Beam Injection and Accumulation Method in Storage Rings for Heavy Ion Fusion

T. Katayama, A. Noda, N. Tokuda and Y. Hirao

Institute for Nuclear Study, University of Tokyo

Tokyo, JAPAN

ABSTRACT

A combination of multiturn injection and RF stacking is proposed as as efficient beam injection method in storage rings for heavy ion fusion. Five turn injection in each transverse phase space and four RF stackings give a total of 100 stacking turns. This represents a compromise between the tolerable emittances and momentum spread in the ring.

Space charge limitations and coherent beam instabilities are investigated. The most severe limit is found to be the transverse coherent instability, but this can be controlled by the use of sextupole and octupole magnetic fields.

Assuming a charge exchange cross section of 1 x 10^{-15} cm^2, the e-folding life time is estimated at 180 ms, while the stacking time is 40 ms.

1. INTRODUCTION.

The heavy-ion inertial fusion program has become more promising through intensive work[1 ~ 3)] on high-energy heavy-ion accelerators during the past three years. The heavy ion method is superior to those of the other particle beams because of its drastic reduction in the peak current requirement to the order of 1 kiloampere (particle current). This reduction of current is allowed by the comparatively high energy per particle in relation to its range-energy behavior. At present it is a consensus among accelerator physicists that such high currents of heavy ions could be produced, handled, transported and focused on a pellet by the use of conventional high energy accelerator techniques, especially RF linacs with storage rings or induction linacs. It is also true, however, that many kinds of research and development should be pursued: for example it is a serious problem to accumulate heavy ion beams for ~ 100 turns in the limited emittances and momentum spread without any significant beam loss, and to compress them into small bunches in the storage rings.

In the present paper, the combination of multiturn injection in the two transverse phase spaces and RF stacking in the momentum space, is proposed as an efficient beam accumulation method which in principle brings about very small beam loss during the accumulation process. Details of the design of the accelerator are given for U^{+1} ions, as at the Workshop, but the proposed method takes a rather long time and might be generally favored for longer life ions such as Xe^{8+}.

2. REQUIREMENTS ON THE STORAGE RINGS

First we will start from R.O. Bangerter's three cases of target data at this workshop, listed in Table 1 for convenience. Emittance considerations in the beam lines give an upper limit of the allowable transverse emittances in the storage ring as 30 π mm·mrad (unnormalized). The momentum spread at ejection from the storage ring should be lower than ± 0.4 % because the momentum spread at the target is assumed to be ± 2 % and the bunch compression factor in the beam transport lines is designed to be 5.

TABLE 1 3 CASES OF TARGET DATA AND BEAM PARAMETERS.

E(MJ)	p_p(TW)	T(GeV)	r(mm)	t(ns)	t_p(ns)	g
1	100	5	2	20	6	8
3	150	10	2.5	40	16	30
10	300	10	4	70	20	120

CASE	A	B	C
E(MJ)	1	3	10
N($\times 10^{15}$)	1.25	1.875	6.25
T(GeV)	5	10	10
E_p(MJ)	0.6	2.4	6
N_p($\times 10^{15}$)	0.75	1.5	3.75
I_p(A)	2×10^4	1.5×10^4	3×10^4
I_{av}(A)	10^4	0.75×10^4	1.43×10^4

Ions are U^{+1} and the following notation is used. E; Beam stored energy, P; Peak power, T; Kinetic energy, r; Target radius, t; Pulse width, g; gain of the pellet, N; No. of ions. Subcript p refers to peak value at the end of pulse.

Table 2 Ring parameters

Case	A	B	C	
Number of rings (N_{SR})	7	4	9	
Harmonic number	6	3	2	
Particles / ring	1.79	4.69	6.94	($\times 10^{14}$)
Particles / bunch	0.30	1.56	3.47	($\times 10^{14}$)
Emittance	30 π	30 π	30 π	($\times 10^{-6}$ m•rad)
Average radius	59.8	83.1	97.1	m
Radius of curvature (B = 5T)	31.6	44.9	44.9	m
Circumference	375.6	522	610	m

Other parameters of the storage rings are given in Table 2 which are determined by considerations of space charge power limit in beam lines (Courant-Maschke formula), limited tune shift in rings for accumulation ($\Delta\nu = 0.25$), and bunch lengths before and after compression.

Momentum spread of the beam from the injector linac is assumed to be $\pm 2 \times 10^{-4}$ after the debuncher, and the phase spread in the ring after the multiturn injection could be 2π, which means that the beam is completely debunched. The longitudinal emittance, ε_L, of the beam in the ring is

$$\varepsilon_L = \Delta\phi \cdot \Delta T = 105.6 \quad (\text{keV} \cdot \text{rad}) \tag{1}$$

where T denotes a kinetic energy of each nucleon in the ion. In the present paper numerical values are calculated for the case A, while the results for other two cases are also listed in Table 5.

3. MULTITURN INJECTION

Ions are first injected for 5 turns in the horizontal phase space of the injection ring, whose diameter is six times larger than that of the storage rings. The reason why 5 turns are used is given in the following paragraph. Beam is ejected from the injection ring by the fast ejection method and its transverse phase spaces are interchanged with each other in the beam transport lines from the injection ring to the storage rings. Then beam is injected for 2 turns in each horizontal space of three storage rings, whose tune values of betatron oscillation are adjusted to a half integer at this time. This process is repeated two times and another four storage rings are filled with two-turn beams. After the two turn injection process, the tune value of betatron oscillation of each storage ring is adjusted to an integer plus three quarters, and beam is injected in each storage ring for three turns. The total lay-out of the injection ring and the storage rings is illustrated in Fig. 1.

