STUDY OF INTER-BEAM INTERACTION IN INJECTION PROCESSES AT THE SPACE CHARGE LIMIT

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1. INTRODUCTION

Augmenting a particle simulation program originally by I. Haber,[1] beam dynamics of the injection process into a storage ring is studied. Although a real storage ring recently designed employs various kinds of magnetic elements, the essential features of its beam dynamics are determined by the quadrupole doublets. The lattice structure of the storage ring consists of 20 FODO periods. Figure 1 shows the lattice, and Table 1 shows the storage ring parameters.

The quadrupoles are assumed to be thin. Thus, by obtaining an FODO transport matrix for one period starting from beam injection point I, acceptance ellipse parameters α_{Aj}, β_{Aj}[2] are calculated at I for both x-x′ and y-y′ planes (j=x,y) for a given phase advance per period. In this paper, we discuss single plane (xx′) injection. We set the beam emittance ellipse parameters for the y-y′ plane (α_{By}, β_{By}) equal to the acceptance ellipse (α_{Ay}, β_{Ay}) and take the center of the beam ellipse in the y-y′ plane at the center of the beam transport system. Thus, the beam ellipse in the y-y′ plane is exactly matched, regardless of its emittance, so the beam ellipse in the y-y′ plane has the same periodicity as the storage ring elements. Thus, the four-dimensional efficiency and dilution of the injection process are equal to the two-dimensional ones in the x-x′ plane. We wish to check whether beam loss or phase space dilution can occur due to multibeam envelope oscillations dependent on the beam stacking method in injection.

The program keeps track of beam loss for a given aperture of a transport line; a special method accounts for septum losses. Chamber wall image force is not included in the calculation, and the septum has thickness equal to zero.

2. BEAM STUDY

We will briefly describe here the circle coordinate transformation[3] which is convenient for optimizing beam shape parameters. For a given injection point I in a storage ring, it is always possible to find a coordinate system so that the acceptance ellipse at I is a circle. The transformation from x-x′ coordinates to circle coordinates is a function of I.

They are given as:

$$T = \begin{pmatrix} \frac{1}{\sqrt{\beta_{AI}}} & 0 \\ \frac{\alpha_{AI}}{\sqrt{\beta_{AI}}} & \sqrt{\beta_{AI}} \end{pmatrix} \qquad \text{Physical} \longrightarrow \text{Circle}$$

$$T^{-1} = \begin{pmatrix} \sqrt{\beta_{AI}} & 0 \\ -\frac{\alpha_{AI}}{\sqrt{\beta_{AI}}} & \frac{1}{\sqrt{\beta_{AI}}} \end{pmatrix} \qquad \text{Circle} \longrightarrow \text{Physical}$$

where α_{AI}, β_{AI} are acceptance ellipse parameters at I.

The following results are obtained by varying mesh size, beam size, and time step length by factor of two. The result is independent of these factors. All calculations were done with 1024 macro particles for each injected turn. The time step for the particle pusher is 1/5 of each lattice period. Thus, 100 time steps make one turn around the ring. The initial particle distribution is chosen as K-V for each injected turn.

a). Four turn, tune $\nu_x = 5 + \frac{1}{4}$ injection

This tune gives a phase advance at injection point I after one turn of the storage ring of $(5\times360+90)^{\circ}$. Thus, in Fig. 2-1, we see the acceptance at point I of the ring filled with the incident beam at position 1; it is expected to come to position 2 after one turn. It clears the thin septum due to betatron oscillation with zero loss. The dilution of this complete injection is theoretically calculated to be 2.0 without loss. Figure 2-2 shows the result for zero current of this injection scheme. Figure 2-3 shows the result with 1. mA in each beam. A total of four turns reaches the space charge limit of 4. mA. The two figures correspond at time step 400.

b). Five turn, tune $\nu_x = 5 + \frac{1}{4}$ injection

As is seen in Fig. 2-1, there is a hole in the center of x-x' acceptance at point I in four turn injection. If we inject a beam in the center of the acceptance at the first turn followed by bumping of the equilibrium orbit, we can inject five turns as seen in Fig. 3-1. Theoretically, the dilution of this injection is calculated to be 1.60 without septum loss. Figure 3-2 shows the result for five turns with zero current. Figure 3-3 shows the result with 0.8 mA each beam. A total of five turns reaches the space charge limit of 4 mA. The two figures correspond at time step 500.

c). Four turn, tune $\nu_x = 5$ injection

This tune gives a phase advance at injection point I after one turn of the storage ring as $(5\times360)^o$. Injection is performed as follows. First, the equilibrium orbit is bumped locally at I such that it is two beam widths to the right of the septum. Inject the first turn. Bump the equilibrium orbit at I toward the left of the septum by the full width of the beam. Inject the second turn. Subsequent turns are injected the same as the second turn. Figure 4-1 shows the acceptance and beams at I. Theoretically, the dilution of this injection is calculated to be 1.57 without septum loss. Figure 4-2 shows the result for four turns with zero current. Figure 4-3 shows the result with 1 mA in each beam. A total of four turns reaches the space charge limit of 4 mA. The two figures correspond at time step 400.

