# LONGITUDINAL BEAM STABILITY IN HEAVY ION STORAGE RINGS

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## 1. INTRODUCTION

This is an attempt to scale conditions for longitudinal beam stability to heavy ion storage rings (HIS) which have been proposed as part of some accelerator schemes to drive pellet fusion [1]. The instability considered has been observed in many high intensity proton machines. In the CERN 25 GeV Proton Synchrotron (PS), it can occur near transition energy [2] as well as during debunching at high energy [3]. In the 30 GeV intersecting storage rings (ISR) similar effects happen to the newly injected beam [4] when too dense bunches are transferred. In all these cases the instability manifests itself by a rapid blow-up of the beam momentum spread and this blow-up is accompanied by RF activity observed on beam current pick-up electrodes at frequencies in the, say, 0.3 - 2 GHz region. The picture is consistent with the assumption (first made in the classical paper by Nielsen, Symon and Sessler [5] and generalized by many subsequent workers) that a longitudinal density modulation $\lambda = \lambda_0 e^{in(s/R - \omega t)}$ develops on the beam and self-amplifies via the interaction with structures surrounding the beam.

## 2. ASSUMPTIONS FOR SCALING

For the present purpose I shall take three sets of observation as established:

i) The instability threshold is described by the "Keil-Schnell" selfbunching criterion [6] with local values for momentum spread and beam current [2,3,7] (although a rigorous derivation only exists for the coasting beam case). By the same token the growth rate is determined by the coasting beam selfbunching rate taking the local value for the current of a bunched beam.

ii) The impedance $Z_n$ describing the coupling of the beam to its environment at a frequency near n times the particle revolution frequency is $|Z_n/n| \simeq 20\ \Omega$ with a real part $R_n/n \simeq 2 - 15\ \Omega$ both in ISR and PS for the 0.5 - 2 GHz region.

iii) Growth times can be as fast as 100 μs in the PS and probably also in the ISR.

As the real part $R_n/n$ is of vital importance for our scaling two comments are in order : $R_n$ has been estimated from measurements [8] of the growth rate of the transverse head tail instability using relations between transverse and longitudinal impedance. Clearly this is an indirect measurement which in addition gives lower limits on $R_n$ to the extent that Landau damping tends to reduce transverse growth.

Values obtained in this way are $R_n/n = 2 - 5$ Ohm in PS and $2 - 15\ \Omega$ in ISR ($f = 0.1 - 2$ GHz). In the ISR, $Z_n$ seems to be inductive rather than capacitive as expected for perfect walls. This permits some cross checks on $R_n$ from the longitudinal growth rate (see following section). Further checks are possible observing that the "imperfect wall contribution" $Z_w$ to $Z_n$ is a physical impedance. Hence transformations[13] can be used which relate $R_w$, $X_w$ and $Z_w$ and permit e.g. to calculate $R_w(\omega)$ if $|Z_w(\omega)|$ is known over a large enough frequency range. From these checks one might speculate that $R_n$ becomes comparable in magnitude to $|Z_n|$ at frequencies around a GHz.

## 3. SCALING RELATIONS

a) Threshold current [6] ("Keil-Schnell") :

$$I \lesssim F \left| \frac{\eta}{Z_n/n} \right| \left( \frac{\Delta p}{p} \right)^2_{FWHM} \beta^2 \gamma U_p \frac{A}{q} \qquad (1)$$

where : $F$ is a form factor depending on the nature of the impedance and the sign of $\eta$. As a rule of thumb $F \sim 1$;

$\eta = \gamma_{transition}^{-2} - \gamma^{-2}$ is the "off energy" function of the storage ring (machine constant at fixed energy depending on the distance from transition energy).

$Z_n = \int_0^{2\pi R} \frac{\langle E_{n,s} \rangle\, ds}{I_n}$ is the coupling impedance defined by the longitudinal electric field $E_{n,s} \exp\{in(s/R - \omega t)\}$ induced by a beam current $I_n \exp\{in(s/R - \omega t)\}$ and summed over one turn and averaged over the beam cross section.

