable, and it is not too sensitive to current variations. However its momentum acceptance is very small and it may behave badly with more realistic distributions. Work is now in progress to produce a superior channel. The main ideas are to increase the density of quadrupoles so that their focussing effect will dominate the space-charge effect, to produce beam envelopes that are on the average more symmetrical, and to avoid very small intermediate waists, such as that near Q3.

Dispersion in Beam with Space Charge

A beam line with sextupole corrections was also calculated by Brown and Peterson.[1] It

Table IV Dependence of Spot-Size at Target on Current
$Q_0 = 6.387\times10^{-5}$ ((I_0 = 780A)

Q/Q_0	a_x(mm)	a_y(mm)
0	64.1	26.0
0.1	51.5	27.1
0.2	40.1	27.7
0.3	30.2	24.8
0.4	22.5	19.0
0.5	16.9	14.0
0.6	12.6	10.3
0.7	9.12	7.69
0.8	6.42	5.88
0.9	4.62	4.66
0.925	4.35	4.45
0.95	4.16	4.26
0.975	4.05	4.12
1.0	4.02	4.00
1.025	4.06	3.92
1.05	4.17	3.87
1.075	4.33	3.85
1.10	4.53	3.86

consists of three modules like the channel shown in Fig. 1 with the center one reflected. Dipoles and sextupoles are placed in the first two modules for chromaticity correction. In this section an approximate method to calculate dispersion in beams with space charge will be outlined, which should be useful in calculating sextupole corrections.

Suppose each momentum component of the beam has a K-V distribution with ellipse axes a_x, a_y. Let there be a rectangular distribution in momentum deviation $\delta = \Delta p/p$, $-\Delta < \delta < \Delta$. If the dispersion (to be calculated) is η, then the horizontal beam dimension will be

$$A_x = a_x + \eta \cdot \Delta \tag{8}$$

We treat the resulting beam as a uniform-density ellipse with axes A_x, a_y. Single particles will then follow the equations

$$\frac{d^2X}{ds^2} + \left(K - \frac{Q}{A_x(A_x + a_y)}\right) X = \frac{\delta}{\rho} \tag{9}$$

$$\frac{d^2Y}{ds^2} + \left(-K - \frac{Q}{a_y(A_x + a_y)}\right) Y = 0 \tag{10}$$

where ρ is the local radius of curvature and X, Y are taken relative to the center of the beam. Decomposing the horizontal motion relative to the center of each momentum constituent,

$$X = x + \eta\, \delta\,, \tag{11}$$

gives the following equations in place of Eq. (9);

$$\frac{d^2x}{ds^2} + \left(K - \frac{Q}{A_x(A_x + a_y)}\right) x = 0 \tag{12}$$

$$\frac{d^2\eta}{ds^2} + \left(K - \frac{Q}{A_x(A_x + a_y)}\right) \eta = \frac{1}{\rho} \tag{13}$$

