EXCITATION OF HIGHER ORDER MODES IN ALVEREZ LINAC STRUCTURES*

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I. INTRODUCTION

For overall system efficiency it is desirable to run the high beta linac as heavily beam loaded as practical. One limitation is the effect of higher order rf mode excitation on beam stability.

The excitation of these modes is reviewed. A possible excitation mechanism with synchronous particles is introduced.

II. RESONANCE POSSIBLE IN HOLLOW CYLINDRICAL GUIDES

Table 1 lists the modes that can propogate in a hollow cylindrical guide. The cutoff frequency, f_c, below which no field can propogate, is dictated by the transverse dimension namely radius, a. When the guide is closed a resonance cavity is formed whose frequency is determined by the longitudinal dimension, z.

TABLE 1

Mode	λ_c	Freq (MHz)	Field Components
TM_{01}	2.61a	200	Ez, Hϕ
TM_{02}	1.14a	457	Ez, Hϕ
TM_{11}	1.64a	318	Ez, Hr, Hϕ
TE_{01}	1.64a	318	Hz, Hr, Er
TE_{11}	3.41a	153	

An Alverez cavity is a cylindrical guide operated in the TM_{010} mode (at cutoff). The drift tube causes a frequency perturbation. A guide operating in the TM_{01} mode can support numerous longitudinal resonances corresponding to guide wavelengths in the longitudinal direction ($\lambda/2$, λ, $3\lambda/2$, etc.). If the length is much greater than the diameter,then their frequency will be close to the TM_{010} mode. Similarly the other cylindrical guides have resonances corresponding to "n" longitudinal wavelengths. (See Figure 1).

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Due to the large frequency separation between the operating mode and the other cylindrical guide modes, only TM_{01n} modes appear to be worrisome. This assumes that the higher mass of heavy ions will exclude the deflection mode (TM_{11}) blowup observed in electron linacs. This should be verified however.[1,2,3]

The field distribution of the TM_{01n} modes are shown in Figure 2.

III. TRANSIENT MODAL EXCITATION BY RF POWER AMPLIFIER

Due to the Fourier components existing during the rf turn on, the TM_{011} and TM_{012} modes have been observed in Alverez accelerators during the rf buildup period.[4] These fields exponentially decay before the beam is turned on.

From Figure 2 it can be seen that by placing the rf drive points at the 1/4 and 3/4 L points these modes cannot be driven. The moding effect has been further reduced by the use of dummy stems which push the frequency of the higher order TM_{01n} modes further away from the TM_{010} operating frequency.[5] The motivation for suppressing these higher order modes is the desire to minimize the transient effect when the rf is increased for beam compensation. The excitation of these modes has been from the generator and not the synchronous beam.

IV. BEAM EXCITATION OF TM_{011} and TM_{012} MODES

The beam which is in synchronism to the TM_{010} operating frequency has not been observed to cause higher order mode excitation. This is to be expected because of the distribution of the H field. Excitation from the synchronous beam in one section of the tank tend to be canceled out by the field induced (180° out of phase) in other sections of the structure (see Figure 2). One means of coupling can be envisioned, however.

Since the frequency of a TM011 mode is slightly higher than that of the TM_{010} mode, a synchronous particle traveling the length of the tank will see a phase advance of any TM_{011} electric field that exists. Figure 3A assumes that the synchronous beam enters the first drift tube gap during the maximum accelerating phase of the longitudinal electric field. If there is a total of 180° phase advance this particle will receive kinetic energy. In Figure 3B the synchronous beam particle enters the first gap during the de-accelerating part of the TM_{011} mode. With a 180° phase advance this particle gives up kinetic energy. Under the condition cited the beam would act as a modulating system, alternately acting as an rf generator and rf load. The modulating rate would be equal to the difference frequency between the TM_{010} and TM_{011} natural excitation frequencies.