In order to reduce the beam loss at the septum of the inflector during the beam injection process and to minimize the dilution factor in each phase space, five turn injection is applied for the injection ring and storage rings, by the following process:

1) The tune value of horizontal betatron oscillation is adjusted to half integer and the beam is injected in the ring during a time 2 τ_o, where τ_o is one revolution period in the ring.

2) After two turn injection, the position of the septum of the inflector is moved a distance of ~ 10 mm in the transverse phase space within a time of 1/100 τ_o in order to reduce the beam loss at the septum to less than 1%. The horizontal tune value of the ring also should be changed from half integer to integer plus three quarters, when the tune shift due to the already injected two turn beams and one turn beam to be newly injected, is compensated.

3) Beam is subsequently injected in the horizontal phase space during the time 3 τ_o instead of 4 τ_o , because the tune shift due to the space charge of successively injected beam is significantly large, and the phase advance of the betatron oscillation per revolution is varied well away from $\pi/2$.

Details of the multiturn injection method are given in the Appendix to the present paper.

The dilution factor of the emittance during the whole process of 5 turn injection is claculated to be 2.4 in each phase space. The emittance of the linac beam is given by

$$5 \times \pi\varepsilon_{linac} \times 2.4 = 30\ \pi \times 10^{-6} \quad (m\cdot rad) \quad , \qquad (2)$$

$$\pi\varepsilon_{linac} = 2.5\ \pi \times 10^{-6} \quad (m\cdot rad) \quad , \qquad (3)$$

and the normalized emittance is

$$\pi\varepsilon_{n} = \pi\varepsilon_{linac}\ \beta\gamma$$

$$= 0.534\ \pi \times 10^{-6} \qquad (m\cdot rad) \quad , \qquad (4)$$

which is smaller than the value estimated by the linac group at this workshop. But the peak current of the linac beam can be reduced to ~ 50 mA in our method, to allow such small emittance to be obtained.

4. RF STACKING

The injected beam in the storage ring by the five turn injection method is completely debunched. It is captured adiabatically by the RF separatrix and is accelerated to the stacking orbit, when the rate of change of momentum for the sychronous particle is given by

$$\frac{dp/dt}{p} = \frac{f_{rev}}{E_N\beta^2}\frac{q}{A} \cdot eV \cdot \Gamma \quad , \qquad (5)$$

where f_{rev} is a revolution frequency around the ring, E_N is a total energy of each nucleon, $\beta = v/c$, q/A is a charge to mass ratio and

$\Gamma = \sin\phi_s$. The fractional momentum difference between the injected orbit and the bottom of the stacked region is designed at 1.5 %, the acceleration period is 5 ms, and the required RF voltage is 356 kV. The period of phase oscillation during the acceleration is 1.48 ms.

During a period of acceleration from the bottom to the top of the stacked region, the RF voltage should be reduced to avoid an undesirable energy spread of the stacked beam in the stacked region. Final RF voltage is determined, as the area of the separatrix is just equal to the longitudinal phase space area of the injected beam, 105.6 keV•rad. In order to cover the longitudinal phase space area, S, of the injected beam by the separatrix, the minimum RF voltage is given by the following relation.

$$S = \left(\frac{hqeV}{A}\right)^{1/2} \alpha(\Gamma)\ \frac{16\beta}{h} \left(\frac{E_N}{2\pi|\tilde{\eta}|}\right)^{1/2} , \qquad (6)$$

where h is a harmonic number and $\tilde{\eta}$ is defined as

$$\tilde{\eta} = \frac{1}{{\gamma_{tr}}^2} - \frac{1}{\gamma^2} . \qquad (7)$$

Other notations concerning the synchrotron oscillation can be found in Reference 4. Substituting numerical values in the relation, a minimum voltage 81.6 keV is obtained. The phase oscillation period at the final voltage is 3.09 ms, and the necessary time to change adiabatically from the initial bucket to the final one is given by

$$T = \frac{1 + \kappa}{2(1 - \kappa)} \left(\frac{1}{\omega_2} - \frac{1}{\omega_1}\right) , \qquad (8)$$

where ω_1 is an angular frequency of phase oscillations associated with the initial bucket, ω_2 is that of the final bucket and κ is a quantity related to the phase space efficiency of the process. Substituting numerical values, κ is assumed to be 0.9, and T is 2.43 ms. Thus the shape of the envelope of the RF voltage is that shown in Fig. 2.

Next we must consider the relation between the number of RF stackings, the compression voltage and the final momentum spread in the storage ring. For simplicity, we assume that the momentum spread after n times RF stacking is

$$n \times \left(\frac{\Delta p}{p}\right)_i \quad , \tag{9}$$

where $(\Delta p/p)_i$ represents an initial momentum spread of $\pm 2 \times 10^{-4}$. The compression voltage including the effect of space charge and momentum spread is given by[5)]

$$\frac{eV}{2\pi\gamma A mc^2} = \frac{3N_b q h^2 r_o g}{A\gamma^3 R\cdot\Delta\phi_o\cdot\Delta\phi_{MIN}\,(\Delta\phi_o + \Delta\phi_{MIN})} + \frac{1}{q\cdot\Delta\phi^2_{MIN}}\, h|\tilde{\eta}|\beta^2\left(\frac{\Delta p}{p}\right)^2 \tag{10}$$

where

$mc^2 = 931.5$ MeV, $\quad r_o = 1.547 \times 10^{-18}$ m

q = charge state (= 1)

g = geometrical factor (= 1.5)

N_b = number of particles/bunch

$\Delta\phi_o$ = initial phase spread (= 2π)

$\Delta\phi_{MIN}$ = final phase spread (= 0.2π) .