3. SUMMARY OF CONDITIONS AND RESULTS

Scheme	a	b	c
Phase Advance/Period	94.5^o	94.5^o	90^o
Quadrupole Thin Lens Power*	0.44776 1/m	0.44776 1/m	0.43116 1/m
No. of Injection Turns	4	5	4
Acceptance α_{Ax}	0.73661	0.73661	0.70711
β_{Ax}	4.67707 m	4.67707 m	4.920 m
Acceptance ε_{Ax}	25 mr cm	25 mr cm	25 mr cm
Beam α_{Bx}	0.36831	0.36831	0.20676
β_{Bx}	2.33854 m	2.33854 m	1.4386 m
Emittance ε_x	3.125 mr cm	3.125 mr cm	3.98 mr cm
No. of macro particles/turn	1024	1024	1024
Beam loss due to septum	30	203	403
Beam loss by wall**	105	75	0
Total loss/Total beam	3.30%	5.43%	9.84%

Acceptance ε_{Ay}	3.125 mr cm	3.125 mr cm	3.125 mr cm
Beam α_{By}	-0.73661	-0.73661	-0.70711
β_{By}	4.67707 m	4.67707 m	4.920 m
Beam Emittance ε_{By}	3.125 mr cm	3.125 mr cm	3.125 mr cm

*Lens power = 1/focal length.

**Vacuum chamber cross section is assumed as 14 cm × 8 cm.

The beam X_e^{+1} is assumed to have 8.5 MeV kinetic energy.

4. DISCUSSION

We notice that, for the beam at the space charge limit, although the turns are deformed in phase space due to mutual beam interaction, they can still be identified individually. A ring large enough to contain the beam with space charge artifically turned off suffers losses on the septum and on the walls at full current. The total magnitude of the loss and its partition among wall and septum both depend strongly on which injection scheme is used. The septum losses differ by a factor of 10, and the best scheme in that regard suffers from significant wall depletion. Scheme b illustrates a trade-off: the incoming current is 20% lower than that of Scheme a, but the septum losses are eight times greater.

The next step is to add wall image forces to the program to make it more accurate. The present results are probably accurate enough to prove that injection schemes cannot be ordered in efficiency and dilution or evaluated properly without this type of computation. The space charge effect is so significant in these high-efficiency, low-dilution injection processes that two-plane vs. one-plane injection schemes may exchange places in merit. We have considered only schemes that have 100% efficiency with zero space charge. Dilution can be improved by accepting losses, but quantitative determinations will require study of a wider class of schemes.

FIGURE CAPTIONS

Fig. 1 The storage ring is represented by a circle, with focusing and defocusing quads indicated. One period consists of two units drift space (2×1.64m) followed by a thin defocusing (x) quad, one unit drift space, thin focusing (x), and two units drift space.

Fig. 2-1 Injection scheme of four turn, $\nu_x = 5.25$ is shown in circle coordinates in x-x′ phase space. The acceptance is represented by a circle. Each beam is represented by an ellipse.

Fig. 2-2 x-x′ phase space at injection point I for four turn $\nu_x = 5.25$ injection with zero current. This figure shows beam phase space at time step 400.

Fig. 2-3 x-x′ phase space at injection point I for four turn $\nu_x = 5.25$ injection with a total current 4 mA. This figure shows beam phase space at time step 400.

Fig. 3-1 Injection scheme of five turn, $\nu_x = 5.25$ is shown in circle coordinates in x-x′ phase space. The acceptance is represented by a circle. Each beam is represented by an ellipse.

Fig. 3-2 x-x′ phase space at injection point I for five turn $\nu_x = 5.25$ injection with zero current. This figure shows beam phase space at time step 500.

Fig. 3-3 x-x′ phase space at injection point I for five turn $\nu_x = 5.25$ injection with a total current 4 mA. This figure shows beam phase space at time step 500.

Fig. 4-1 Injection scheme of four turn, $\nu_x = 5.0$ is shown in circle coordinates in x-x′ phase space. The acceptance is represented by a circle. Each beam is represented by an ellipse.

Fig. 4-2 x-x′ phase space at injection point I for four turn $\nu_x = 5.0$ injection with zero current. This figure shows beam phase space at time step 400.

