SPHERICAL ABERRATION FROM NON-UNIFORM SPACE-CHARGE

J. D. Lawson
Rutherford Laboratory

As an illustrative example of the effect of non-uniform space-charge, we calculate the spherical aberration coefficient of a lens, which has its focusing power reduced by a space-charge distribution that is slightly non-uniform,

$$n = n_0 - \Delta n \frac{x^2}{a^2}, \quad x < a \qquad (1)$$

$$= 0 \qquad , \quad x > a$$

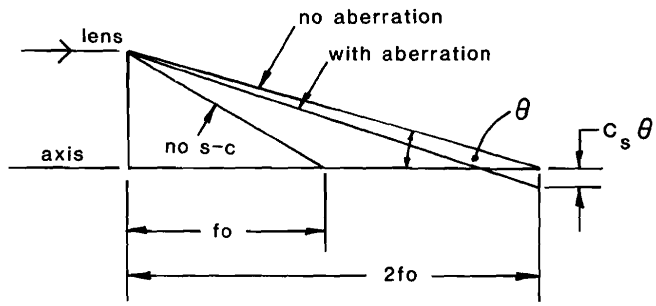
where n is the density and a is the beam radius.

For a density distribution of the form given in equation (1), the outer rays will be brought to a focus before the inner rays, since the defocusing effect of the space-charge will be less than it would be if Δn were zero. We consider a system such that, when $\Delta n = 0$, the space-charge just doubles the focal length of the lens. This represents a "tune depression" factor of 1/2. The space charge is assumed localized at the lens.

If the lens focal length is f_0, then the "focal length" of the space-charge contribution is $-2f_0$ when $\Delta n = 0$, and for a parallel beam of radius a incident on the lens, all rays will meet that axis at a distance $2f_0$ from the lens at an angle $a/2f_0$. For finite Δn, the outer rays will travel at an increased angle, $(a/2f_0)(1 + \Delta n/2n)$. At the focus for rays near the axis, the spot size will be $a\Delta n/2n$. Setting this equal to $C_s\theta^3$, where C_s is the spherical aberration coefficient of the combined lens and space-charge, we get

$$C_s = \frac{(2f_0)^3}{a^2} \frac{\Delta n}{2n} . \qquad (2)$$

The figure below illustrates the argument.



We estimate this effect first for a periodic FODO lattice. For thin lenses with spacing L and tune 60° without space charge, f = L, and the maximum β function value is 3.46L. If space charge depresses the tune to 24° then $\beta = 8.2L$, and $a = \sqrt{\beta\varepsilon}$ where ε is the emittance/π. From equation (2) we then have

$$C_s = \frac{L^2}{2\varepsilon} \frac{\Delta n}{n} \qquad (3)$$

Taking L = 5m, $\varepsilon = 6 \times 10^{-5}$ mrad and $\Delta n/n = 0.1$, we get $C_s = 2 \times 10^4$ m/rad^3 $= 2 \times 10^{-2}$ mm/mr^3. Since $a \approx 50$ mm and $\theta = a/\beta \approx 1$ mr, the effect here is negligable.

Secondly, for the final focus case the circle of least confusion is $1/4\ C_s\theta^3 = 1/4\ a(\Delta n/n)$. This suggests a serious effect, (several mm even for $\Delta n/n = 0.1$). This conclusion is pessimistic, however, since the space-charge is not concentrated at the lens; it becomes most effective only near the focus, where small angular deflections have relatively little effect. Nevertheless, it is clear that future consideration is needed.