The final phase spread is determined so that the phase compression factor in the ring is 10, when the tune shift, $\Delta\nu$, during the compression is assumed to be 2.5. The compression voltage is

$$eV = 0.1424\ (n^2 + 0.1798) \quad (\mathrm{MeV}) \tag{11}$$

Next we should calculate the separatrix height, H, related to the compression voltage and the final momentum spread, $\Delta p/p$, by using the following formulae,

$$H = \left(\frac{hqeV}{A}\right)^{1/2} Y\ \frac{\beta}{h} \left(\frac{E_N}{\pi|\tilde{\eta}|}\right)^{1/2} (\mathrm{keV}) \quad , \tag{12}$$

$$\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{2H}{E_N} \quad . \tag{13}$$

Numerical results are given in Table 3.

TABLE 3

n	eV (MeV)	H (keV)	Δp/p (%)
0	0.025	22.29	0.107
2	0.595	107.48	0.517
4	2.304	211.46	1.016
6	5.152	316.21	1.520
8	9.139	421.15	2.024
10	14.266	526.17	2.529

n: Number of RF stackings
eV: Compression voltage
H: Separatrix half height
$\Delta p/p$: Momentum spread (full width) after the compression

If we assume that the final momentum spread in the storage ring should be less than 1 %, the maximum number of RF stackings is 4.

As mentioned in preceding Sections, the number of multiturn injections is 5 in each transverse phase space. Then the required peak current, I_p, for the linac is given by

$$5^2 \times I_p \tau_o n = eN \qquad (14)$$

where N is a number of the particle in each storage ring, and

$$I_p = \frac{1.6 \times 10^{-19} \times 1.79 \times 10^{14}}{5^2 \times 4 \times 6 \times 10^{-6}} = 47.7 \quad \text{mA} \ . \qquad (15)$$

5. BEAM INSTABILITIES

5.1) Space charge limit

The space charge limit in a circular ring is given by

$$N = \frac{2\pi\Delta\nu}{B \cdot r_p} \left(\frac{A}{q^2}\right) \varepsilon\beta^2\gamma^3 \quad , \qquad (16)$$

where B is a bunching factor, r_p is a classical proton radius 1.547×10^{-18} m and ε is an unnormalized emittance. In the injection ring, emittance should be averaged over horizontal and vertical phase spaces, each of which has a numerical values of 30 π mm·mrad and 2.5 π mm·mrad. The averaged emittance is

$$\pi\varepsilon = \pi\sqrt{\varepsilon_x \cdot \varepsilon_y} = 8.66\,\pi \times 10^{-6} \quad (\text{m} \cdot \text{rad}) \ . \qquad (17)$$

If we take a bunching factor 1.0, the space charge limit in the injection ring is 9.8×10^{13} particles. In the storage ring, emittance is 30 π mm·mrad both in the horizontal and vertical phase spaces and the space charge limit is 3.38×10^{14} particles. In both rings, space charge limit exceeds the designed circulating currents.

5.2) Resistive wall instability

Next we will consider the longitudinal and transverse coherent resistive wall instabilities. The longitudinal coherent limit is given by the Keil-Schnell criterion[6),]

$$\left|\frac{Z_L}{n}\right| < F \frac{\beta^2 \gamma E_o}{e} \frac{|\tilde{\eta}|}{I} \left(\frac{\Delta p}{p}\right)^2 \qquad (18)$$

where Z_L/n is a longitudinal coupling impedance. Its numerical value should be examined further for heavy ion machines. However we will adopt here the value of 25 Ω which is scaled from the experimental values at ISR & CPS[7)]. Thus the longitudinal coherent limit is 59 A for a momentum spread of $\pm 2 \times 10^{-4}$, and there would be no problem related to the longitudinal coherent instability in the injection ring and the storage rings.

On the other hand the transverse coherent instability limit is given by

$$\left|\frac{Z_\perp}{n}\right| < 4F \frac{AE_o}{qe} \frac{\nu\beta\gamma}{I_o R} \left(|(n - \nu)\tilde{\eta} + \xi| \frac{\Delta p}{p} + \frac{\partial \nu}{\partial a^2} \Delta a^2 \right) \qquad (19)$$

where $Z_{\perp}/n$ is a transverse coupling impedance and ξ is a chromaticity. The first term in the bracket shows the effects of sextupole fields, and the second term the octupole fields. In the storage ring the momentum spread is fairly large, ~ 1 %, and the correction due to the sextupole fields is much more efficient than that of octupole fields. When we introduce a chromaticity of -10, the intensity limit is 0.54 A or 2.0×10^{13} particles, which is much smaller than the space charge limit. The e-folding growth time of this instability is given by [8)]

$$\tau = \frac{4\pi\nu\cdot\gamma AE_0/qe}{cI\ \mathrm{Re}(Z_{\perp})} \simeq 46\ \mathrm{ms}\ , \tag{20}$$

if we assume the radius of the vacuum chamber to be 5 cm and the stored current to be 4.78 A. This formula, however, holds under the condition that there is no sextupole and no octupole corrections. We expect that TCI can be controlled by their corrections during a total accumulation time of ~ 40 ms.