Fig. 4-3 x-x′ phase space at injection point I for four turn $\nu_x = 5.0$ injection with a total 4 mA. This figure shows beam phase space at time step 400.

The code on the scatter plots in Figs. 2-2, 2-3, 3-2, 3-3, 4-2, and 4-3 indicates the number of macro particles per printing bin; we have not, as yet, used these details.

REFERENCES

1. I. Haber, NRL Memorandum Report 3705.

2. The acceptance ellipse is defined as:

$$x^2 + (\alpha_{Ax} x + \beta_{Ax} x')^2 = \beta_{Ax} \varepsilon_{Ax}$$

$$y^2 + (\alpha_{Ay} y + \beta_{Ay} y')^2 = \beta_{Ay} \varepsilon_{Ay}$$

where ε_{Ax}, ε_{Ay} are acceptances for x, y, respectively.

3. C. Bovet, R. Gouiran, I. Gumowski, and K. H. Reich, CERN/MPS-SI/Int. DL/70/4, p. 17.

ACKNOWLEDGMENT

We deeply appreciate I. Haber for his offer to use his computer program freely, and we thank him for the many discussions for the details of the program and the time he spent with us.

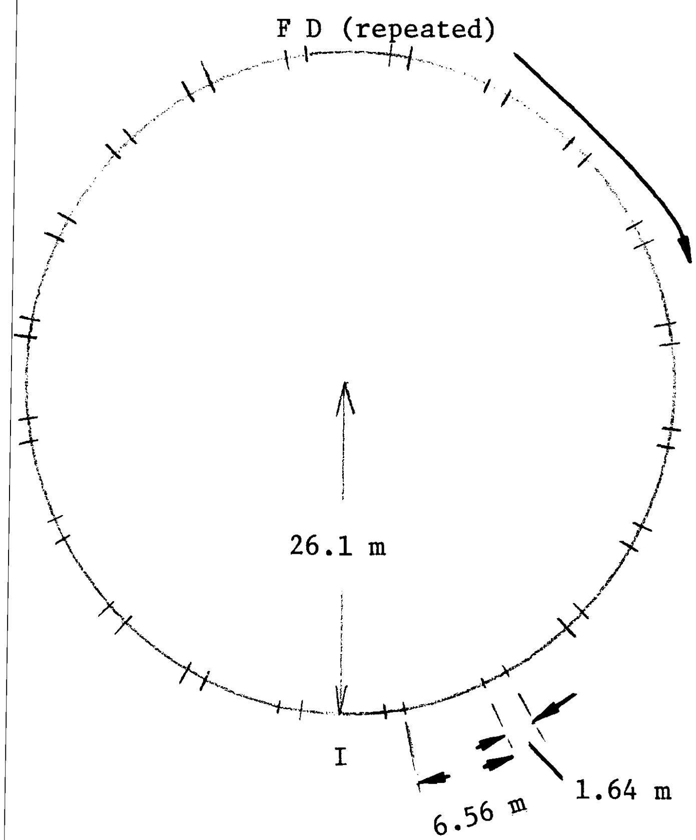

Fig. 1

Storage ring lattice D and F corresponds to defocusing and focusing quadrupoles.

Table 1. Storage Ring Parameters

Method	1	2	3
Quadrupole thin lens power	0.44776 1/m	0.44776 1/m	0.43116 1/m
Acceptance X	25 mr cm	25 mr cm	25 mr cm
α_{Ax}*	0.73661	0.73661	0.70711
β_{Ax}*	4.67707 m	4.67707 m	4.920 m
Acceptance Y	3.125 mr cm	3.125 mr cm	3.125 mr cm
α_{Ay}*	-0.73661	-0.73661	-0.70711
β_{Ay}*	4.67707 m	4.67707 m	4.920 m

*See Ref. 2.

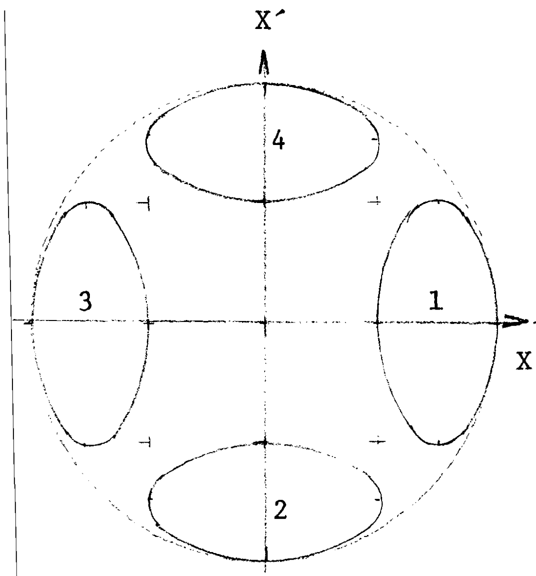

Fig. 2-1

Four turn injection, tune $\nu_x = 5 + 1/4$, X-X´ phase space in circle coordinates. Outer circle is X acceptance. Inner ellipses are injected turns.
Dilution = 2.

