Coherent Space Charge Instability of a Two-Dimensional Beam

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Space charge induced coupling between different degrees of freedom can be responsible for emittance growth or transfer of emittance from one phase plane to another. The underlying instability mechanism is coherent if it depends primarily on the electric field due to the collective motion, in contrast with an incoherent growth that can be described as a single particle effect. Such coherent phenomena become increasingly important if one studies beams where the space charge force is no longer negligible compared with the external focusing force, as in Heavy Ion Fusion applications.

In this note we present results of analytic calculations on the coherent space charge instabilities of a beam with initial Kapchinskij-Vladimirskij distribution and unequal emittances, rsp. average energy in the two transverse phase planes x-p_x and y-p_y. We note that in computer simulation calculations evidence has been given for rapid emittance transfer to occur if the initial emittances ε_x, ε_y are noticeably different [1,2]. We have not attempted to make a quantitative comparison of our results with those from computer simulation. The main purpose of this study is to give some insight into the instability mechanism, the dimensionless parameters that characterize the situation and the growth rates one may expect to find.

Dimensionless Parameters

For the round beam case with equal emittances, which was studied by Gluckstern [3], stability is described by one single parameter, the space charge depressed tune ν/ν_o. For the anisotropic beam three parameters are required, instead, which we have chosen to be

1 P. Lapostolle, this workshop
2 R. Chasman, IEEE Trans.Nucl.Sci., NS-16, 202 (1969)
3 R.L. Gluckstern, Proc.of the 1970 Proton Lin.Acc.Conf., Batavia, p.811

$\alpha \equiv \frac{\nu_y}{\nu_x}$ tune ratio; $I \equiv \frac{\omega_p^2}{\nu_x^2}$ intensity; $\eta \equiv \frac{a}{b}$ excentricity

with $\omega_p^2 = 4q^2N/(v^2ab)$ the "plasma frequency" and a, b the x, y semi axi of the elliptic cross section uniformly charged unperturbed beam. The tune depressions are found as

$$\frac{\nu_x^2}{\nu_{ox}^2} = \frac{1 + \eta}{1 + \eta + I}, \quad \frac{\nu_y^2}{\nu_{oy}^2} = \frac{1 + \eta}{1 + \eta + I\eta/\alpha^2}$$

with ν_{ox}, ν_{oy} the zero intensity tunes.

Results

The dispersion relation has been calculated by integrating the Vlasov equation along unperturbed orbits (details see elsewhere[4]). Eigenmodes of the perturbed electrostatic potential V can be written as finite order polynomials in x, y with a distinction between "even" and "odd" (as describing the symmetry in the angle if elliptic coordinates are introduced):

even	odd	mode
x	y	dipole
$x^2+A_2y^2$	xy	quadrupole (envelope)
$x^3+A_3xy^2$	$y^3+B_3x^2y$	sextupole
$x^4+A_4x^2y^2+C_4y^4$	$x^3y+B_4xy^3$	octupole

etc.

Anisotropy lowers the stability threshold considerably. Instability occurs as a result of (linear) mode coupling or depression of ε^2 to zero and negative values. Modes that are stable in a round beam for arbitrary tune depression, like the sextupole mode (Fig.1a), can become unstable with anisotropy. A general rule of stability in terms of η, α has not been found. It may be of interest to note that we have calculated a number of cases and found that imbalance in energy as well as in emittance can give rise to instability. As a general feature, however, instabilities with noticeable growth rates were found only if the tune in one of the directions is sufficiently much depressed. A lower bound of .75 for the two depressed tunes seems to be quite safe from this point of view. Future work should consider non-KV distributions and r-z ellipsoidal geometry.

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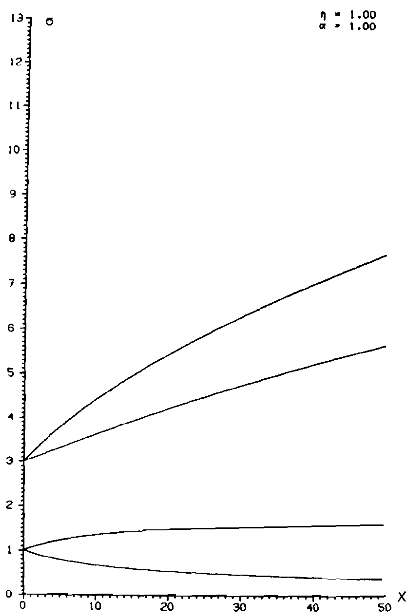
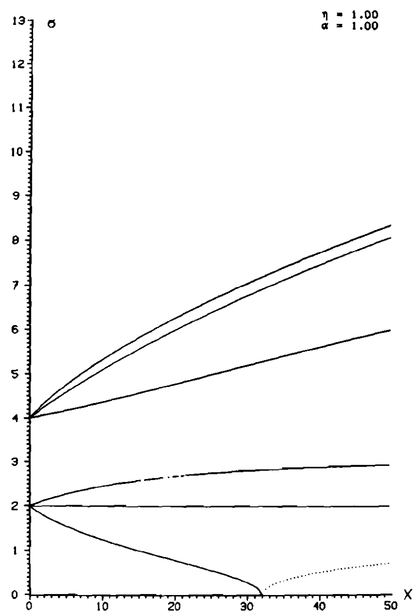
4 I. Hofmann, to be published

