BUNCH COMPRESSION IN HEAVY ION FUSION STORAGE RINGS

M. Cornacchia (BNL) and G. H. Rees (Rutherford Laboratory)

1. INTRODUCTION

Bunch compression in the proposed heavy ion fusion storage rings results in large transverse and longitudinal space charge forces. In this note parameters are derived for the bunch compression in three reference designs and aspects of the transverse and longitudinal motion are discussed.

Transverse and longitudinal space charge forces on an ion depend on both the transverse and azimuthal co-ordinates of the ion in the bunch. It appears therefore to be insufficient to treat the transverse and longitudinal motions separately in the non-adiabatic compression and that a detailed evaluation of the problem requires elaborate numerical simulation or an experimental test facility.

2. REFERENCE STORAGAE RING DESIGNS

Parameters used for the three reference designs are:

	CASE A	CASE B	CASE C
Ion	U^+	U^+	U^+
Kinetic Energy	5 GeV	10 GeV	10 GeV
Target Bunch duration	20 ns	40 ns	70 ns
Initial Bunch duration	1 μs	2 μs	3.5 μs
Revolution period	6 μs	6 μs	7 μs
Bunches/ring	6	3	2
Bending radius (B = 5T)	31.6 m	44.9 m	44.9 m
Mean radius	59.8 m	83.1 m	97.1 m
Emittance/π	60×10^{-6} rad m	37.5×10^{-5} rad m	60.0×10^{-6} rad m
I_{DC} ($\Delta\nu$=1/4)	18 A	23.3 A	32 A
50 I_{DC}	900 A	1165 A	1600 A
Number of bunches	24 (or 42)	12	18
Number of rings	4 or 7 ($\Delta\nu$ = 1/7)	4	9
Particles/ bunch, N	1.12 or 0.64×10^{14}	2.91×10^{14}	7×10^{14}
Energy in target	> 1 MJ	> 3 MJ	> 10 MJ
P(TW)	100 TW	150 TW	300 TW

The total bunch compression of a factor 50 is assumed to consist of a factor 7 in the storage rings and a factor 50/7 in the transport line to the target. The maximum momentum spread allowed at the target has been specified as $\Delta p/p = \pm 1\%$ and this infers, in the absence of dilution, a momentum spread for the unbunched storage ring beam of $\Delta p/p = \pm 2B_f \times 10^{-4}$ where B_f is a final bunch shape factor (approximately 2/3).

3. TRANSVERSE MOTION DURING COMPRESSION

Reference design B will be used to indicate the scale of the effect of the transverse space charge forces on the betatron motion. Ring B has a mean radius of 83.1 m and the following lattice design was assumed:

Betatron tunes ν_x, ν_y	8.3, 8.3
Number of cells	28
Cell length	18.65 m
Integrated quadrupole strength $\int g d\ell$	41.5 T
Peak value of beta-function, $\hat{\beta}$	33.1 m
Peak value of dispersion function, $\hat{\eta}$	2.2 m
Gamma transition, γ_t	7.7
Minimum value of beta-function, $\check{\beta}$	4.0 m
Minimum value of dispersion function, $\check{\eta}$	1.0 m

At a bunch compression of 7 and an assumed bunch shape factor of 2/3 the resulting bunching factor B = 0.0952 = 1/10.5 for a uniform transverse density.

The parameters above have been used in a program of A. Garren that solves the equation of the betatron envelope in the presence of linear transverse space charge forces. The program calculates the equivalent modifications to the lattice functions, and for B = 1/10.5 the following results were found (when B = 1 corresponded to $\Delta\nu = 0.25$):

$\nu = 8.3 \rightarrow 6.05$

$\hat{\beta}_x = 33.1 \rightarrow 38.6$ m

$\check{\beta}_x = 4.0 \rightarrow 6.23$ m

$\hat{\eta} = 2.2 \rightarrow 4.0$ m

$\check{\eta} = 1.0 \rightarrow 1.85$

$\gamma_t = 7.7 \rightarrow 5.6$

Results for other values of $\overline{B}$ are given in Fig. 1.