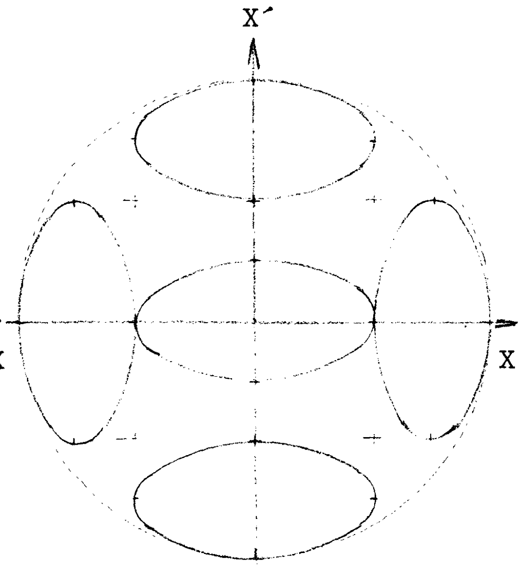

Fig. 3-1

Five turn injection, tune $\nu_x = 5 + 1/4$, X-X´ phase space in circle coordinates. Outer circle is X acceptance. Inner ellipses are injected turns.
Dilution = 1.6

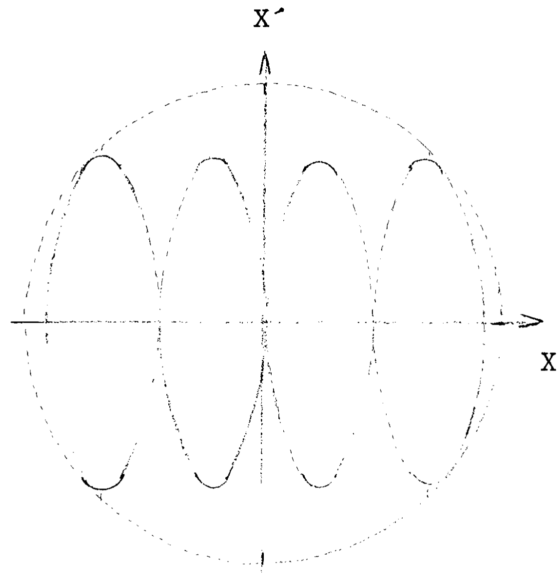

Fig. 4-1

Four turn injection, tune $\nu_x = 5.0$, X-X´ phase space in circle coordinates. Outer circle is X acceptance. Inner ellipses are injected turns.
Dilution = 1.57

Fig. 2-3. x vs. x′ phase space plot for 4 turn injection, tune $\nu_x = 5 + 1/4$. Each beam current = 1 mA.