IV.A DETERMINING DIFFERENCE FREQUENCY

The required beta and accelerator length to produce 180^0 phase shift can be found.

$$\Delta f = f_{011} - f_{010}$$

where

f_{011} = frequency of TM_{011} mode

f_{010} = frequency of TM_{010} mode

$\left(\frac{\Delta f}{f_0}\right)(360°)$ = degree advance/drift tube

Assuming that the relative beam velocity, β, is constant through the accelerating structure.

$$\frac{\Delta f/f(360^0)}{\beta\lambda} = \text{degree advance/cell length}$$

The difference frequency required to produce 180° phase shift in an accelerator tank of length, Z, is found to be

$$\Delta f = \frac{\beta c}{2Z} \qquad c = \text{velocity light} \qquad (1)$$

The cavity dimension required to set up a particular Δf can be approximated by assuming an unperturbed cylindrical guide.

$$f_{01n} = \left(\frac{2.405}{a}\right)\left(\frac{c}{2\pi}\right)\left[1 + \left(\frac{n\pi a}{2.405Z}\right)^2\right]^{\frac{1}{2}}$$

where

a = radius

Z = length

Since a << Z

$$f_{011} \cong (f_{010})\left[1 + \frac{1}{2}\left(\frac{\pi}{2.405}\frac{a}{Z}\right)^2\right]$$

$$\Delta f = \left(\frac{1}{8\, f_{010}}\right)\left(\frac{c}{Z}\right)^2 \qquad (2)$$

By equating 1 and 2 the appropriate β and λ to set up a 180^0 phase shift are found.

$$\Delta f = \frac{\beta c}{2Z} = \frac{c^2}{L^2 \; (8 \; f_{010})}$$

$$\beta = \frac{\lambda}{4c} \qquad (3)$$

where λ = free space wavelength of TM_{010} mode

$$\beta\lambda = \frac{\lambda^2}{4Z}$$

Practical values of $\beta\lambda$ are between .1 and 1 meter and Z between 10 and 20 meters which would suggest a frequency range of between 30 to 150 MHz. This corresponds to a range of betas from .025 to .1. The most likely range is the lower beta, higher frequency range. Picking values of Z = 15 meter, f_{010} = 100 MHz, from equation 3 we obtain:

$$\beta = \frac{3}{60} = .05$$

For this case the frequency difference would be as follows:

$$\Delta f = \frac{\beta c}{2Z} = \frac{(.05)\;(3 \times 10^8)}{30} = 500 \text{ kilohertz}$$

The bandwidth of an Alverez structure is approximately 5 kilohertz however. Since energy is alternately being supplied to and delivered by the beam at a 500 kilohertz rate it is most unlikely that the higher mode would build up. If for some reason the loading effect on the beam is not equal to the generating effect of the beam the mode could build up.

V. CASE OF CHANGING BETA

The above analysis was not rigorous since it assumed constant beta through the accelerating tank and did not consider the effect of dummy stems in the structure. It did indicate areas where problems could occur.

To see the effect of changing beta in an acccelerating section let the following specifications apply:

Input Energy = 2 MeV
Output Energy = 20 MeV
f_{010} = 80 MHz
Z = 18 meters
β_{in} = .046
β_{out} = .146
# of Cells = 55
Beam Current = 100 milliamps
Rf Power = 3.5 megawatts

The approximate separation of the TM_{011} mode from the operating mode is found by using Eq. (2).

$$\Delta f = \frac{c^2}{2^2(8\ f_{010})} = 434 \text{ kilohertz}$$

The phase advance per cell is given as follows:

$$\frac{\Delta f}{f_{010}}\ 360^0 = 1.95^0/\text{cell}$$

55 cells x 1.950/cell = 107° total advance

If the actual Δf is higher because of dummy stems the phase advance per cell will be larger. Since the beam kinetic energy is low any moding will effect the particle velocity.

Figure 4 suggests a mechanism for initiating the TM_{011} buildup. It is noted that there can be unsymmetrical excitation from the beam at 90° phase advance. Figure 5 indicates the excitation versus entrance angle at the first gap. The phase advance is reduced by the particles gaining kinetic energy thus tending to approach synchronism with the TM_{011} wave. If synchronism is reached, these particles no longer act as an absorber since energy gain will equal energy lost. Particles losing energy in the first gap, however, will tend to increase their phase advance toward the 180° required for strong coupling. For this case symmetry between the driving function (decreasing beam K.E.) and the absorption function does not exist. Particles entering the 1st gap at 270° could loose as much as 2 MeV kinetic energy (200 kilowatt instantaneous) while particles entering at 360° could loose 10 MeV kinetic energy (1 megawatt instantaneous).