$\left(\frac{\Delta p}{p}\right)_{FWHM}$ is the beam momentum spread (full width at half maximum)

$\beta = v/c$, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ are the usual relativistic factors

$U_p \simeq \frac{m_o c^2}{e}$ = 980 MV is the "proton rest voltage" and

A, q are the mass number and the charge state of the ion (A = 238, q = 1 for $U_{238}^{+1}$).

If (1) is violated for any mode number n, the corresponding beam density modulation will self amplify. The e-folding time for conditions for above the threshold (1) is obtained [5,6,9], noting that $1/\tau = \mathrm{Im}\,(n\,.\omega)$ and solving

$$(n\,\omega - n\,\omega_{rev})^2 = n^2\,\omega_{rev}^2\,\frac{i\,\eta\,I\,Z_n/n}{2\pi\,\beta^2\,\gamma\,U_p(A/q)} \qquad (2)$$

where $\omega_{rev} = \beta c/R$ is the (angular) particle revolution frequency. For protons conditions near threshold and more details are described, e.g. in ref. 9).

We shall be interested in cases where $Z_n = R_n + i\,X_n$ is such that $|X_n| >> R_n$. Then (see ref. 5) for the "good" sign of $X_n$ and $\eta$ (capacitive $X_n$ below, inductive above transition*) we find from (2)

$$\frac{1}{\tau} \approx n\,\omega_{rev}\,\sqrt{\frac{|\eta|\,I}{2\,\pi\beta^2\,\gamma\,U_p\,(A/q)}}\;\frac{R_n/n}{2\sqrt{X_n/n}} \qquad (2a)$$

And for the "bad" sign ("negative mass region" [5])

$$\frac{1}{\tau} \approx n\,\omega_{rev}\,\sqrt{\frac{|\eta|\,I}{2\,\pi\beta^2\,\gamma\,U_p(A/q)}}\;\sqrt{\frac{|X_n|}{n}} \qquad (2b)$$

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* Following bad tradition [5] we use the theorists ($e^{-i\omega t}$) rather than engineers ($e^{j\omega t}$) convention. Hence capacitive impedance means positive Im(Z) etc.

For coasting proton beams, the threshold and growth rate conditions are discussed in literature[9]. The generelization to bunched beams is done - somewhat in an ad hoc manner - in references 2, 3, 7 and this bunched beam theory seems to fit with observation [2,3,4]. The generalization to heavy ions can be readily done following e,g. Hereward's "old fashioned" derivation [10]. He observes that threshold corresponds to conditions where the RF potential ("self bucket") induced by the perturbed beam is just deep enough to hold the beam momentum spread. In a similar fashion the growth time is related to the period of synchrotron oscillation in the self bucket.

## 4. IMPEDANCES IN AN HIS

Relations to estimate $Z_n/n$ for many structures are compiled in reference 11. Two things are important for our scaling, namely the $\gamma$ dependence of the impedances and the cut off wave number beyond which the beam ceases to couple to the walls.

In the long wavelength limit (see below) the basic contribution to $Z_n$ namely the impedance of a beam (radius b) in a perfectly conducting smooth chamber (radius h) is

$$\frac{Z_n}{n} = i \frac{377\ \Omega}{\beta \gamma^2} \{\frac{1}{2} + \ln (\frac{h}{b})\} \tag{3}$$

Note the difference between high energy protons ($\beta\gamma^2 >> 1$) and heavy ions ($\beta\gamma^2 < 1$).

Additional contributions to $Z_n$ due imperfect walls (cavities, cross section variations, ferrite structures, etc.) are similar in proton and heavy ion storage rings up to the cut off wave number which can be shown to be of order (Appendix) $n_c \simeq \gamma \frac{\text{storage ring circumference}}{2\pi \times \text{chamber half height}}$

For above this cut off the beam fails to couple to the wall. In the cut off region (say, up to 2 $n_c$) the growth rate is roughly constant (Fig. 1) for constant wall impedance $Z_n/n$ rather than to increase linearly with frequency as suggested by (2).