The dependence of the lattice functions on the bunching factor, depicted in Fig. 1, results effectively in a spread of values of these functions along the bunch, due to the longitudinal variation of the space charge density. Since different particles have different optics, it is impossible to completely match the beam ejected from the storage ring into the transfer line. The result is a blow-up of emittances. For the case B under consideration the maximum increase in horizontal betatron amplitude due to the dispersion mismatch is, for a bunching factor

$\overline{B} = \frac{1}{10.5}$:

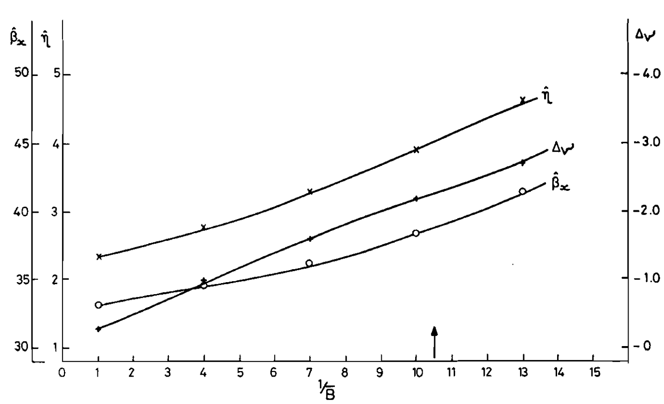

FIG.1. CHANGE IN LATTICE FUNCTIONS FOR CASE B (ν=8.3)

$$\Delta x_{disp} \simeq (\eta_1 - \eta_0) \cdot \frac{\Delta p}{p} = (4 - 2.2) \times 4 \times 10^{-3} = 7.2 \text{ mm},$$

where

η_0 = maximum value of the unperturbed dispersion function in the ring = 2.2 m

η_1 = dispersion in presence of space charge = 4 m

$\frac{\Delta p}{p}$ = maximum momentum deviation at the end of the compression = $\pm 4 \times 10^{-3}$. (This value is discussed in Section 4).

The mismatch of the β functions also introduces an increase in the horizontal and vertical betatron amplitudes of the order of

$$\Delta x_{bet} = \sqrt{\varepsilon_0 \beta_1} - \sqrt{\varepsilon_0 \beta_0} = \sqrt{37.5 \times 38.5} - \sqrt{37.5 \times 33} = 2.8 \text{ mm},$$

where

ε_0 = injected beam emittance = $37.5\pi \times 10^{-6}$ rad m

β_0 = unperturbed β function = 33 m

β_1 = β function in presence of space charge = 38.5 m.

It is debatable whether the blow-up of betatron amplitudes discussed above should be added linearly to the unperturbed amplitudes of the injected beam in order to estimate the resultant emittance blow-up. If we do so, we find, for the relative emittance increase,

$$\frac{\Delta\varepsilon}{\varepsilon_0} = \frac{x_1^2 - x_0^2}{x_0^2} = \frac{45.2^2 - 35.2^2}{35.2^2} = 0.65 ,$$

where x_1 and x_0 are the maximum perturbed and unperturbed betatron amplitudes.

If, instead, we add the amplitude increase quadratically, the relative emittance blow-up is only 4.8%. A statistically significant analysis of this effect would require further study; here, we shall arbitrarily settle for approximately half the value calculated from the linear addition of betatron amplitudes, and assume:

$$(\Delta\varepsilon/\varepsilon_0)_{mismatch} = 30\% .$$

Figure 1 shows that a considerable fraction of particles (those in the higher charge density region) will cross two integer and four 1/2 integer resonances during the compression. We estimate first the blow-up in crossing the integer resonance.

