LONGITUDINAL MICROWAVE INSTABILITY

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A small deviation from the nominal distribution of particles in longitudinal position and momentum creates fields that may or may not increase this deviation; in the former case, a longitudinal instability occurs. They have been observed at microwave frequencies and their growth rate may be calculated in terms of the longitudinal impedance, which is the ratio of the negative of the voltage induced once around the ring to the perturbation current. This voltage is obtained by integrating the axial longitudinal electric field along the particle trajectory over one turn. The relevant frequencies are those above the cutoff for chamber propagation.

A derivation of the relationship between growth rate and impedance is given here, starting from a chosen point in Ref. 1. The only other instability considered here is the resonance effect produced by excessive tune shift due to space charge. We assume, without discussion, that $(\Delta\nu)_{max} = .25$; that is, this resonance growth is avoided (stability) by limiting the ring charge. On the other hand, the longitudinal microwave instability is assumed to be present (cannot be stabilized). Thus, the latter involves a limiting impedance to keep the growth rate low enough. The maximum allowed impedance for a maximum allowed growth rate is listed below.

Both instabilities are amenable to calculation only if quasistatic conditions obtain. Thus, the formulas below will be considered to apply during the following stages: post-injection debunching with BF = 1 and $\Delta\nu = 1/8$; and adiabatic rebunching with BF = 1/2 and $\Delta\nu = 1/4$.

We have altered the ring parameters from those of the workshop in accordance with these specifications.

Now, we will derive the relation between growth rate and resistive impedance.

Notation: MKSA units

q	= charge state	A	= atomic weight
m_p	= proton mass	e	= proton charge
c	= speed of light	β	= v/c
$\Delta p/p$	= momentum spread		
I	= electrical ring current		
Z_c	= capacitive longitudinal impedance		

N = no. particles on target
I_{av} = electrical current on target averaged over pulse shape
N_{bun} = number of bunches per ring
l_b = length of bunch on target
N_R = number of storage rings
C = ring circumference
E = energy on target
T = kinetic energy per beam ion
Z_r = resistive longitudinal impedance
Z_r' = resistive longitudinal impedance per meter
U' = normalized capacitive longitudinal impedance
V' = normalized resistive longitudinal impedance
n = number of instability density oscillations per turn
Z_o = $120\ \pi$
R = ring radius   BF = bunching factor
ε_{sr} = transverse geometrical emittance (area/π) in ring
$\Delta\nu$ = allowed tune shift
ω_o = particle revolution frequency = BC/R
$\Delta\Omega_r$ = real frequency shift
$\Delta\Omega_i$ = instability growth rate
τ = instability rise time
$\Delta\Omega$ = $\Delta\Omega_r + i\Delta\Omega_i$
z = $x+iy$ = normalized $\Delta\Omega$
$f(z)$ = normalized longitudinal distribution function
$I_D'(z)$ = normalized dispersion integral
I_{sci} = space-charge limited ring instantaneous electrical current
n_{sc} = space-charge limit number of ions
f = frequency of the instability
ω = $2\pi f$
λ = wavelength of the instability
$\Delta\Omega_r$ = $\omega - n\omega_o$

Normalization Definitions[1]

$$V' + iU' = \frac{2}{\pi} \cdot I \cdot \frac{q}{A} \cdot \frac{e}{m_p c} \cdot \frac{1}{c} \cdot \frac{1}{\beta^2} \frac{1}{(\Delta p/p)^2} \cdot \frac{1}{n} (Z_r + iZ_c)$$

$$z = \frac{2 \cdot \Delta\Omega}{n\omega_o(\Delta p/p)}$$

Relations

Space charge limit

$$n_{sc} = \frac{\pi}{15e} \cdot \Delta\nu \cdot \frac{A}{q^2} \cdot \frac{m_p c}{e} \cdot \varepsilon_{sr}\ \beta^2\gamma^3 \cdot BF$$

$$I_{sci} = \frac{\beta cqe}{2\pi R} \cdot n_{sc} \cdot \frac{1}{BF}$$

Mode number $\qquad n = 2\pi R \cdot f/c = 2\pi R/\lambda$

Ring and target energy balance

$$N_R \beta cqe n_{sc} = I_{av} l_b$$

Compression factor = 49

Note: $\frac{1}{30e} \cdot \frac{m_p c}{e} = \frac{1}{1.5347 \times 10^{-18}}$

$$C = 49\ l_b \times N_{bun}/BF$$

By Ref. 2,

$$Z_c/n \simeq Z_o/\beta$$

The dynamics of growth may be obtained from the dispersion relation. The time-dependent factor is exp $(-i \cdot \Delta\Omega \cdot t)$ and $\Delta\Omega$ is decomposed into its real and imaginary parts.