## 5. SCALING

We conclude from section 4 that the HIS will be dominated by the large capacitive impedance (3). [This was first pointed out to me by Graham Rees.] Since probably all HIS will work below transition, we take equation (2a) to work out the growth rate. Guided by experience from PS and ISR we take for the real part $R_n/n = 2\ \Omega$ (optimistic ?) or $R_n/n = 15\ \Omega$ (pessimistic ?) . With the parameters of Table 1 we find

$$\tau \simeq 800\ \mu s \qquad \frac{R_n}{n} = 2\ \Omega$$

$$\tau \simeq 100\ \mu s \qquad \frac{R_n}{n} = 15\ \Omega$$

at the cut off $n_c = \frac{\gamma R}{h} = 2{,}5 \times 10^3$.

It is possible in principle to reduce $1/\tau \propto |X_n|^{-\frac{1}{2}}$ by increasing the capacitive space charge impedance (increase of h). This would however further complicate the RF manipulations required for controlled beam bunching because the external RF has to counteract space charge.

It might also be argued that Z/f rather than Z/n should be kept constant scaling wall imperfections between machines of similar size. This would improve the HIS growth times by 2 - 3.

## 6. HOW MANY E-FOLDINGS ARE TOLERABLE ?

For a perfect coasting beam the initial perturbation is the Schottky noise [12)] due to the finite particle number. The corresponding current may be written in terms of the average beam current $I_o$ as

$$I_n = \sqrt{2\ e\ q\ f_{rev}\ I_o}$$

where we assume high mode number such that $n\ |\eta|\ \frac{\Delta p}{p} \gtrsim 1$.
"Catastrophic" growth has occurred when this current becomes comparable to the DC component i.e. when

$$I_n\ e^{t/\tau} = I_o$$

From which one obtains:

$$t/\tau = \tfrac{1}{2}\,\ell n(I_o/2\ e\ q\ f_{rev}) \simeq 17 \text{ in HIS}$$

This would suggest that, say, 15 growth periods are acceptable.

On the other hand assume that a 1 per mille high frequency modulation remains as a memory of the linac bunch structure. Then, $\ell n\ 10^3 \approx 7$ e-foldings lead to large blow-up, $t/\tau \approx 5$ may be acceptable.

As a result with "pessimistic scaling" one expects deterioration of the HIS beam after, say, 1 ms. With "optimistic scaling" the corresponding time is $\approx$ 10 ms. Beam may be required in an HIS for several ms. Assume for instance $N_{SR} = 5$ rings with $S = 60$ turn injection in each ring from the same linac. Then, the first ring has to hold beam for at least $t_{rev} \times N_{SR} \times S \approx 2$ ms before all rings can simultaneously eject onto the pellet.

## CONCLUSION

Longitudinal stability in a HIS is an important and challenging problem with possible repercussion on parameters like the number of rings, the aperture of the vacuum chamber (cost !) , the ion charge state, the storage time, etc.

A beam environment with a low resistance is important to permit safe beam storage for several milliseconds. Care has to be taken about cavities, ferrite structures, ceramics etc. to keep their coupling resistance $R_n/n$ as low as a few Ohms even for singly charged ions. For higher charge states the tolerable impedances are lower (in proportion to the square of the ion charge if the same number of ions is used).

Impedance values of a few Ohms are at least as good as those obtained (after work !) in PS and in ISR where $|Z_n/n|$ is estimated to be about 20 $\Omega$ and R/n of the order of 2 - 15 $\Omega$ over the frequency range of concern.

There are interesting differences between the HIS and the proton case, amongst them :

The $1-\beta^2$ cancellation of space charge forces is ineffective in a HIS (typically $\beta \approx 0.3$). Hence the capacitive impedance due to space charge (direct and images on perfect walls) is large, typically 1,5 kΩ compared to a few Ohms in ISR and PS at high energy .

To the extent that this impedance cannot be compensated by inductive walls (a difficult task for 1500 Ω) a "typical" HIS beam will always be unstable with growth proportional to the resistive "wall imperfection". What one can hope for then is slow enough growth within the required few milliseconds of storage time. On the positive side, the frequency band of importance, is, say, 1 GHz in a HIS rather than 20 GHz or more in PS and ISR, because due to Lorentz contraction the cut off wavelength is $\gamma$-times shorter in high energy proton machines. The smaller frequency band might make it easier to improve the effective coupling resistance and/or device feedback stabilization.