For N random dipole errors in the ring, the oscillation amplitude increase is given by (r.m.s value):

$$\left\langle\frac{d\bar{x}}{d\theta}\right\rangle = \frac{\sqrt{N}\cdot\ell_{dip}\sqrt{\beta_{av}}}{2\sqrt{2}\pi\rho}\left\langle\frac{\Delta B}{B}\right\rangle \sin\phi = \frac{\sqrt{\beta_{av}}}{\sqrt{2N}}\left\langle\frac{\Delta B}{B}\right\rangle \sin\phi , \quad (1)$$

where $\bar{x}$ = normalized maximum oscillation amplitude, horizontal or vertical (i.e. max. ampl. $\div \sqrt{\hat{\beta}}$).

θ = circumferential angle around the accelerator, in radians.

ℓ_{dip} = length of a single dipolar perturbation.

β_{av} = average value of horizontal or vertical β function at the peturbation location.

ϕ = betatron phase angle at a reference point in the ring.

ρ = bending radius.

$\left\langle\frac{\Delta B}{B}\right\rangle$ = r.m.s. value of random dipole errors.

In equation (1), we have made the simplifying assumption that the sources of errors come exclusively from the main dipoles. We assume that the tune change occurs linearly. Thus the non-integer part of the tune at the reference point in the ring is given by $\nu_f = \nu_{of} + A\theta$ and equation (1) becomes:

$$\left\langle\frac{d\bar{x}}{d\theta}\right\rangle = \frac{\sqrt{\beta_{av}}}{\sqrt{2N}}\left\langle\frac{\Delta B}{B}\right\rangle \sin\left(\nu_{of}\theta + \frac{A\theta^2}{2} + \phi_0\right) , \quad (2)$$

where ϕ_0 is a constant.

The expectation value of the normalized betatron amplitude after crossing is:

$$\langle\bar{x}\rangle_{final} = \frac{\sqrt{\beta_{av}}}{\sqrt{2N}}\left\langle\frac{\Delta B}{B}\right\rangle\int_{\theta_0}^{\theta}\sin\left(\nu_{of}\theta + \frac{A\theta^2}{2} + \phi_0\right)d\theta . \quad (3)$$

If the points θ_0 and θ are chosen well away from the resonance, the integral can be approximated by:

$$\sqrt{2\pi/A}\,\sin\left[\phi_0 - (\nu_{of}^2/2A) + (\pi/4)\right]$$

Thus, the final amplitude after crossing for the most unfavorable phase is:

$$\langle\bar{x}\rangle_{final} \simeq \frac{\sqrt{\beta_{av}}}{\sqrt{2N}}\left\langle\frac{\Delta B}{B}\right\rangle\sqrt{\frac{2\pi}{A}}$$

It is indicative to relate the residual dipole field perturbation $\langle\Delta B/B\rangle$ to a realistic value of the closed orbit distortions after correction for the unperturbed working point ν_0. The expectation value of the orbit distortion is given by, in normalized amplitude:

$$\langle\bar{x}_{co}\rangle = \frac{\sqrt{\beta_{av}}}{\sqrt{2}\,\sin\pi\nu_0}\,\frac{\pi}{\sqrt{N}}\left\langle\frac{\Delta B}{B}\right\rangle . \quad (5)$$

Expressed in terms of the above closed orbit error, equation (4) becomes:

$$\langle\bar{x}\rangle_{final} = \langle\bar{x}_{co}\rangle \sin\pi\nu_0 \sqrt{2/\pi A} . \quad (6)$$

Then, for $\langle\bar{x}_{co}\rangle$ assume 1 mm $\div \sqrt{\beta}$.