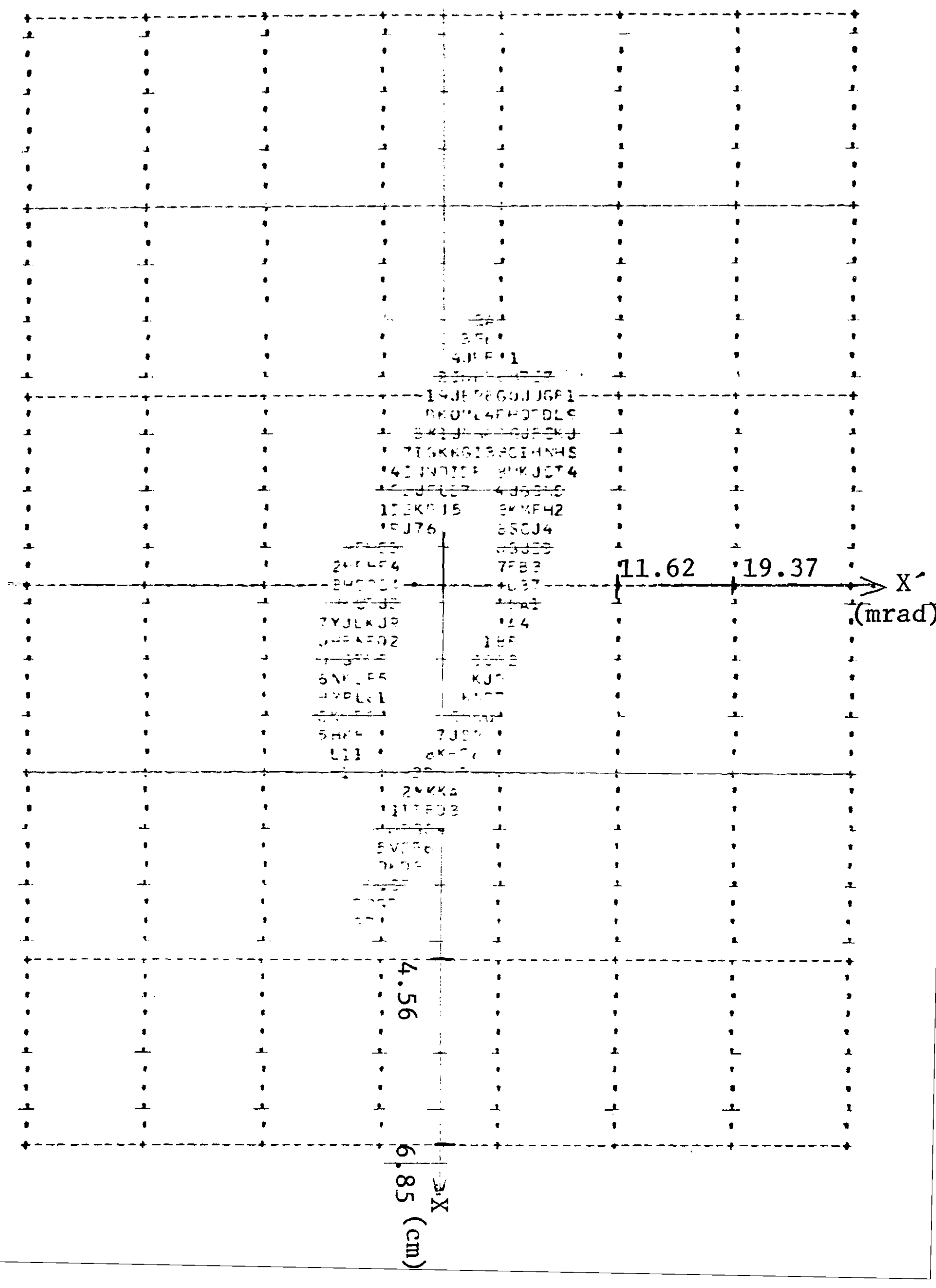

Fig. 2-2. x vs. x´ phase space plot for 4 turn injection, tune ν_x = 5 + 1/4. Each beam current = 0 mA.