Figures:

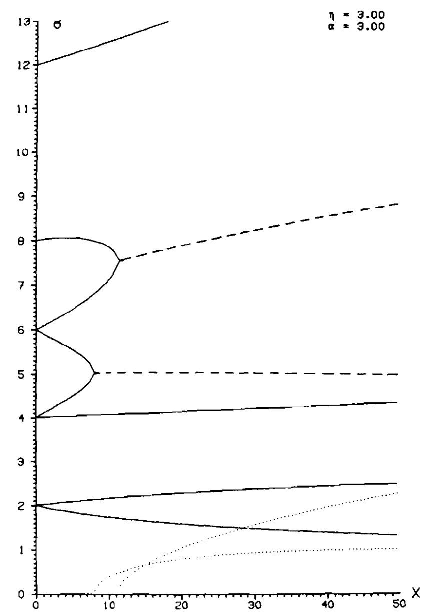
The normalized mode frequency $\sigma \equiv \omega/\nu_x$ is plotted against intensity I. In case of complex solutions Re σ is shown by a dashed line and Im σ (instability growth rate) by a dotted line. Examples are given for even modes and for different sets of α, η, which are readily converted into the ratio of emittances, $\epsilon_x/\epsilon_y = \eta^2/\alpha$, and the ratio of single particle energies $E_x/E_y = \eta^2/\alpha^2$.



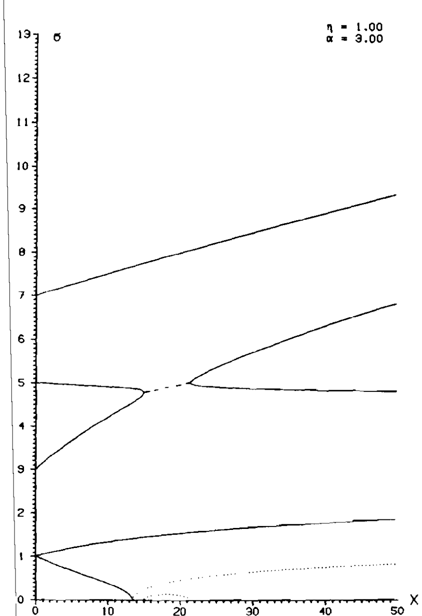
a) sextupole
(round beam)



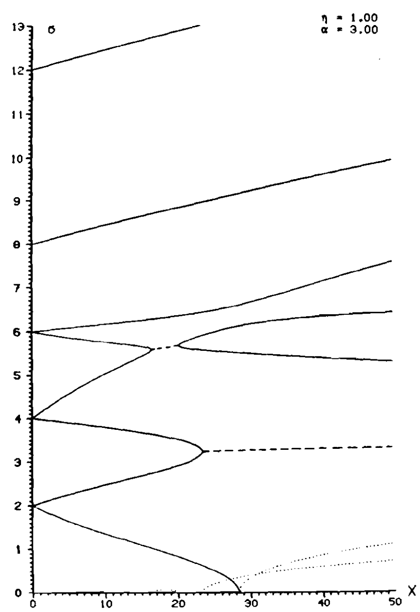
b) octupole
(round beam)



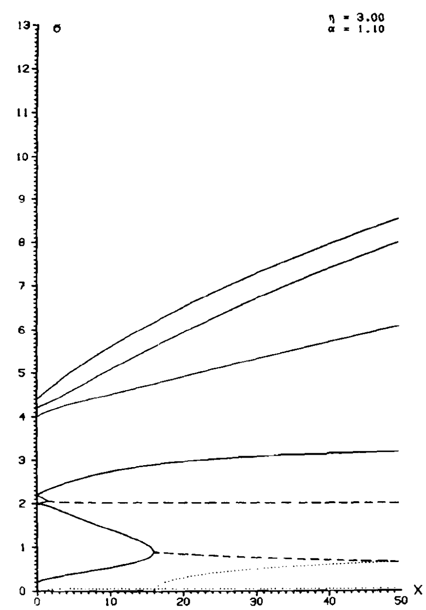
c) octupole



d) sextupole



e) octupole



f) octupole