In estimating the rate of tune change, we ignore the sinusoidal variation of the bunch length and take the average over the whole tune-span. This approximation is justified by the fact that the integer resonance is crossed twice and it is not possible to choose the fastest rate of tune change for both crossings. We choose, therefore $A = (8.3 - 6.1)/(2\pi \times 68) = 0.0051\ rad^{-1}$ where the number 68 is the number of bunch rotations during compression: see Section 4. Eqn. (6) gives the final normalized betatron amplitude after one crossing of the integer resonance. For two crossings, the value increases by $\sqrt{2}$, and we obtain:

x_{final} (not normalized) $= \sqrt{\hat{\beta}} \left\langle \bar{x} \right\rangle_{final} = 12.8$ mm

For the half-integer resonance we first calculate the stop-band half-width(1):

$$\left\langle \delta\nu \right\rangle = \frac{1}{4\pi B\rho} \left[\sum_i \beta^2_{xi} \right]^{1/2} \left\langle \Delta g \right\rangle \ell_q ,$$

where $\left\langle \Delta q \right\rangle$ is the r.m.s. of the gradient errors in the quadrupole of length ℓ_q. Assuming, for the lattice under consideration:

$$\left\langle \frac{\Delta g}{g} \right\rangle = 10^{-3} ; \frac{g\ell_q}{B\rho} = 0.18 \text{ m}^{-1}, \; g = \text{main}$$

quadrupole gradient,

$$\left\langle \delta\nu \right\rangle = \frac{0.18}{4\pi} \left[\sum_i \beta^2_{xi} \right]^{1/2} \left\langle \frac{\Delta g}{g} \right\rangle \approx \frac{0.18}{4\pi} \sqrt{28} \times 33 \times 10^{-3}$$
$$= 0.0025 .$$

The amplitude blow-up after crossing is given by:

$$\frac{x}{x_0} = \exp \left[\frac{2\pi\delta\nu}{\left(2\Delta\nu_r\right)^{1/2}} \right] ,$$

where x_0 = initial betatron amplitude

$\Delta\nu_r \doteq$ average tune change per turn

$$= \frac{8.3 - 6.1}{68} = 0.032 .$$

From eqn. (6) we find the blow-up per crossing:

$$\frac{x}{x_0} = 1.06 .$$

For 4 crossings, the amplitude growth increases by $\sqrt{4}$:

$$\left(\frac{x}{x_0} \right)_{final} = 1.12 .$$

If we calculate the relative emittance blow-up, assuming that the betatron amplitudes are added linearly, we find:

For integer resonance: $\frac{\Delta\varepsilon}{\varepsilon_0} = 90\%$,

For $\frac{1}{2}$-integer resonance: $\frac{\Delta\varepsilon}{\varepsilon_0} = 26\%$.

(1) C. Bovet et al., A Selection of Formulae and Data Useful for the Design of AG Synchrotrons, CERN/MPS-SI/Int. DL 70/4, (1970).

For the integer resonance, the linear addition of amplitudes is justified by the fact that there is a coherent displacement of a cross-section slice of the beam: the 90% emittance blow-up concerns only that fraction of the beam which crosses the resonance twice, which is also, unfortunately the region with higher particle density. For the half-integer resonance, the blow-up is incoherent and a quadratic increase is probably more statistically realistic, which makes this resonance considerably less dangerous than the integer resonance. Finally, the magnetic field quality we have assumed for the dipole and quadrupole errors is, of course, subject to discussion.

The change in the bunching factor as the bunch compression proceeds is shown in Fig. 2 for the reference Case B.

The magnitude of the effects indicates that it may subsequently be necessary to reduce somewhat the number of particles per ring and increase the number of rings.

4. LONGITUDINAL MOTION DURING COMPRESSION

The reactive impedance due to the longitudinal space charge forces is:

$$\frac{Z}{n} = - jgZ_0/2\beta\gamma^2 . \qquad (9)$$

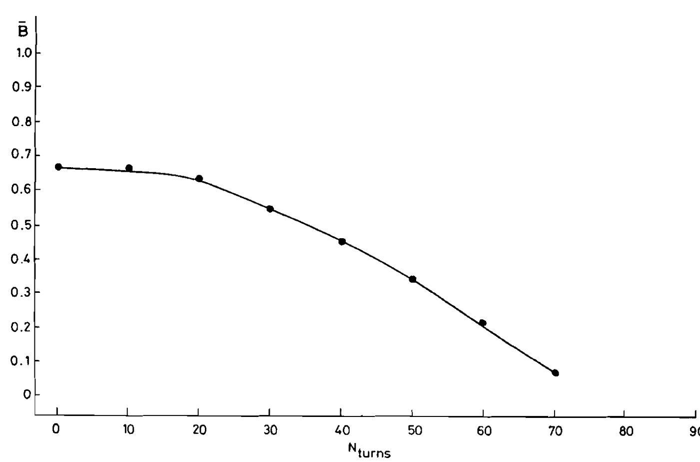

FIG. 2 CASE B: BUNCHING FACTOR Vs NUMBER OF COMPLETED REVOLUTIONS

Assuming a value g = 2

$$\frac{Z}{n} \text{ (at 5 GeV)} = -j\,(1728)\Omega$$

$$\frac{Z}{n} \text{ (at 10 GeV)} = -j\,(1189)\Omega \quad .$$

The equivalent negative inductance values are:

	Case A	Case B	Case C
L =	-1.650 mH	-1.134 mH	-1.325mH .

The large values of Z/n and L are a consequence of the low β-values. To estimate the magnitudes of the longitudinal space charge forces, the storage ring bunches are assumed to have a parabolic form in the azimuthal direction so that the maximum energy gain or loss per turn is:

$$eV_{sc} = \pm\, 3\, NeL/2T^2, \qquad (10)$$

where T is half time duration of the bunch. The energy gains/losses per turn for the three reference designs are:

	Case A	Case B	Case C
-L	1.650 mH	1.134 mH	1.325 mH
N	0.64×10^{14}	2.91×10^{14}	7×10^{14}
T (min)	71.4 ns	142.8 ns	250 ns
eV_{sc}(max)	±4.98 MeV	±3.89 MeV	±3.57 MeV

The maximum levels are achieved just prior to ejection. The forces vary linearly along the extent of the bunch, being zero at the center. The energy gains will differ from the above values for bunch shapes other than parabolic and the variation with position will then become non-linear.

For rapid bunch compression it is necessary to compensate for the longitudinal space charge forces and to provide additional focusing fields. A closed curve in (Δp/p, ϕ) space containing all the beam particles may be made to rotate and reach a minimum phase extent after a quarter of a synchrotron period. For a linear rotation, the RF voltage waveform must be of a saw-tooth form and not sinusoidal. The bunch is extracted before reaching the quarter wavelength point to allow further rotation and compression in the external transfer line to the target.

When the RF waveform is of a saw-tooth form, the number of rotations of the bunch in the ring during a quarter of a synchrotron period is given by n_q:

$$n_q = \pi/[4(\gamma^{-2} - \gamma_t^{-2})\, h\, (\Delta p/p)] \, ,$$

where h is the harmonic number of the saw-tooth wave (= no of bunches),

γ_t is gamma transition, and

(Δp/p) is the peak momentum spread at the quarter synchrotron point.

The peak voltage of the saw-tooth waveform for the rotation is given (when the space charge forces are compensated separately) by V:

$$eV = \pm\, 2E\pi^2\beta^2/(\gamma^{-2} - \gamma_t^{-2})\, h\, (4n_q)^2 \quad . \qquad (12)$$

E is γE_0 where E_0 is the rest energy of the ion.

V is proportional to $h\,\beta^2\,(\Delta p/p)^2\,(\gamma^{-2} - \gamma_t^{-2})$.