$$\Delta\Omega = \Delta\Omega_r + i\ \Delta\Omega_i$$

Two significant properties of the distribution function are:

a). $\int_{\text{real axis}} f(z)dz = \int_{-\infty}^{\infty} f(x) \cdot dx = 1$

b). f is nonzero on approximately $-\frac{1}{2} < x < \frac{1}{2}$.

The dynamical theory shows that the normalized dispersion integral

$$I_D'(z) = -i \int_{\text{real axis}} \frac{1}{u-z} \cdot \frac{df}{du} \cdot du$$

determines U', V' through the dispersion relation

$$(V' + iU') \cdot I_D' = 1.$$

and that x has the significance of normalized real frequency shift as defined above. This variable is eliminated in favor of the others below.

To find the relation among Z_r, $\Delta p/p$ and the growth rate $\Delta\Omega_i$, one must choose an f. Simplest is the step function:

$$f(x) = 1 \qquad -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$f(x) = 0 \qquad \text{elsewhere.}$$

The derivative of f is equal to ± the unit delta function at $x = \mp\frac{1}{2}$ and we find

$$I_D' = \frac{-i}{z^2 - 1/4}$$

$$V' + iU' = -2xy + i\,(x^2 - y^2 - \frac{1}{4})$$

Assume, for the moment, that U' is a given number. Then,

$$x = \pm\sqrt{y^2 + \frac{1}{4} + U'}$$

$$V' = 2y\sqrt{y^2 + \frac{1}{4} + U'}$$

where we choose the sign to give growing waves. This expression may be substituted into the above normalization definition to express Z_r in terms of y and hence the growth rate. The value of U' is determined in terms of Z_c/n by the same definition. Numerical values below show $U \gg \frac{1}{4}$, so $\frac{1}{4}$ can be dropped. Then, $\Delta p/p$ cancels out, and one finds a quadratic equation for $w \equiv y^2$ as:

$$w^2 + U'w - (\frac{V'}{2})^2 = 0$$

with roots

$$w_\pm = -\frac{1}{2}U' \pm \frac{1}{2}(U'^2 + V'^2)^{\frac{1}{2}}$$

Root w_- may be discarded as negative. One notes that $V'^2 \ll U'^2$ because $Z_r \ll Z_c$. The latter inequality is clearly true with the assumption $Z_c \simeq nZ_o/\beta \geq 10^6$. Thus, we find:[4]

$$y = \sqrt{w_+} = \frac{1}{2}\frac{V'}{\sqrt{U'}}$$

or

$$Z_r = (8\pi \cdot \frac{m_p c}{e} \cdot \frac{A}{q} \cdot \frac{1}{c} \cdot \frac{1}{I} \cdot \frac{Z_c}{n})^{\frac{1}{2}} \cdot R \cdot \Delta\Omega_i$$

This expression may be derived directly from a δ-function distribution taken for f. The result, in different notation, is:[1]

$$\Delta\Omega_i = \left[\frac{-e\eta\ \omega_o^2\ n\ I_o Z_i}{2\pi\beta^2 E}\right]^{1/2}$$

Put $\eta\to-1$, $E\to Am_p c^2$, $I_o\to I$, $e\to qe$, $Z_i\to Z_c+iZ_r$, $\omega_o\to\beta c/R$ to obtain

$$\Delta\Omega = \frac{nc}{2\pi R}\left[2\pi\cdot\frac{q}{A}\cdot\frac{e}{m_p c}\cdot\frac{I}{c}\cdot\left(\frac{Z_c}{n}+i\,\frac{Z_r}{n}\right)\right]^{1/2}$$