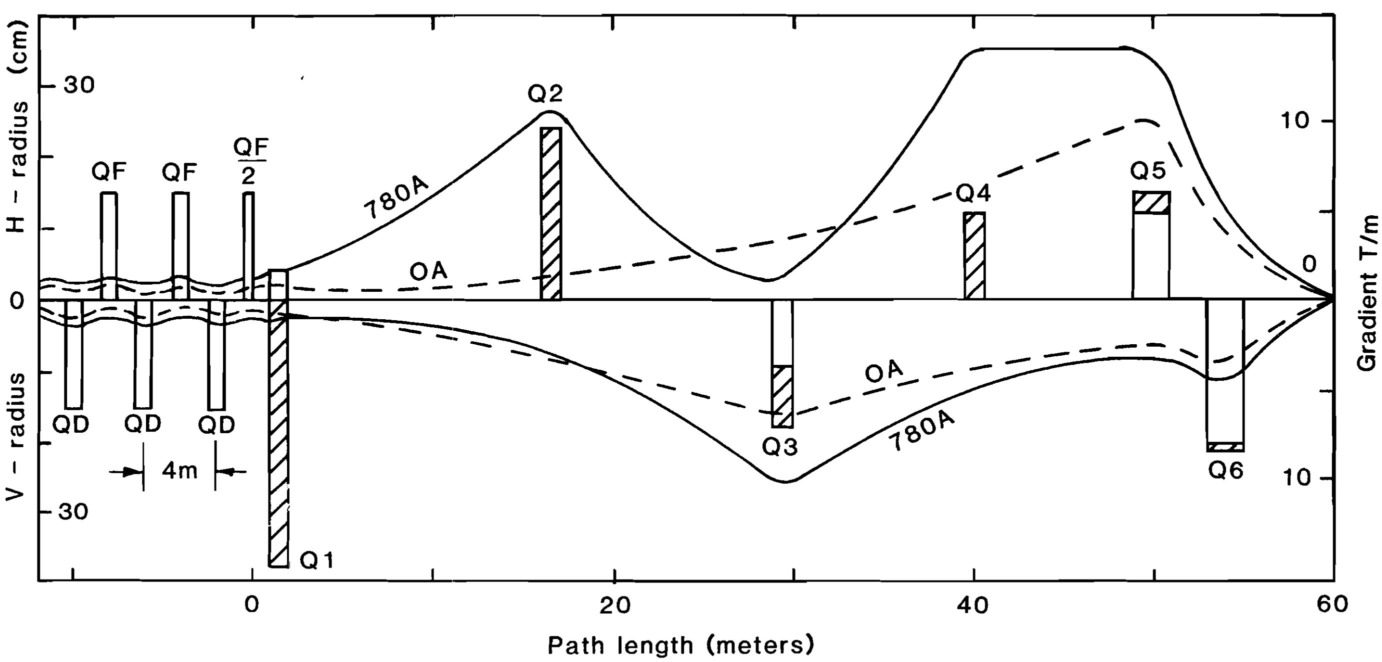

Fig. 1 Final Transport System for 10 GeV, U^{+4} Beams. Beam envelopes and quadrupole gradients designed for zero currents (dashed curves, open bars) and for 780A (solid curves, cross hatched bars). Beam is focussed to 4mm radius waist at 60m, the target position.

Equations (10), (12) lead to the envelope equations

$$\frac{d^2a_x}{ds^2} + Ka_x - \frac{Qa_x}{A_x(A_x + a_y)} - \frac{\varepsilon_x^2}{a_x^3} = 0 \qquad (14)$$

$$\frac{d^2a_y}{ds^2} - Ka_y - \frac{Q}{A_x + a_y} - \frac{\varepsilon_y^2}{a_y^3} = 0 \qquad (15)$$

The beam evolution is traced by simultaneously integrating Eqs. (13), (14), (15) with A_x given by (8). Single particle behavior can be obtained by also integrating Eqs. (10) and (12). For a periodic lattice one must find periodic solutions in a_x, a_y, and η.

It is assumed in the above derivation that the momentum of individual particles does not change significantly and that particles do not move longitudinally to parts of the bunch with very different momentum spread Δ during the period of interest.

After carrying out the integrations, one should estimate the true charge distribution resulting from superposition of the beamlets and estimate the non-linear forces arising from this distribution. If these do not seem serious, it may be possible to apply sextupole chromatic corrections. To calculate these, appropriate non-linear kicks can be applied at the sextupole locations and a set of single rays traced through the system corresponding to a small δ value together with a non-zero initial value of either x, x', y, or y'. The initial and final values of these rays give the second order transport coefficients T_{ij6}, where i and j = 1, 2, 3, 4. The correction consists of reducing the largest of these coefficients to zero.

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Reference

1. K.L. Brown and J.M. Peterson, "Chromatic Correction for the Final Transport System", these proceedings.