6. LIFE TIME OF THE BEAM IN THE STORAGE RING.

In high-intensity heavy-ion storage rings, a beam loss due to an electron transfer process between ions in the beam, $A^{n+} + A^{n+} \rightarrow A^{(n+1)+} + A^{(n-1)+}$, may be a severe problem. The loss rate α is estimated as follows:

$$\alpha \equiv \frac{1}{N}\frac{dN}{dt} \tag{21}$$

$$= n_{1ab} v_{cm} \sigma_{cm}\ . \tag{22}$$

The symbols are defined in Table 4, where machine parameters are also listed. The density of ions in the ring is

$$n_{lab} = \frac{N}{2\pi RS} \quad , \tag{23}$$

on the assumption that the beam is completely debunched. The beam is to be stored in the ring as shown in Fig. 3. Then the cross section of the beam is

$$S = \pi ab + b\Delta x_p \ , \tag{24}$$

where a and b are obtained from the beam emittance ε and the average betatron amplitude function, $\bar{\beta}$,

$$a = \sqrt{\varepsilon_x \bar{\beta}} \ , \tag{25}$$

$$b = \sqrt{\varepsilon_y \bar{\beta}} \ , \tag{26}$$

The beam spread due to a momentum dispersion is

$$\Delta x_p = \eta \frac{\Delta p}{p} \ . \tag{27}$$

The dispersion function is approximately

$$\eta = \bar{\beta}^2/R \ . \tag{28}$$

$$= 1.81 \quad (m)$$

Then the beam cross section is numerically calculated with values listed in Table 4, and the density is

$$n_{lab} = 4.62 \times 10^{14} \quad (m^{-3}) \ . \tag{29}$$

The speed of the ion in the center of mass frame is given by

$$\beta_{cm}^2 = \left(\frac{\beta}{2} \frac{\delta' p}{p} \right)^2 + \left(\beta\gamma \sin\frac{\theta}{2} \right)^2 \quad . \tag{30}$$

As the ion momenta are considered to be distributed as in Fig. 4, the typical momentum difference between the ions which will collide with each other, is

$$\frac{\delta' p}{p} = \frac{\Delta p}{p} \frac{2a}{2a + \Delta x_p} \quad , \tag{31}$$

where $\delta' p/p$ is determined so that the areas of the parallelogram and the rectangle are equal. Then the first term of eq.(30) is 1.65×10^{-4}. The maximum collision angle in the laboratory is evaluated by

$$\theta = 2\sqrt{\varepsilon_x/\bar{\beta}} \quad , \tag{32}$$

and the second term is numerically 3.61×10^{-4}. Then the velocity in the c.m. frame is

$$v_{cm} = 1.19 \times 10^5 \text{ (m/s)} \quad . \tag{33}$$

which corresponds to a kinetic energy of 75 eV.

According to papers[9,10] the cross sections for the electron transfer process of various ions are estimated to be of the order of 10^{-15} cm^2. Therefore a value of 1×10^{-15} cm^2 is appropriately adopted here for U^{1+}.

Now the loss rate can be numerically calculated, and

$$\alpha = 5.50 \text{ (s}^{-1}) \quad . \tag{34}$$

The life time, the inverse of the loss rate, is

$$\tau = 0.182 \quad (s). \tag{35}$$

which means that the beam will be lost by the amount of 20 % during stacking process of $\sim$ 40 ms. Therefore if such an amount of beam loss is serious, even though it does not occur at localized positions such as the inflector septum, but could be uniformly lost around the ring, another kind of ion of low intrabeam charge exchange cross section such as Xe^{8+} should be used.

Table 4 List of symbols and machine parameters for case A

N	number of ions in the ring	1.79×10^{14}
n_{lab}	density of ions	
R	mean radius of the ring	59.8 m
S	cross section of the beam	
v_{cm}	velocity of ions in the center of mass frame	
α	loss rate	
β	ratio of ion velocity to that of light	0.208 (21 MeV/u)
$\bar{\beta}$	average betatron amplitude function	10.4 m
γ	$1/\sqrt{1 - \beta^2}$	1.0224
$\frac{\delta' p}{p}$	momentum difference between colliding ions	
$\frac{\Delta p}{p}$	total momentum spread	1.6×10^3
ε_x	emittance in the horizontal direction	30×10^{-6} m·rad
ε_y	emittance in the vertical direction	30×10^{-6} m·rad
η	dispersion function	
θ	collision angle in the laboratory frame	
σ_{cm}	cross section of the electron transfer process	1×10^{-19} m^2
τ	life time	

Table 5 Summary of the calculations for three cases.

Case	A	B	C
No of multiturn	5 × 5 = 25	25	25
Dilution factor for multiturn	2.4	2.4	2.4
Normalized emittance of the injected beam from the linac	$0.534\pi\times10^{-6}$ (m·rad)	$0.759\pi\times10^{-6}$ (m·rad)	$0.759\pi\times10^{-6}$ (m·rad)
LINAC peak current	48 mA	125 mA	158 mA
RF stacking number	4	4	4
Momentum spread after the compression in the ring	1 %	1 %	0.899 %
required period for one RF stacking	10 ms	28 ms	47 ms
Total injection period for each ring	40 ms	112 ms	188 ms
Total accumulation period for n_r rings	∿40 ms	∿112 ms	∿200 ms
Fusion repetition rate	20 Hz	8 Hz	5 Hz
Space charge limit in the			
stacking ring	8.6×10^{13}	1.8×10^{14}	5.7×10^{14}
storage ring	3.3×10^{14}	8.7×10^{14}	1.4×10^{15}
Longitudinal coherent limit	300 A	more safe than in case A	more safe than in case A
Transverse coherent limit	0.54 A	"	"
Growth time of transverse instability	46 ms	"	"
Compression voltage	2.3 MV	2.2 MV	1.5 MV
Compression time	102.4 μs (17 turns)	∿100 μs	∿100 μs

APPENDIX

Scheme of multiturn injection into transverse phase space

For the purpose of reducing the beam loss due to the collision with an inflector septum, the following process was studied.