VI. CONCLUSIONS

The excitation of higher order TM_{01} modes produced during the rf build-up period has been observed and compensated for over the years.

Synchronous beam excitation of these modes has not been observed. A possible, though unlikely, mechanism for beam excitation has been presented. This would occur only in a very specific section of the machine (low beta).

There is no strong reason to expect poor beam quality for currents near 160 milliamperes but further studies into the effect of heavy beam loading should be persued.

REFERENCES

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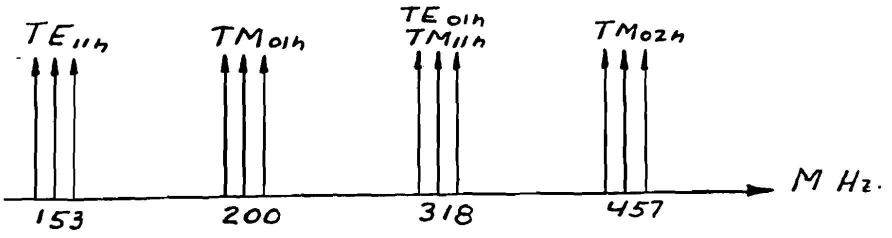


Figure 1. Modal Separation of Cylindrical Guide (2 = .57 Meters).

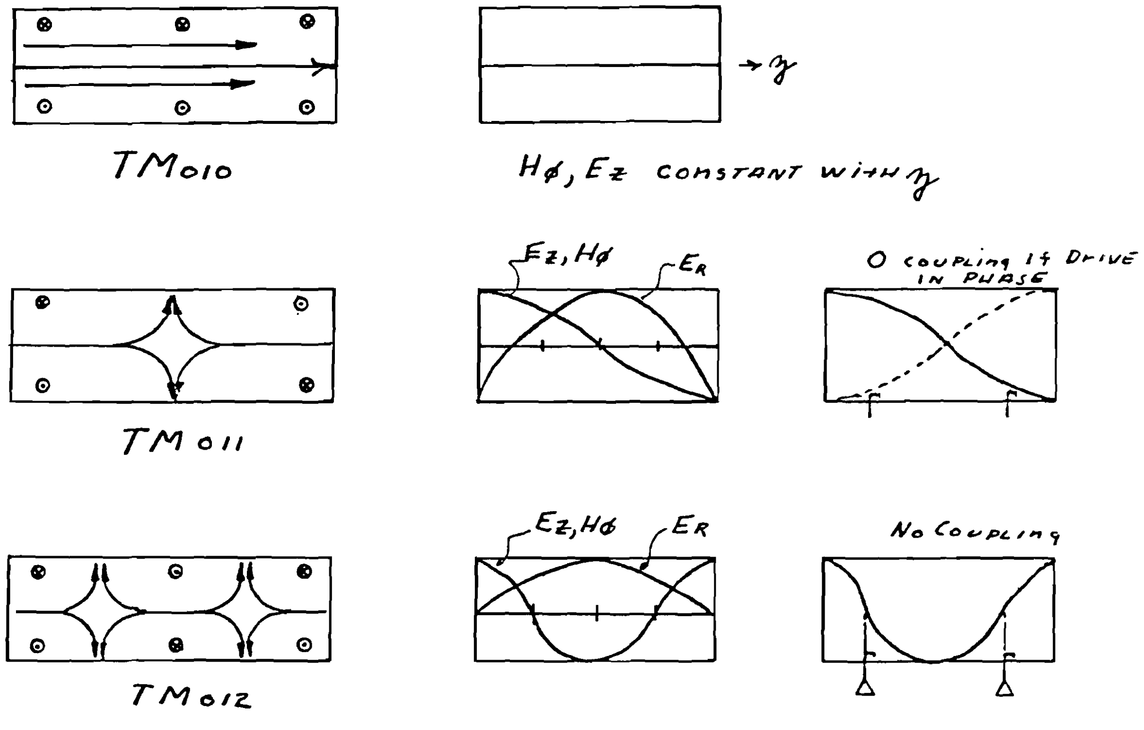
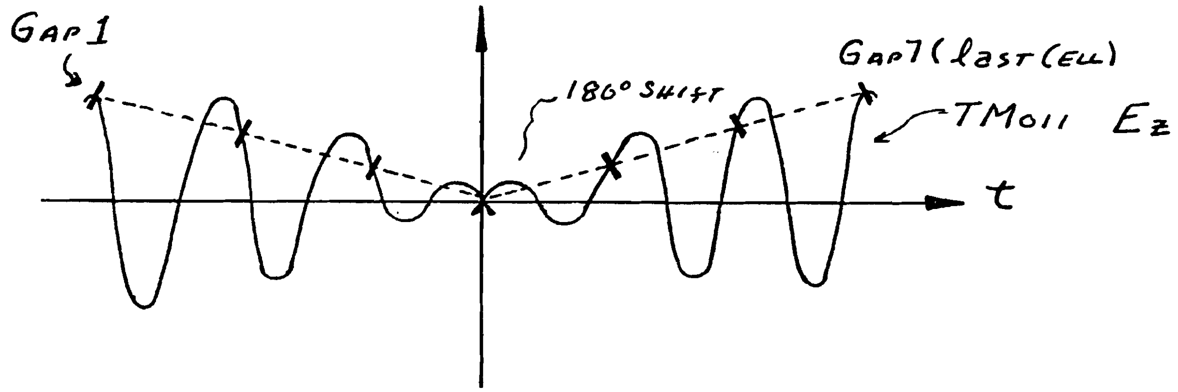
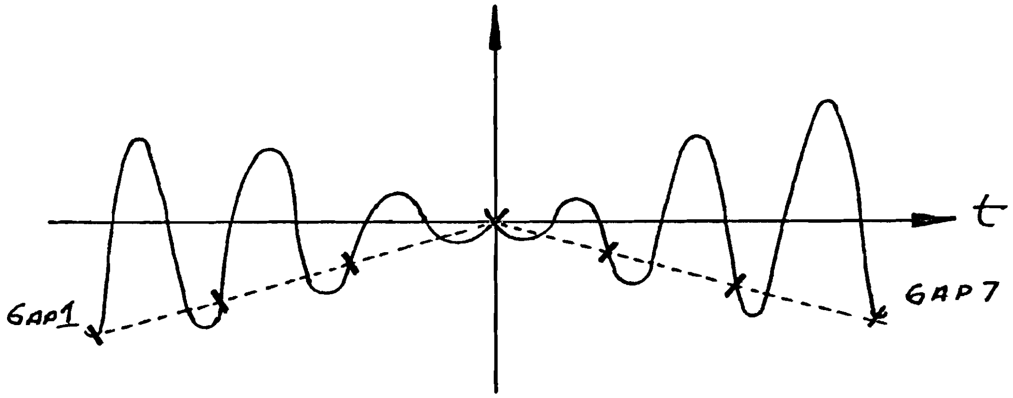


