# CHROMATIC CORRECTION FOR THE FINAL TRANSPORT SYSTEM

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### I. INTRODUCTION

The final transport and focusing of the heavy-ion beam onto the fusion pellet in vacuum is complicated by several non-linear effects --namely, chromatic (momentum dependent) effects, geometric aberrations, and space-charge forces. This paper gives an example of how the chromatic effects can be nullified, at least to second order. Whether third- or higher-order terms are important is not yet clear. Space-charge effects are important but are not considered here.

#### II. THE NEED FOR A CHROMATIC CORRECTION SYSTEM

Consider the problem of bringing a 10 GeV U<sup>4+</sup> beam from the end of a FODO-type transport lattice and focusing it onto a 4 mm radius pellet. Let the emittance in each transverse plane be 60  $\pi$  mm-rad. We take for the values of the betatron functions at the end of the transport lattice  $\beta_x = 6.93$  m and  $\beta_y = 2.31$  m (corresponding to a 4-meter FODO period with 60 degrees phase shift per period). The matched beam spot at this point is 20.4 mm (horizontal radius) by 11.8 mm (vertical radius).

To match this beam to the 4 mm pellet  $(\beta_x = \beta_y = 0.27 \text{ m})$  at the center of a 5-meter-radius reaction chamber, it is convenient to use a half-wave optical system consisting of four quadrupole magnets as shown in Figure 1. This focusing array was devised using the TRANSPORT program.<sup>1</sup>



Figure 1. A Half-wave Section Matching the Beam from a 4-meter FODO Transport System to a Round, 4-mm-Radius Target Pellet

This half-wave matching section, although achromatic in first order, has significant second-order chromatic coefficients:

Coefficient	<u>Contribution to Spot</u> Radius for 1%Ap
(x:x'\Deltap) = 2.02 mm/ (mrad-%)	5.9 mm
$(y:y'\Delta p) = 0.369$	1.9

where the symbol  $(x:x'\Delta p)$  represents the coefficient multiplying the input horizontal angular width x' and the momentum spread  $\Delta p$  of the beam in the second-order expansion for the horizontal radius x at the pellet, et cetera. These coefficients indicate that a momentum spread of more than 0.5 percent will seriously degrade the focal spot at the pellet. It would be advantageous for a high-current accelerator system to be allowed an energy spread of several percent. Thus we are led to consider a system for correcting the chromatic aberration in the final transport.

#### III. AN EXAMPLE OF A CHROMATIC-CORRECTION SYSTEM

It is common in synchrotron/storage ring technology to reduce chromatic effects by the use of sextupole magnets at points of relatively high dispersion. Such arrangements produce

<sup>&</sup>lt;sup>1</sup>K. L. Brown, D. C. Carey, Ch. Iselin, and F. Rothacker, "TRANSPORT, a Computer Program for Designing Charged Particle Beam Transport Systems", SLAC 91, (1973 Rev.), NAL91, and CERN 73-16.

momentum-dependent focusing effects which can cancel, to first order, the chromatic effects of the basic lattice. The extent to which this technique is useful depends on the strength of the second and higher-order effects produced in the dipole and sextupole magnets.

Methods have been developed<sup>2</sup> for introducing the sextupole magnets in such a way that the net second-order geometrical aberrations are relatively small. The basic strategy is to use pairs of sextupoles with half-wavelength separations so that their higher-order effects tend to cancel. As an example of such a chromatic-correction system, consider the arrangement shown in Figure 2.

This system consists of three half-wave sections with each section being optically like that shown in Figure 1. The last half-wave section is exactly the one shown in Figure 1. The first half-wave section has the same quadrupole arrangement but has in addition two dipole magnets to create dispersion and two sextupole magnets to produce momentum-dependent focussing in the two transverse planes. The second half-wave section is a mirror image of the first -- i.e., it has identical elements but in the inverse order. With this symmetry the higher-order effects of the sextupoles tend to cancel. The amount of dispersion required is such that the beam width due to momentum spread is about the same as that due to the transverse emittance at the sextupole magnets. Note that the beam is focused to a 4-mm round spot at the



Figure 2. A 3X/2 Chromatic-Correction and Matching Section

independent of the exact location of the sextupole magnets.