1st) Before the beam injection the tune value of the betatron oscillation should be adjusted to a half integer taking account of the space charge effect due to the intensity of a single turn.

2nd) The beam from the linac is injected by a two turn injection method during the time $2\tau_o$, where τ_o is the revolution time of the beam.

3rd) The position of the septum in the phase space should be moved in a time of $\frac{1}{100}\tau_o$ in order to reduce the beam loss to around 1 %.

4th) The horizontal tune value is shifted to an integer +3/4, taking account of the effect of the space charge force due to the two turn beam already stacked in the ring, and the first one turn beam to be injected in the next step.

5th) The beam from the linac should be three turn injected during the time interval $3\tau_o$, and just before the three turn the position of the septum is moved from x = 11 mm to x = 21 mm in a time of $\frac{1}{100}\tau_o$.

The acceptance of the ring and the emittance of the beam from the linac were assumed to be $30\pi \times 10^{-6}$ and $2.5\pi \times 10^{-6}$ m·rad (unnormalized), respectively.

In the second process the transfer matrix of one turn, M_o, can be written as

$$M_o = \begin{pmatrix} \cos\{2\pi(N + \frac{1}{2} + \Delta\nu)\}, & \beta\sin\{2\pi(N + \frac{1}{2} + \Delta\nu)\} \\ -\frac{1}{\beta}\sin\{2\pi(N + \frac{1}{2} + \Delta\nu)\}, & \cos\{2\pi(N + \frac{1}{2} + \Delta\nu)\} \end{pmatrix}, \qquad \text{(A-1)}$$

where N is an integer, $\Delta\nu$ is the tune shift due to the space charge effect of the beam, and $\alpha \left(= -\frac{1}{2}\beta'\right)$ is assumed to be zero. We represent the beam ellipse in the phase space just one turn after injection as $(a\cos\theta + x_c,\ b\sin\theta)$, where a and b are the length of horizontal and vertical axes of the beam ellipse and x_c is the position of the center of the beam as shown in Fig. A-1. Using

$$M_o = \begin{pmatrix} -\cos\Delta\mu, & -\beta\sin\Delta\mu \\ \frac{1}{\beta}\sin\Delta\mu, & -\cos\Delta\mu \end{pmatrix}, \qquad \text{(A-1)'}$$

where $\Delta\mu = 2\pi\cdot\Delta\nu$, the position of the beam with respect to the closed orbit after another revolution is given by

$$x = -\cos\Delta\mu(a\cos\theta + x_c) - \beta\, b\sin\theta\cdot\sin\Delta\mu\ . \qquad \text{(A-2)}$$

The maximum value of x is obtained for the value of θ which satisfies

$$\frac{dx}{dx'} = 0 \qquad \text{(A-3)}$$

and

$$\tan\theta = \frac{\beta b}{a}\tan\Delta\mu\ . \qquad \text{(A-4)}$$

For such a value of θ,

$$x = -x_c\cos\Delta\mu \pm \sqrt{a^2\cos^2\Delta\mu + \beta^2 b^2\sin^2\Delta\mu}\ . \qquad \text{(A-5)}$$

The maximum value of x in the equation (A-5) when $\Delta\mu$ is varied, is obtained as

$$x_{max} = \beta \cdot b \sqrt{\left| 1 - \frac{x_c^2}{a^2 - b^2\beta^2} \right|} \qquad (a \neq b\beta) \qquad \text{(A-6)}$$

$$= |x_c| + a \qquad (a = b\beta)$$

In our case the numerical values are as follows;

$$\begin{aligned} \beta &\simeq 15 \text{ m}, \\ a &= 4.0 \times 10^{-3} \text{ m} \\ b &= 6.25 \times 10^{-4} \\ x_c &= -4.5 \times 10^{-3} \text{ m} \quad . \end{aligned} \qquad \text{(A-7)}$$

and the maximum value is

$$x_{max} = 0.0106 \text{ m} \quad . \qquad \text{(A-8)}$$

Therefore when the position of the inflector septum is shifted outward as far as 11 mm after 2 turn injection in a time $\frac{1}{100}\tau_o$, no further beam collision with the septum is expected.

In the third process, it is necessary to estimate the required high voltage of the pulsing system for bump magnets. The bump magnets should be located 90° up and down stream of the inflector. In Case A, the required deflection angle of the bump magnet is estimated to be 6.733×10^{-4} rad in order to distort the closed orbit by 11 mm at the position of the inflector. If each bump magnet is divided into 6 units which are excited in parallel, then the necessary deflection angle for each unit is $\frac{1}{6} \times 6.733 \times 10^{-4} = 1.122 \times 10^{-4}$ rad. The field strength of the bump magnet is calculated at 177 G for case A, where the total momentum of U^{1+} is 47349 MeV. The required current for each bump magnet unit is calculated to be 704.8 A if a single turn coil is used.