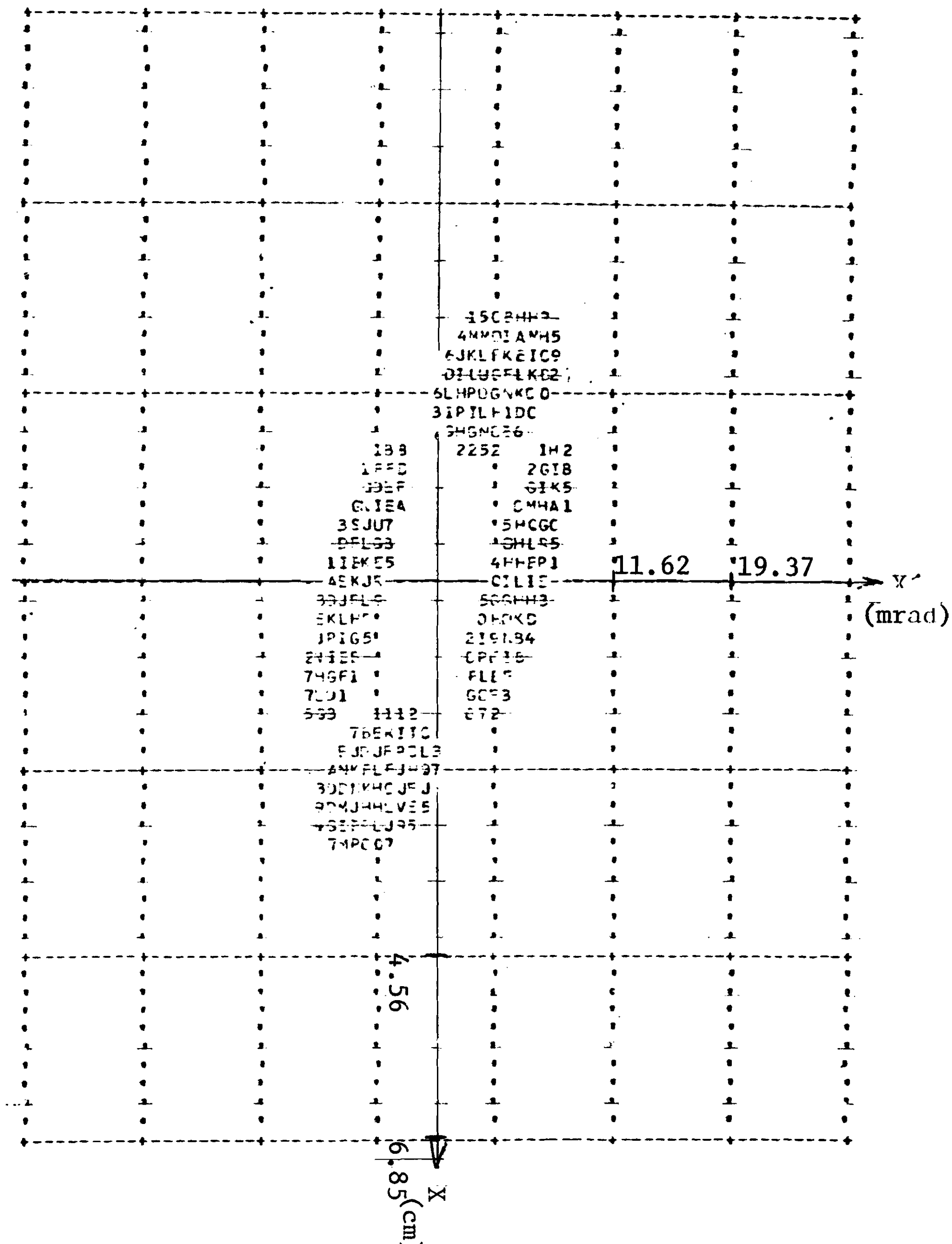

Fig. 3-2. x vs. x´ phase space plot for 5 turn injection, tune $\nu_x = 5 + 1/4$. Each beam current = 0 mA.