Figure 2. Field Distribution of TM_{01n} Modes.

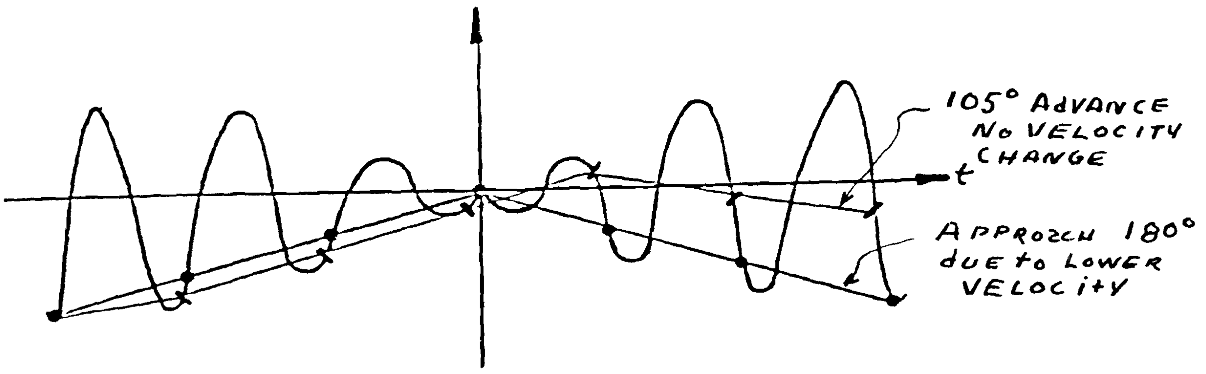


A. Particle Gaining Kinetic Energy - 90^{0} Entrance Angle

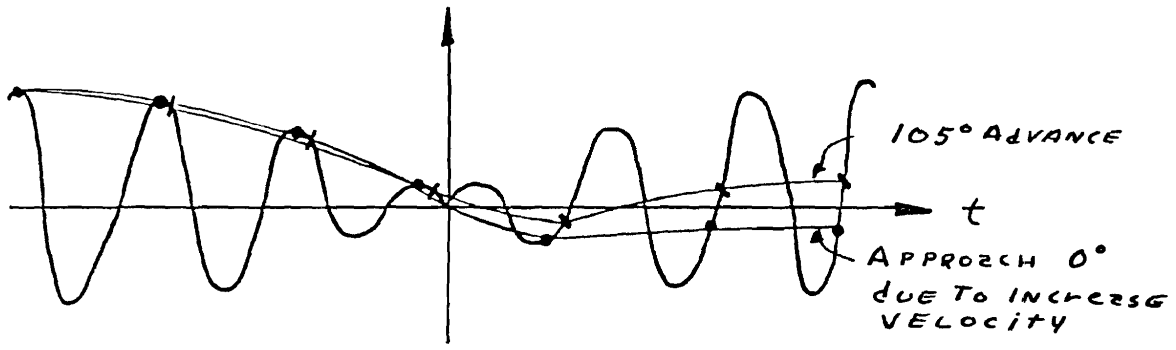


B. Particle Loosing Kinetic Energy - 270^{0} Entrance Angle.

Figure 3. 180^{0} Phase Advance of TM_{011} Longitudinal Electric Field As Synchronous Particle Passes Through Structure.



A. Effect of Reduced Velocity on Driving Particle



B. Effect of Increased Velocity on Absorbing Particle

Figure 4. Velocity Change Resulting in Anti-symmetric Driving and Absorbing Function (105° Phase Advance).

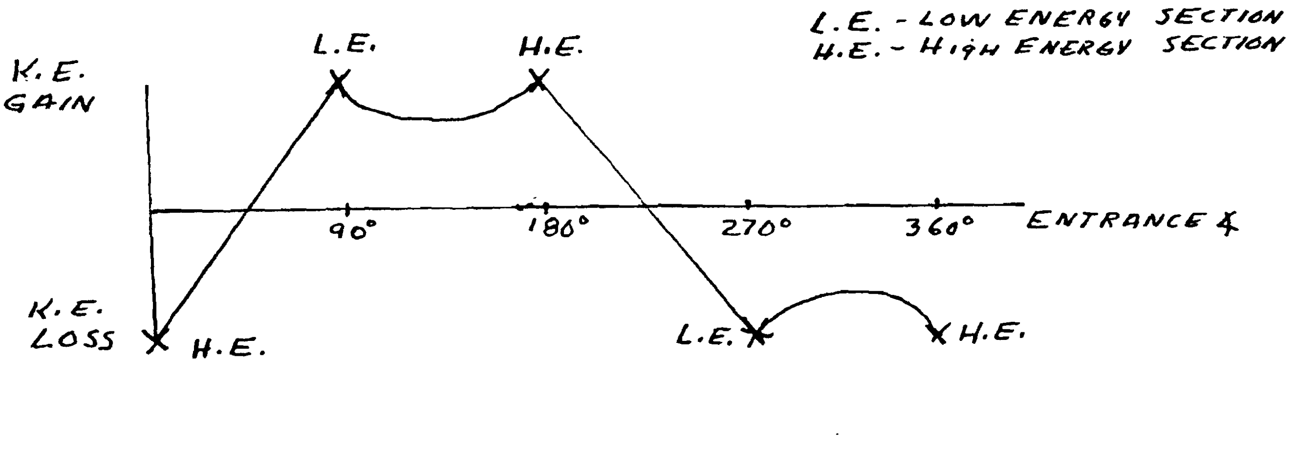


Figure 5. Location of Energy Interchange Versus Entrance Angle at First Gap (90° Phase Advance)