If we assume a critical damping, the rise time t_r from 5 % to 95 % of the maximum value is given by

$$t_r = 1.14 \frac{L}{Z_o} \quad , \tag{A-9}$$

where L and Z_o are the inductance of the coil and the characteristic impedance of the circuit, respectively. The inductance of the magnet is given by the relation

$$L = N^2 \cdot \mu_o \frac{w \cdot \ell}{d} \cdot F \quad , \tag{A-10}$$

where μ_o is the permeability of air, w, ℓ, d are the width of the pole, the length of the magnet, and the gap height of the magnet, respectively, and F is the ratio of the leakage flux defined by

$$F = \frac{\phi}{\phi_1} \quad , \tag{A-11}$$

where ϕ and ϕ_1 are the total flux in the iron yoke and the total flux which goes through the pole face, respectively. Assuming the following values

$$\begin{aligned} w &\simeq 0.07 \text{ m} \\ \ell &\simeq 1.0 \text{ m} \\ d &= 0.05 \text{ m} \\ F &\simeq 2.0 \quad , \end{aligned} \tag{A-12}$$

L is 3.52 μH and the characteristic impedance Z_o should be 67 Ω so as to make the rise time t_r as short as 60 μs. The required high voltage V_o is given by

$$V_o = 2 Z_o \cdot I$$

and is 94.44 kV, which is a manageable value.

Then the horizontal tune of the betatron oscillation is moved to an integer +3/4, including the space charge effect due to the beam already injected into the ring and the beam which will be injected in the next turn.

After tuning, the beam from the linac is injected by three-turn injection as is illustrated in Fig. A-2 (a) ∿ (c). In the calculation, the additional effect of space charge due to beam newly injected into the ring is taken into account. The tune shift is given by the formula

$$\Delta\nu = \frac{-NB \cdot r_p \, q^2}{2\pi\varepsilon\beta^2\gamma^3 A} \quad , \tag{A-14}$$

where B, r_p and ε are the bunching factor, the classical proton radius and the unnormalized emittance of the beam. In case A, this effect is estimated to be $\Delta\nu_1 = -0.07$ and the transfer matrices of the 1st 2nd and 3rd turns, M_1, M_2 and M_3, are given by

$$M_i = \begin{pmatrix} \cos 2\pi\,(N + \frac{3}{4} + (i - 1)\Delta\nu_1), & \beta \sin 2\pi\,(N + \frac{3}{4} + (i - 1)\Delta\nu_1) \\ -\frac{1}{\beta} \sin 2\pi\,(N + \frac{3}{4} + (i - 1)\Delta\nu_1), & \cos 2\pi\,(N + \frac{3}{4} + (i - 1)\Delta\nu_1) \end{pmatrix} . \tag{A-15}$$

$i = 1, 2, 3.$

Due to space charge tune shift, the beam will come back to the septum position after three turns, as is shown in Fig. A-2. Hence it is needed to shift the septum position to x = 21 mm in the time interval $(2 + \frac{99}{100})\,\tau_o < t < 3\tau_o$. The required current for the bump magnet is 576.7 A, and the high voltage is calculated at 77.3 kV.

During the multiturn injection process, beam with an emittance of $2.5\,\pi \times 10^{-6}$ m·rad (unnormalized) is injected into the ring with an acceptance of $30\ \pi \times 10^{-6}$ m·rad during $\{(2+3) - \frac{1}{100} \times 2\}$ turns. Then the dilution factor due to this multiturn injection is

$$D = \frac{30}{2.5 \times \{(2+3) - \frac{1}{100} \times 2\}} = 2.41 \quad . \qquad \text{(A-16)}$$

This factor is close to the value of the usual multiturn injection, but in this scheme beam loss due to collision with the inflector septum is reduced to 2 % of the total beam.

References

1) ERDA Summer Study of Heavy Ions for Inertial Fusion, 1976, LBL-5543.

2) Proceedings of the Heavy Ion Fusion Workshop, 1977, BNL-50769.

3) Proceedings of the Heavy Ion Fusion Workshop, 1978, ANL-79-41.

4) C. Bovet et al., A selection of formulae and data useful for the design of A. G. synchrotrons, CERN/MPS-SI/Int. DL/70/4, 1970.

5) T. K. Khoe, "Longitudinal Bunching:, Proc. of the Heavy Ion Fusion Workshop, 1977, BNL-50769.

6) W. Schnell, Coherent instabilities of proton beams in accelerators and storage rings-experimental results, diagnosis and cures CERN 77-13, 1977.

7) D. Möhl, Talk at this workshop.

8) B. Zotter and F. Sacherer, Transverse Instabilities of Relativistic Particle Beams in Accelerators and Storage Rings, CERN 77-13, 1977.

9) K. T. Cheng, G. Das, Y. K. Kim and R. C. Raffenetti, Proc. of the Heavy Ion Fusion Workshop, 1977, BNL-50769.

10) J. Macek, Proc. of the Heavy Ion Fusion Workshop, 1978, ANL-79-41.

Figure captions

Fig. A-1. Beam injection is carried out with the elliptically-shaped beam shown in the figure. The number of betatron oscillations per revolution is tuned to a half integral value, including the space charge effect due to the one-turn injected beam. During the first turn, the beam ellipse revolves by 180° in phase space, but during the second turn the beam ellipse rotates as illustrated in the figure due to the space charge effect of another one-turn beam. In order to avoid beam loss, the septum is shifted to x = 11 mm in the time interval $(1 + \frac{99}{100})\tau_o < t < 2\,\tau_o$, where τ_o is a revolution time of the beam.