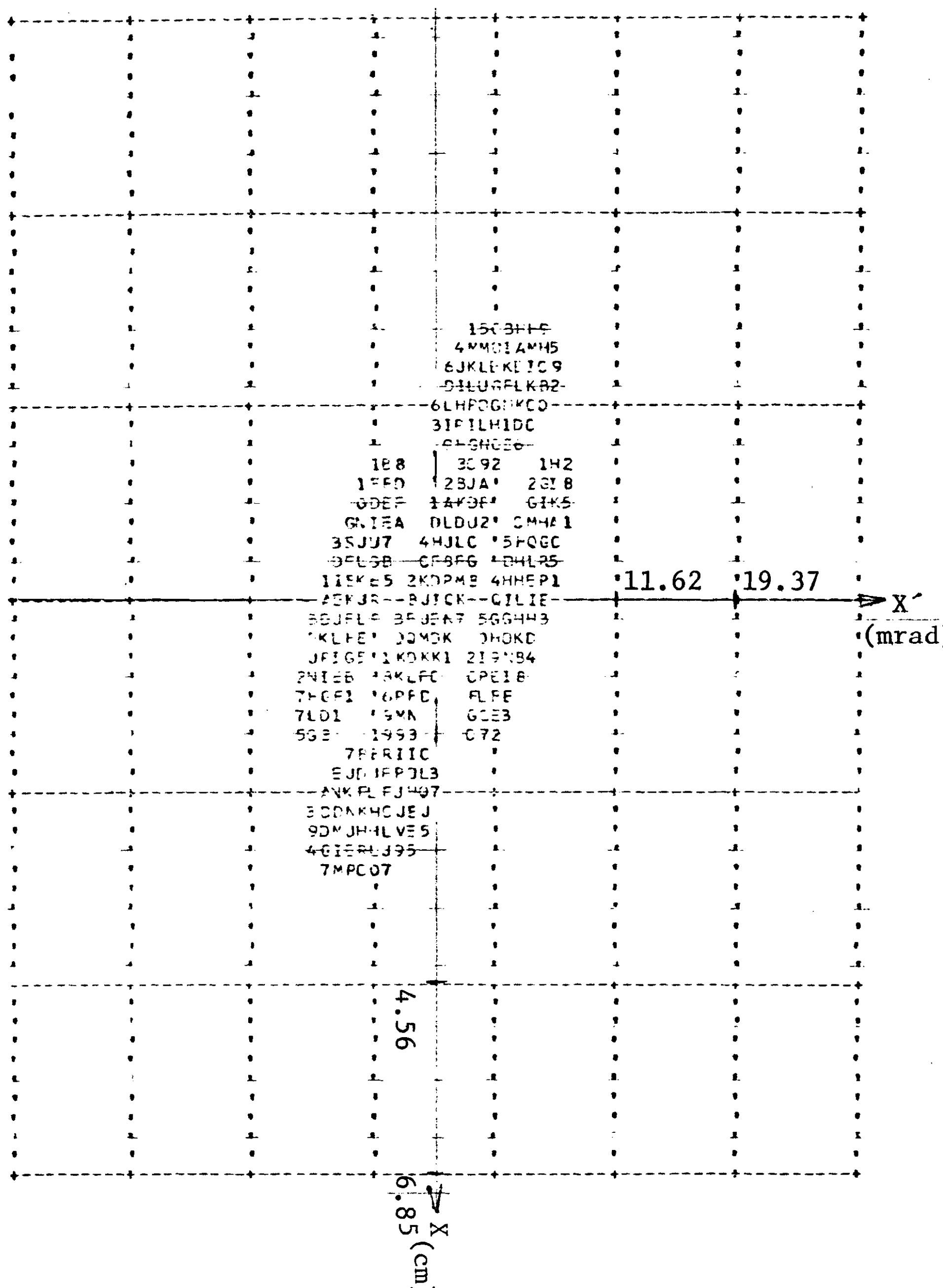

Fig. 3-3. x vs. x´ phase space plot for 5 turn injection, tune ν_x = 5 + 1/4. Each beam current = 0.8 mA.

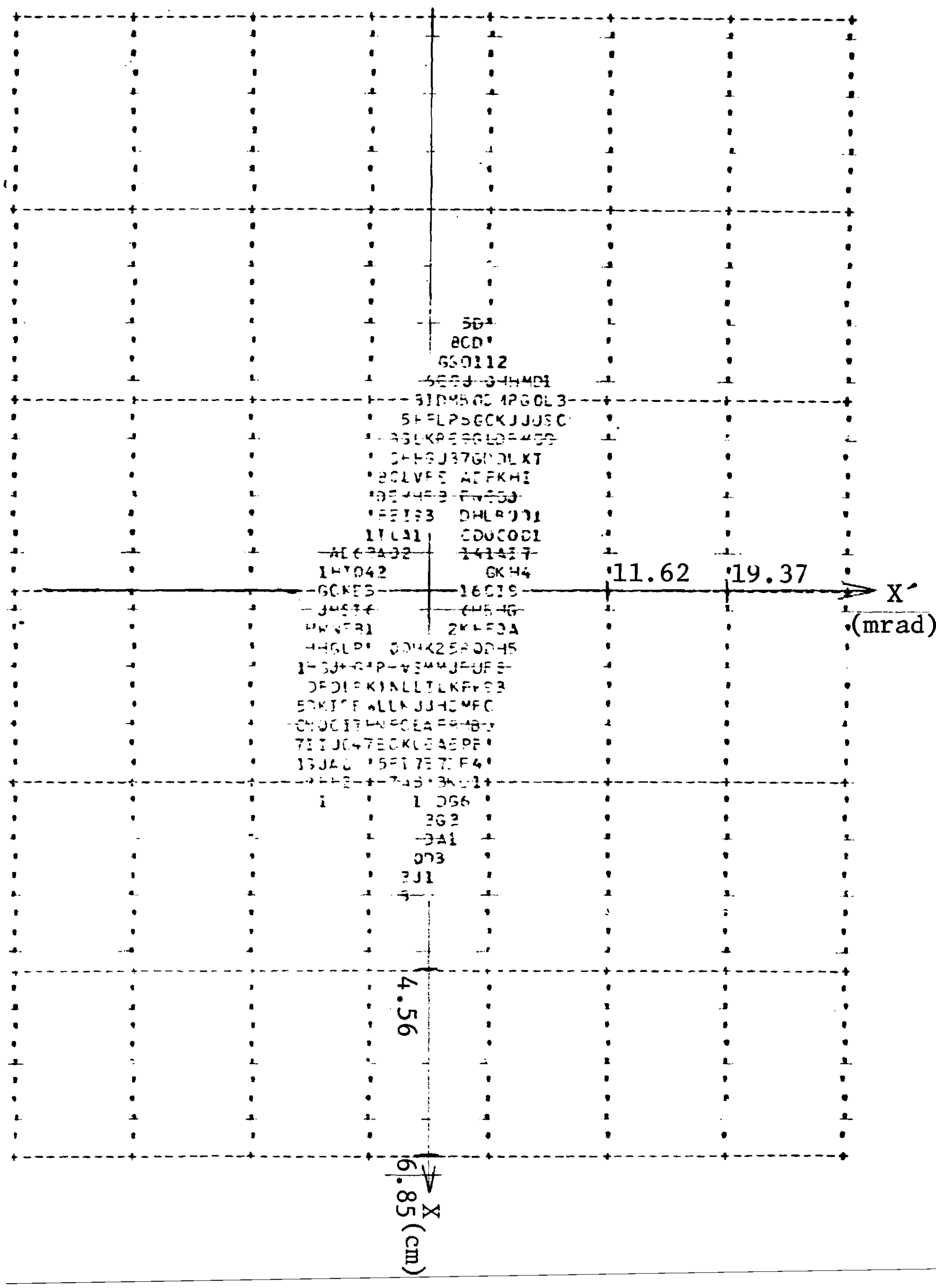

Fig. 4-2. x vs. x′ phase space plot for 4 turn injection, tune ν_x = 5.0. Each beam current = 0 mA.