Assume $Z_r \ll Z_c$ and expand the sum in the root; the imaginary part is

$$\Delta\Omega_i = \frac{1}{2R}\left(\frac{1}{2\pi}\cdot\frac{q}{A}\cdot\frac{e}{m_p c}\cdot cI\cdot\frac{Z_c}{n}\right)^{1/2}\cdot\frac{Z_r}{(Z_c/n)}$$

which can be solved for Z_r as above. A useful form for systems work approximates:[2]

$$\frac{1}{n}Z_c \approx \frac{1}{\beta}\gamma^2 Z_o, \qquad I \approx I_{sc}$$

and reads:

$$\tau = \left(\frac{2R}{\Delta\nu\cdot\varepsilon_g\cdot}\right)^{1/2}\cdot\frac{R}{c}\cdot\frac{1}{\beta^2}\cdot\frac{Z_o}{Z_r}$$

In this estimate, the inductive effect of circular vacuum chamber, cross section variation and bellows, and plates formed by clearing electrodes or pick-up stations[2] have been neglected, as they can only improve the situation in a minor way.

Results are given in the following table, which differs from the Storage Ring Group Summary[3] because the charge balance equation is strictly kept at the sacrifice of a variation in ε_{sr}, and BF = .5 was chosen instead of BF = 1. The rise time τ has been taken as .01. Note that the factor γ^2 in the denominator of Z_c/n makes the crucial difference between Isabelle and the rings considered here.

Table of Heavy Ion Fusion Parameters, MKSA Units

Case	A	B	C
E	$1. \times 10^{6}$	$3. \times 10^{6}$	$1. \times 10^{7}$
N	1.25×10^{15}	1.875×10^{15}	$.625 \times 10^{16}$
T	0.5×10^{10}	$1. \times 10^{10}$	$1. \times 10^{10}$
I_{av}	$1. \times 10^{4}$	0.75×10^{4}	1.43×10^{4}
γ	1.0226	1.0451	1.0451
β	0.2089	0.2906	0.2906
$\beta\gamma$	0.2136	0.3037	0.3037
BF	.5	.5	.5
ℓ_b	1.25	3.485	6.10
$49\ell_b$	61.40	170.8	298.8
N_{sr}	4	3	9
N_{bun}	5	2	2
N_b	20	6	18
(N_b)min	17	6	12
$(\Delta p/p)$min	$\pm 2. \times 10^{-4}$	$\pm 2. \times 10^{-4}$	$\pm 2 \times 10^{-4}$
ε_{sr}	5.478×10^{-5}	5.313×10^{-5}	5.911×10^{-5}
n_{sr}	3.1335×10^{14}	6.2386×10^{14}	6.9401×10^{14}
C	614.0	683.2	1195.2
R	97.72	108.73	190.2
(I_{sr})peak	10.25	25.51	16.22
U'	30.	28.	18.
n(2.5 GHz)	5117	5693	9960
Z_r	1024.7	612.7	1344.2
Z_r/n	0.2002	0.1076	0.1350
Z'_r	1.6688	0.8968	1.1247

CONCLUSION

Heavy ion beam fusion must operate at large U', far from the Keil-Schnell stability region, because a large $\Delta p/p$ (Landau damping) is prevented by limited ring bending magnet strength, injection problems, and economic need of a large BF. Also, one cannot increase the final $\Delta p/p$ by increasing the longitudinal emittance due to the final focussing limitations. Since Landau damping cannot be utilized, there is no point in placing $\Delta p/p$ at the high end of its allowed range. Here, upper limits have been obtained for the resistive impedance corresponding to an assumed upper limit on growth rate. At this point, the longitudinal resistive impedance Z_r needs to be determined for candidate rings via a theoretical and experimental program.

REFERENCES

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2. G. Guignard, CERN 77-10, p. 87 (1977).
3. Heavy Ion Fusion Accelerator Study Storage Ring Group, 1979.
4. V. K. Neil and A. M. Sessler, Rev. Sci. Inst., 36, 429 (1965), Eq. (4.5)