Fig. A-2. In this process the tune value of betatron oscillation is adjusted to integer plus three quarters including the space charge effect due to the one-turn beam.

(a) In the first turn, the beam ellipse rotates in the phase space by 90°.

(b) In the second turn, the tune is shifted by the space charge force due to another one-turn beam and the beam ellipses rotate as is shown in the figure.

(c) After three turns, the first beam comes back to the septum position as is illustrated in the figure because of the tune shift due to additional space charge effect. In order to reduce beam loss, the septum is shifted from x = 11 mm to x = 21 mm in the time interval $(2 + \frac{99}{100})\,\tau_o < t < 3\,\tau_o$.

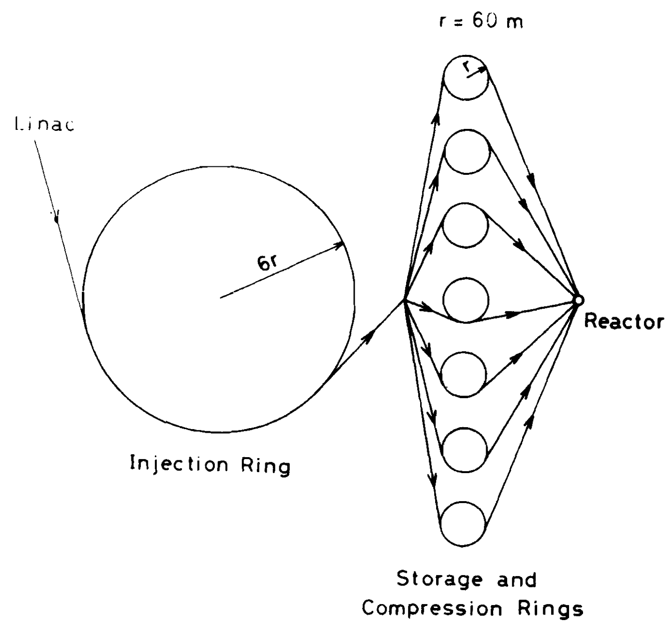

Fig.1 Schematic layout of the injection ring and storage rings for case A.

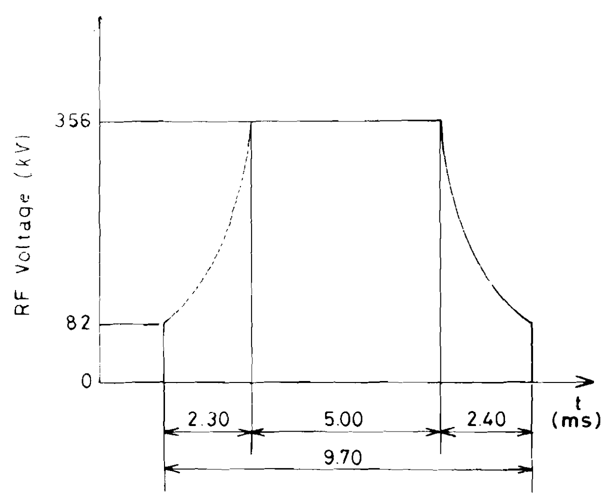

Fig.2 An envelope of the RF field for the momentum stacking.

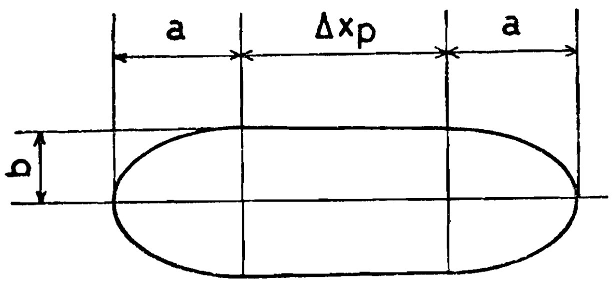

Fig. 3. The beam profile in the storage ring.

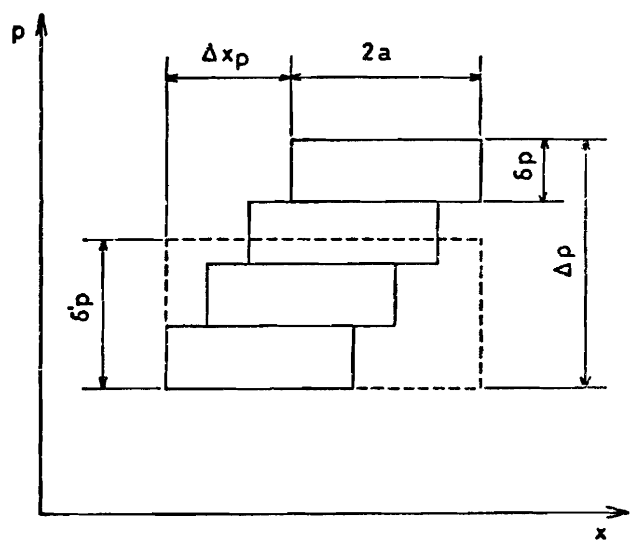

Fig. 4. Four beam pulses of different momenta are stacked in the storage ring. The typical momentum spread $\delta'p$ is determined so that the area of the ractangle (dashed line) and that of the four pulses are equal.