# T A B L E 1

## Assumed Parameters

| | | | PS (at high energy) | ISR (injected pulse) | HIS (typical) |
|---|---|---|---|---|---|
| Circumference/$2\pi R$ | | (m) | 100 | 150 | 100 |
| Relativistic factors | $\beta$ | | 1 | 1 | 0.3 |
| | $\gamma$ | | 20 | 25 | 1 |
| Beam Current | I | (A) | 1 | 1 | 60 |
| Revolution Frequency | $f_{rev}$ | (kHz) | 475 | 315 | 140 |
| Energy parameter | $\frac{\beta^2\gamma\, U_p A}{q}$ | (GeV) | 20 | 25 | 20 |
| Off energy function | $\eta = \gamma_t^{-2} - \gamma^{-2}$ | | 1/38 | 1/100 | - 1 |
| Half height of beam chamber | h | (mm) | 35 | 30 | 40 |
| Cutt off mode number | $n_c = \gamma R/h$ | | $6 \times 10^4$ | $1,2 \times 10^5$ | $2,5 \times 10^3$ |
| Space charge impedance | $\frac{377\ \Omega}{\beta\ \gamma^2}(\frac{1}{2} + \ln \frac{b}{h})$ | ($\Omega$) | 1 | 0.6 | 1500 |
| Wall impedance | $\frac{R_n}{n}$ | ($\Omega$) | 2-15 | 2-15 | 2-15 |
| | $\frac{X_n}{n}$ | ($\Omega$) | 20 | - 20 | << 1500 |

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# APPENDIX

## Coupling impedance of a beam in a circular tube

The fields can readily be obtained e.g. from the static field of a ring of charge in a perfect tube as given in text books. A Lorentz transformation yields the field of a moving ring. Integrating over a perturbed coasting beam, assuming uniform radial density and working out the average electric field yields the coupling impedance (in terms of modified Bessel functions I and K) as

$$\frac{Z_n}{n} = i\,\frac{2 \times 377\ \Omega}{\beta\,\gamma^2\ \ x^2}\ \{ 1 - 2\,I_1(x)\,[K_1(x) + \alpha I_1(x)]\ \} \qquad \text{(A1)}$$

where $\alpha \approx \dfrac{K_o(x_w) - rK_1(x_w)}{I_o(x_w) + rI_1(x_w)}$

$$r = \frac{\beta\ \gamma\ Z_s}{i\ 377\ \Omega}$$

$$Z_s = -\left.\frac{E_z}{H_\phi}\right/_{wall} \qquad \text{(wall "surface impedance")}$$

$$x = \frac{n \text{ beam radius (b)}}{\text{orbit radius (R)}}$$

$$x_w = \frac{n \text{ chamber half height (h)}}{\text{orbit radius (R)}}$$

For low modes ($x_w < 1$) this yields

$$\frac{Z_n}{n} = \frac{i\ Z_o}{\beta\ \gamma^2}\ [\ell n(\frac{h}{b}) + \frac{1}{4}] + \frac{Z_s\ R}{n\ \ h}$$

The factor ¼ rather than ½ as assumed above comes from the fact that we average over a uniform beam. For the central field in a uniform beam or the average over a Gaussian beam ½ is more realistic.
[If one wants to work in terms of the central field one can still use (A1) but replace the term $2I_1(x)$ in front of the innermost bracket by $x$]

Note that for $x_w \ll 1$ the wall impedance $Z_w = \frac{Z_s R}{h}$ simply adds to $Z_n$. The behaviour over a wider frequency range is illustrated in Fig. 1 where $R_n/n$ and the "growth rate"

$$\left( \frac{R_n/n}{2\sqrt{X_n/n}}\, x \right)$$

are drawn from (A1) assuming parameters such that the wall imperfections give $Z_w/n = 15\ \Omega$ independent of frequency and space charge yields $Z_n/n = i\ 1.5\ k\Omega$ for low modes. Equation (A1) can also be used to calculate the surface impedance required for perfect compensation and the "residual" impedance $Z_n$ (which can become resonant ! ) for non perfect compensation of the space charge terms. Note that (A1) is an approximation valid for small surface impedance. More general results are given by Keil and Zotter[14].