Parameters for the three reference cases are:

	Case A	Case B	Case C
h	6	3	2
f_0	166.7 kHz	166.7 kHz	142.8 kHz
hf_0	1 MHz	500 kHz	285.7 kHz
γ_t	5.54	7.7	9.0
Δp/p	$\pm4\times10^{-3}$	$\pm4\times10^{-3}$	$\pm4\times10^{-3}$
n_q	35.4	72.8	108.7
eV	±1.76 MeV	±1.69 MeV	±1.13 MeV
n	33	68	102

Here n is the actual number of bunch rotations in the storage ring and is given approximately by:

$$n = 2n_q \cos^{-1}(C^{-1} - D^{-1})/\pi$$

where C is the bunch compression factor 7, and D is the ratio of the peak $\Delta p/p$ after and before compression (a factor of 20 for $\Delta p/p = \pm 4 \times 10^{-3}$).

The value of $\Delta p/p = \pm 4 \times 10^{-3}$ has been assumed instead of the maximum allowed in the transfer line ($\pm 10^{-2}$) for two reasons:

a) to limit the amplitude of the saw-tooth RF waveform,
b) to limit one of the emittance blow-up effects discussed in Section 3.

The choice of $\Delta p/p = \pm 4 \times 10^{-3}$ infers that additional compression is undertaken in the transfer line to the target station. As an example parameters are given for Case B, assuming an external transfer line of length twice the circumference of ring B and the momentum spread increasing to $\pm 10^{-2}$.

T(1/2 bunch duration)	142.8 ns (initially)	20 ns (at target)
Space charge compensation	± 7.45 keV/m	± 380 keV/m
Additional compression	± 330 keV/m	± 46.2 keV/m,

where energy gains quoted correspond to the particles at the tail and head of the bunch.

At the end-point of compression in the storage ring the ratio of the saw-tooth compression forces to the space charge forces is:

	Case A	Case B	Case C
Ratio	5.0 %	6.2 %	4.5 %

Thus an error in the space charge compensation can readily lead to incorrect rotation of the bunch. Such an error may arise if the bunch shape is not of the assumed parabolic form, a probable event since the compression is non-adiabatic. For this reason it appears advantageous to compensate for the longitudinal space charge forces by including passive inductive cavities in the ring to cancel the equivalent negative inductance of the space charge forces. For Case B this would infer a total inductance of 1.134 mH.

A possible system for Case B would have 378 ferrite loaded cavities, each 3μH, with a maximum voltage per unit of ± 10 kV. The lowest cavity resonance would have to be > 15 MHz and all the resonances would have to be heavily damped. Ferrite Q values in the frequency range up to 15 MHz can be of the order 100 so that the total resistive component of $\frac{Z}{n}$ due to the inductive cavities may be approximately 10 Ω provided it proves possible to damp adequately the resonant modes.

The RF system to provide the saw-tooth waveform has received insufficient study. It may be adequate to approximate the saw-tooth by adding an appropriate number of harmonics, and if this is so, the later stages of the compression may be more readily controlled by switching to one of the higher harmonic RF systems at a lower peak voltage level. The effective $\frac{Z}{n}$ values for the individual cavities may be reduced by feed-forward techniques and by damping of higher modes. The limit to the reduction in $\frac{Z}{n}$ from a fed-forward system is set by the system-gain stability. Because of the peak beam current levels involved, the gain may have to be reduced as the compression proceeds. Without further sutdy of the RF systems and the storage rings it is not possible to say if there is adequate straight section space for the RF. There is probably not for the reference Case A which has the smallest ring circumference and largest RF requirements.

Longitudinal emittance dilution in the bunch compression is very important to evaluate since in the reference cases there is no allowance at all for any dilution. The most likely cause of emittance dilution is the effect of the variation of the longitudinal space charge forces across the transverse dimensions of the beam. In the derivation of the longitudinal space charge forces it has been assumed that the parameter g, is a constant equal to 2. The exact expressions for g depend on the chamber dimensions and of the transverse density distribution in the beam.

For a circular beam centered in a circular chamber with a uniform transverse density distribution:

$$g = 1 + 2\,\ell n\left(\frac{b}{a}\right) - \left(\frac{r}{a}\right)^2 \qquad (14a)$$

where b is the chamber radius, a the beam radius and r the individual particle radius.