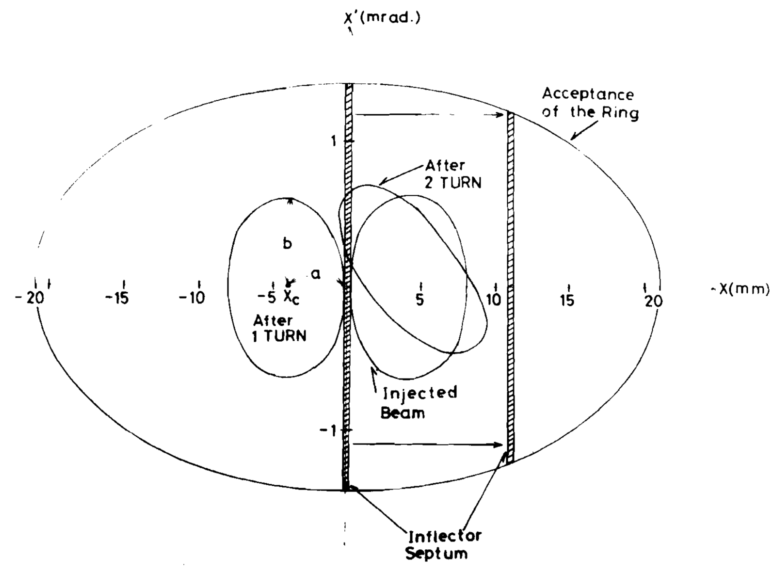

Fig. A-1 Schematic diagram of the first 2-turn-injection with the half integral tune value.

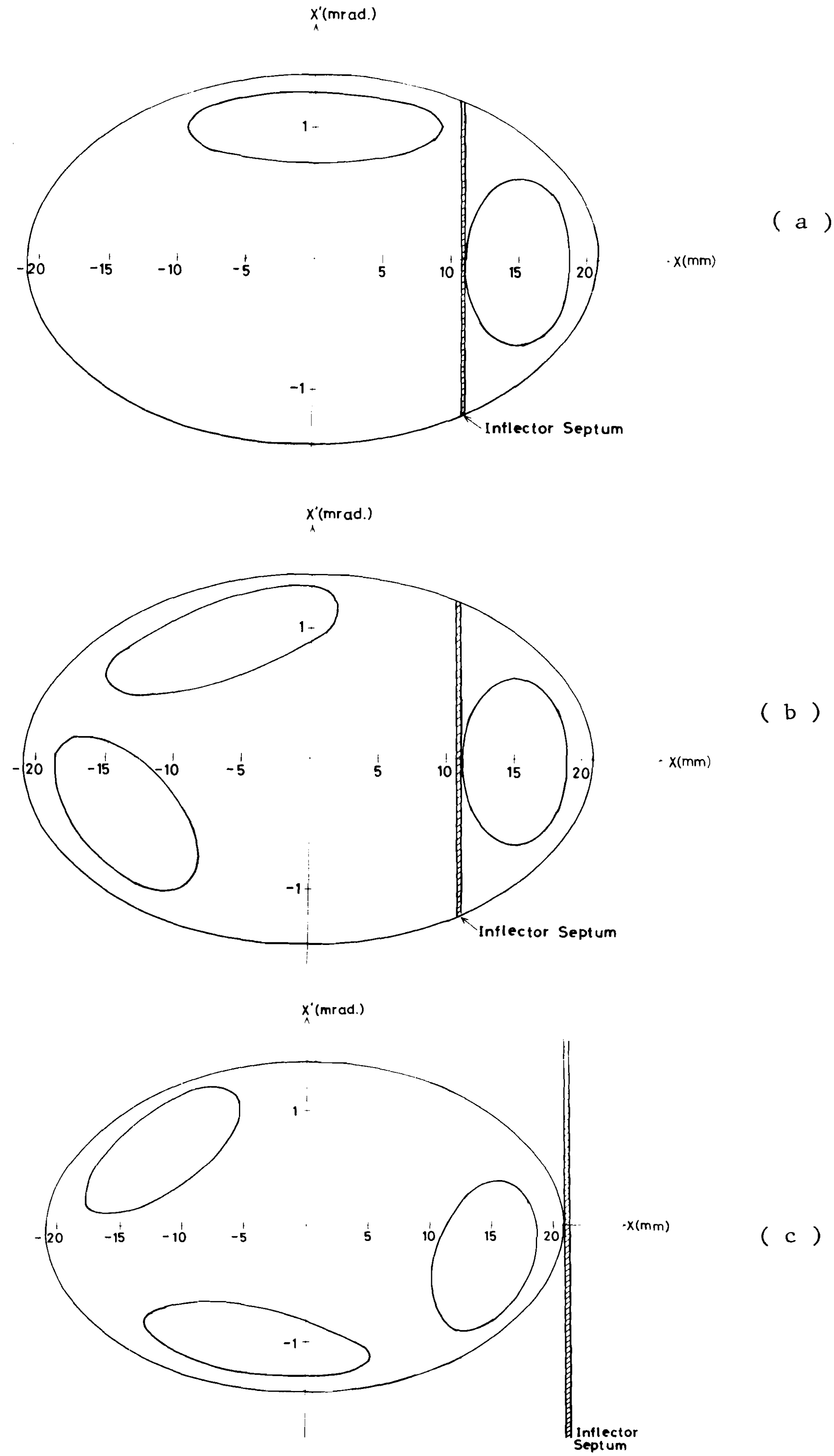

Fig. A-2. Schematic diagram of the second 3-turn-injection with the tune-value of integer plus three quarters.