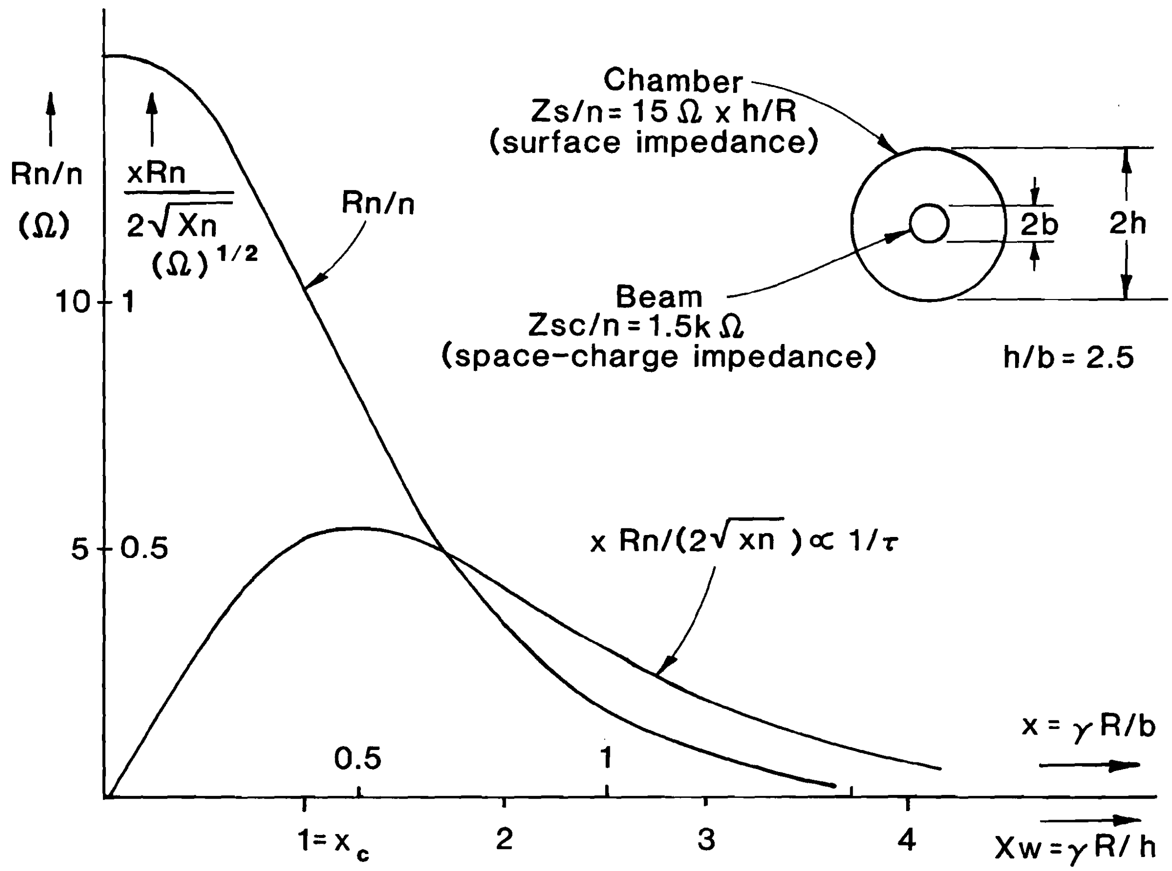
Chamber
Zs/n = 15 Ω x h/R
(surface impedance)
Beam
Zsc/n = 1.5k Ω
(space-charge impedance)
2b
2h
h/b = 2.5
Rn/n
(Ω)
xRn / 2√Xn
(Ω)^1/2
Rn/n
x Rn/(2√xn) ∝ 1/τ
10
1
5
0.5
0.5
1
x = γR/b
1 = x_c
2
3
4
Xw = γR/h

Fig. 1