For a parabolic density distribution, the formula is modified to:

$$g = \left(\frac{3}{2}\right) + 2\,\ell n\left(\frac{b}{a}\right) - 2\left(\frac{r}{a}\right)^2 + \frac{1}{2}\left(\frac{r}{a}\right)^4 . \qquad (14b)$$

For an elliptical beam in an elliptical chamber, with the elliptical contours confocal, and for a uniform transverse density distribution:

$$g = \frac{2b_1b_2}{b_1^2 + b_2^2} + 2\,\ell n\left(\frac{b_1 + b_2}{a_1 + a_2}\right) - G , \qquad (14c)$$

$$\text{where } G = 2\left(\frac{2b_1b_2}{b_1^2 + b_2^2} - \frac{a_2}{a_1}\right)\left(\frac{x^2}{a_1^2 - a_2^2}\right)$$

$$+ 2\left(\frac{a_1}{a_2} - \frac{2b_1b_2}{b_1^2 + b_2^2}\right)\left(\frac{y^2}{a_1^2 - a_2^2}\right) ,$$

(x,y) are particle transverse co-ordinates, (a_1, a_2) are semi-axes of the beam elliptical cross-section, and (b_1, b_2) are semi-axes of the chamber elliptical cross-section.

All three expressions for g indicate a large variation of the longitudinal space charge forces across a transverse section of beam. For example, in the first expression, eqn. (14a), g varies from

$$g = 1 + 2\,\ell n\,\frac{b}{a} \qquad \text{at } r = 0$$

$$\text{to } g = 2\,\ell n\,\frac{b}{a} \qquad \text{at } r = a$$

Since the space charge forces are larger than the saw-tooth compression forces there must be a pronounced effect on the bunch compression.

Numerical simulation is required to evaluate the problem. A realistic simulation must include both longitudinal and transverse motions, and at present it is unknown if realistic 3-D tracking programs can be developed.

5. CONCLUSIONS

1) Large transverse and longitudinal space charge forces are present in the beam during the bunch compression in the reference storage rings. Some dilution in the transverse and longitudinal emittances must therefore be expected.

2) For the transverse motion, dilution occurs due to crossing of betatron resonances and mismatch in transfer to the external beam line. For the Case B considered the transverse emittance/π was assumed to be 37.5×10^{-6} rad m and it is probable that a larger emittance could be tolerated in the target chamber for this case. Emittance dilution of the order 2 has been estimated but the estimates are crude and should be checked by numerical simulation.

3) Some dilution in the longitudinal emittance must be expected because of the variation of the longitudinal space charge forces across the transverse dimensions of the beam; thus it is not possible to have exact compensation of the longitudinal forces for all beam particles. It is important to evaluate the magnitude of the dilution because there are restrictions on: $\Delta p/p$ in the transfer line (transport difficulties for $\Delta p/p > \pm 1\%$) and $\Delta p/p$ at injection in the storage ring (longitudinal instability considerations). Longitudinal motion is dependent on the transverse and

longitudinal beam distributions and a realistic numerical simulation of bunch compression must include both longitudinal and transverse motions.

4) It is desirable to restrict the maximum momentum spread in the storage ring (to $\pm 4 \times 10^{-3}$ just prior to extraction) in order to limit the size of the RF system for bunch compression and to limit the transverse emittance dilution at transfer to the beam line.

5) It appears to be advantageous for the RF system design to have two RF systems, one to compensate for the longitudinal space charge forces and one to provide the bunch rotation/compression. The system suggested for space charge compensation is to use a large number of passive ferrite loaded cavities (whose resonant frequency is well above the harmonic components of interest in the beam current). The RF system for compression has ideally a saw-tooth waveform.

6) The effects of non-linear transverse density distributions have not been considered but could be important.

7) The compression leads to a large variation of the γ_t function along the bunch which will contribute to the dilution of the longitudinal emittance.