INTEGER RESONANCE CROSSING IN H.I. ACCUMULATOR RING

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1. INTRODUCTION

The R.F. Linac scheme requires current multiplication in order to reach the necessary amount of peak power on the target. As part of this multiplication, a complementary accumulator ring is filled up to its current limitation within the usual Laslett tune-shift limit $\Delta\nu = 0.25$. However, during compression the tune can spread over, at least, one integer. The compression time is of the order of several tens of revolutions: (the revolution period is of the order of 5 μs).

Experimentally, on existing circular machines, relatively fast crossing of half-integer resonances has been achieved, which is not true for integer resonances. However, in the case of a heavy ion accumulator ring there is concern with relatively fast crossing of such resonances during the compression time (a fraction of a synchrotron oscillation period), the beam being ejected after this operation.

In what follows, two simple approaches are considered for the transient particle dynamics when the operating tune is very close to an integer resonance and assuming dipole defects are distributed around the ring.

2. INTEGER RESONANCE BEHAVIOR IN CASE OF STATIC TUNE ($\nu \approx k$)

Let's assume a single defect corresponding to a field error $\frac{\delta B}{B} = 10^{-4}$ over a magnetic length of 6 meters. For $\nu = k$, a particle initially moving on the reference closed orbit (axis of the magnetic structure) will get, at each revolution, the same angular kick at a constant betatron phase. Obviously the amplitude of the induced betatron motion will increase linearly with the number of revolutions

$$\hat{x}_n = n \frac{\delta B \Delta s}{B\rho} \beta_x$$

where $\frac{\delta B}{B} \Delta s$ is the integrated field error, n the number of revolutions, β_x the envelope function $\left(\beta_x \approx \frac{R}{\nu_x}\right)$, ρ the bending radius.

Using previous numbers with, in addition, $\beta_x = 10$ meters and $\rho = 50$ meters, one gets

$$\hat{x}_n = \frac{n}{10} \text{ (mm)} .$$

An amplitude growth of 1 cm is then obtained after 100 revolutions.

Assume now that N dipole defects are randomly distributed around the circumference. Considering only the bending magnets as error sources, a realistic value is $N \approx 50$. With no correllation between errors, this leads to:

$$\hat{x}_n = \frac{n\sqrt{N}}{10} \text{ (mm)}$$

which now corresponds to an amplitude growth of 7 cm after 100 turns.

This is certainly unacceptable. However, in the absence of non-linearities, the tune will vary across the integer during compression which increases the space charge forces progressively. This is the object of the next Section.

3. INTEGER RESONANCE CROSSING: VARIABLE TUNE

Using the smooth approximation for the optics, the particle motion in the transverse plane can be written as follows:

$$\frac{d^2x}{d\theta^2} + \nu^2(\theta)\cdot x = F(\theta) , \quad ds = Rd\theta ,$$

where $F(\theta)$ represents the dipole defects distributed along the circumference.

$$F(\theta) = R^2 \frac{\delta B(\theta)}{B\rho} .$$

As usual, let's take the main component of its Fourier expansion:

$$\frac{d^2x}{d\theta^2} + \nu^2(\theta)\cdot x = R^2 F_k \cos(k\theta + \alpha_k) \quad ,$$

where k is the integer close to the tune. One will assume that k is different from a multiple of $\frac{M}{2}$, where M is the periodicity of the magnetic structure. Then the envelope function β_x is not expected to change when ν varies.

Assuming also that ν varies slowly, one can get the adiabatic solution for the homogeneous equation (absence of defects), through the W.K.B. approximation:

$$x(\theta) = Ax_1(\theta) + Bx_2(\theta) \quad ,$$

where:

$$x_1(\theta) = \nu(\theta)^{-1/2} \cos\{\int^{\theta} \nu(\theta)\, d\theta\}$$

$$x_2(\theta) = \nu(\theta)^{-1/2} \sin\{\int^{\theta} \nu(\theta)\, d\theta\}$$

In the presence of defects, the general solution can be obtained by using the method of variation of parameters, which leads to:

$$\frac{dA}{d\theta} = - R^2 F_k\, x_2(\theta) \cos k\theta$$

$$\frac{dB}{d\theta} = R^2 F_k\, x_1(\theta) \cos k\theta$$

With the following assumptions in addition:

i) $x(0) = 0$

ii) $\nu = \nu_0 + k - a\theta$ — where ν_0 is the non integer part of the tune at $\theta = 0$.

iii) $\Delta\nu << k$ — total tune variation $(0 \leq \theta < \infty)$,

it comes out that:

$$A(\theta) = -\frac{R^2F_k}{\sqrt{k}} \int_0^{\theta} \sin\left\{(\nu_0 + k)\theta - a\theta^2\right\} \cos k\theta . d\theta$$

$$B(\theta) = \frac{R^2F_k}{\sqrt{k}} \int_0^{\theta} \cos\left\{(\nu_0 + k)\theta - a\theta^2\right\} \cos k\theta . d\theta \quad ,$$

and $\quad x(\theta) = A(\theta).x_1(\theta) + B(\theta)\, x_2(\theta) \quad .$

It looks realistic to integrate for A and B up to infinity, because as soon as the tune has moved away from the integer the amplitudes will stop growing.

The fast varying terms in the previous equations can be neglected, keeping in mind that half of the effect will happen before the integer is crossed:

$$A(\infty) \approx \frac{R^2F_k}{\sqrt{k}} \int_0^{\infty} \sin(a\theta^2) d\theta$$

$$B(\infty) \approx \frac{R^2F_k}{\sqrt{k}} \int_0^{\infty} \cos(a\theta^2) d\theta \quad .$$

According to the well known Fresnel's integrals, after a while the amplitude of the particle motion becomes:

$$\boxed{\hat{x}(\infty) = \frac{1}{2}\, \frac{R^2F_k}{k} \sqrt{\frac{\pi}{a}}}$$

which is inversely proportional to the square root of the crossing speed.

For comparison let's consider the case a = o and $\nu = k$. From previous equations one gets:

$$A(\theta) = 0$$

$$B(\theta) = \frac{R^2F_k}{2\sqrt{k}}\, \theta$$

$$\hat{x}(\theta) = \frac{1}{2}\, \frac{R^2F_k}{k}\, \theta$$

showing that the amplitude is now increasing linearly with θ.

From the first section, and assuming $R = 1.5 \times \rho$, one can roughly deduce:

$$F_k \approx 2.8 \times 10^{-7} (m^{-1})$$

The variable tune case then gives:

$$\hat{x}(\infty) \simeq \frac{1.25 \times 10^{-6}}{\sqrt{a}} \quad (m)$$

For a total tune shift of 0.5 over 100 turns one gets:

$$\hat{x} = 4.5 \times 10^{-2} \text{ mm}$$

which looks quite reasonable.

However in practice, the non-linearities of the space charge forces may keep the tune on the integer, or very close, as soon as the amplitude starts to grow. The truth may be somewhere between the two approaches.

It will certainly be very interesting to study integer resonance crossing on an existing machine experimentally as a function of closed orbit correction. A crossing time less than 1 ms is required to answer our present worry.

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