The TRANSPORT program was used to find magnet parameters such that the coefficients  $(x:x'\Delta p)$  and  $(y:y'\Delta p)$  became negligibly small (on the order  $10^{-5}$  mm per mrad-percent). The most important remaining second-order coefficients that affect the beam size at the pellet are:

Coefficients	Contribution to Spot
	<u>Radius (Δp = 1%)</u>
-2	
$(x:x\Delta p) = 1.24 \times 10^{-2}$	0.25 mm
$(x:\Delta p^2) = 2.86 \times 10^{-2}$	0.03
$(x:xx') = 2.75 \times 10^{-2}$	1.65
$(x:yy') = 2.46 \times 10^{-2}$	1.48
$(y:x'y) = 4.26 \times 10^{-2}$	1.47
$(y:xy') = 0.99 \times 10^{-2}$	1.03
$(y:y\Delta p) = 0.84 \times 10^{-2}$	0.10

These coefficients were evaluated under the approximation of "square-edge" magnetic fields -- i.e., negligible fringe fields at the ends of the magnets.

This chromatic-correction system was evaluated using the TURTLE<sup>3</sup> program, in which individual rays are traced using matrix elements good to second order for each component of the transport line. The results are shown in Figure 3, in which the rms horizontal and vertical widths and the fraction of the particles striking the 4 mm-radius pellet are plotted as a function of the momentum half width. In transverse phase space, the particle densities used in these calculations were uniform in x-y

<sup>3</sup>K. L. Brown and Ch. Iselin, CERN 74-2, "Decay Turtle", 1974.

<sup>&</sup>lt;sup>2</sup>K. L. Brown, "A Second-Order Magnetic Optical Achromat", SLAC-PUB-2257, 1979.

and in x'-y' spaces out to the the elliptical boundaries  $(x/a)^2 + (y/b)^2 = 1$  and  $(x'/\theta_x)^2 + (y'/\theta_y)^2 = 1$ .

The plots in Figure 3 show that the chromatic effects in this corrected system are relatively small for a momentum spread of less than ± 2 or 3 percent. That there is some spreading of the beam with no momentum spread and consequent loss at the 4 mm radius target is attributable mostly to the second-order geometric aberrations produced by the sextupoles. The question of how much these second-order geometric terms can be reduced by alternative arrangements of the lattice has not been pursued.

To illustrate the efficacy of this chromatic-correction system, similar ray-tracing calculations were made for the simple one-half-wave matching section without chromatic correction (Figure 1) between the end of the transport lattice and the 4 mm pellet. The results are plotted in Figure 4. Comparison with Figure 3 shows that this chromatic-correction system improves the momentum acceptance of the final transport by about a factor of 2. It shows also that for a momentum spread of less than one percent the chromatic correction is probably unnecessary.



Figure 3. The Momentum Sensitivity of the 3x/2 Chromatic-Correction System



## IV. SUMMARY

We have illustrated by means of an example that the inherent momentum sensitivity of a simple, four-quadrupole-magnet final transport for focusing a heavy-ion beam on to a small pellet can be improved through the use of a particular arrangement of sextupole magnets.

The relative importance of third- and higher order terms in such a chromatic-correction system is not yet clear. Preliminary estimates<sup>4,5</sup> have given inconsistent results; further analysis is in progress.

One of the weaknesses of this chromatic correction system is that it does not easily lend itself to the inclusion of space-charge effects. Subsequent to this work, new optical arrangments have been developed which appear to be more amenable to the inclusion of both chromatic and space-charge corrections in the same system. However, no significant results in this regard are now available.

- <sup>4</sup>E. Colton, private communication.
- <sup>5</sup>J. Spencer, private communication.