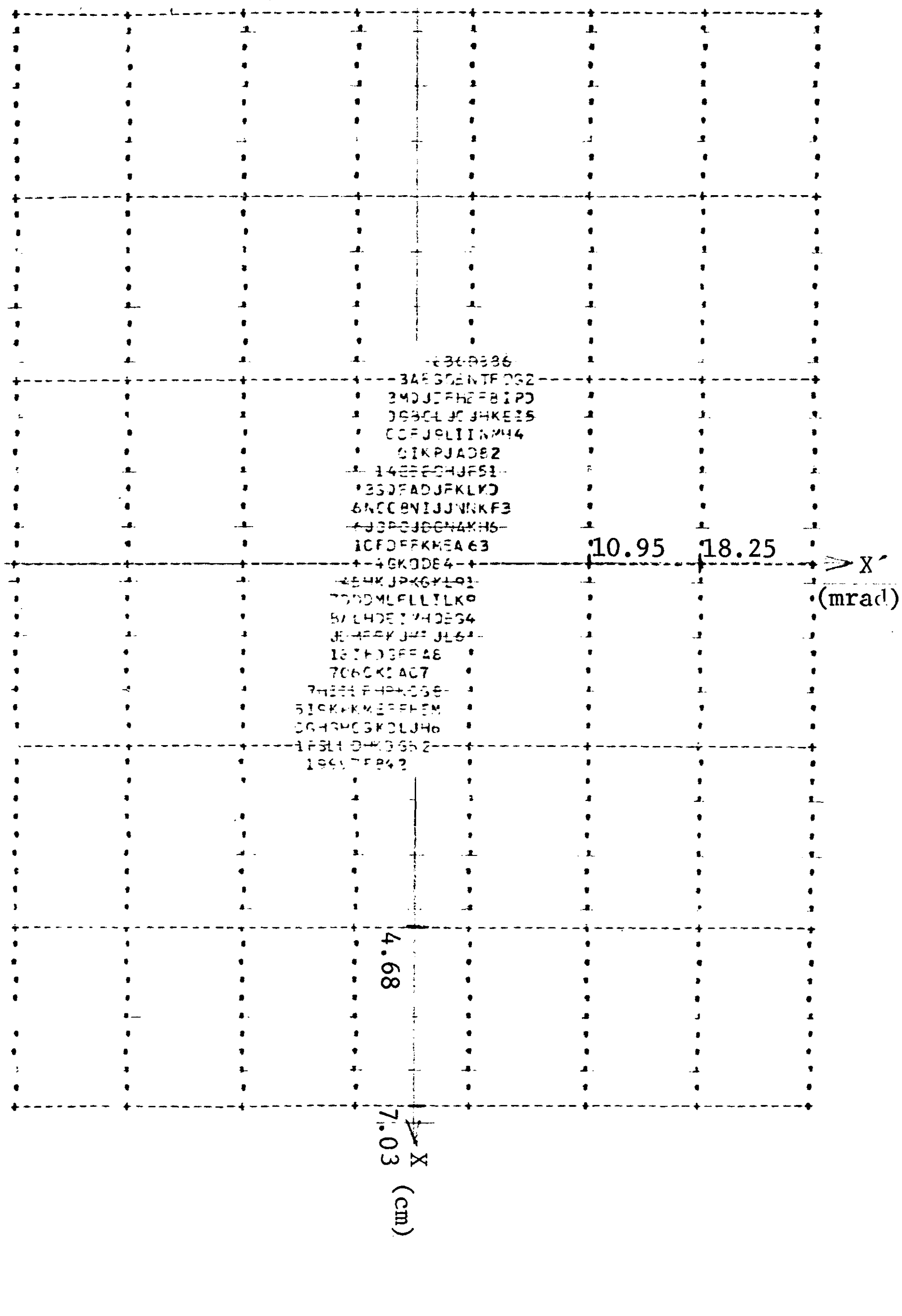

Fig. 4-3. x vs. x′ phase space plot for 4 turn injection, tune $\nu_x = 5.0$. Each beam current = 1 mA.