

SINGLE BUNCH BEAM LOADING IN LINACS

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ABSTRACT

This paper examines the energy loss distribution in a single bunch travelling in a linear accelerator and the way this can be compensated by a suitable choice of peak accelerating field and bunch phase. The example chosen is appropriate to a super-conducting linac in which higher order modes are coupled out of the system. However, there is some discussion of the possibility of not using h.o. mode damping, in which case such modes may not be excited, when the energy spread would be significantly less. Suggestions are made for further study.

1. INTRODUCTION

The possible use of linacs to provide very high energy electrons and positrons for colliding beam experiments calls for the use of very intense bunches of particles. It is well known that the loss of energy from intense single bunches may be much higher than in the multi-bunch situation due to radiation from the bunch which resolves itself into higher order e.m. modes. The field radiated by early particles in the bunch acts on later particles leading to a considerable variation of energy loss throughout the bunch. This may be partially compensated by suitable phasing of the bunch in relation to the accelerating wave.

This note gives the appropriate parameters and compensation for the linac considered at this Workshop<sup>1)</sup>. It must be noted that the theory and calculations are applicable either for a linac operated at normal temperature or for a superconducting linac in which the higher order modes are damped by being coupled out of the structure very efficiently, as they must be if damping is attempted at all.

The intriguing possibility remains, however, that, if no attempt is made to couple out h.o. modes, thus simplifying the structure enormously, then such modes may not be excited. It is easy to show that, if the bunch repetition rate is high compared to the mode decay rate, then there will be no energy loss to these modes, in the steady state, unless their frequencies are at or extremely close to integral harmonics of the accelerating r.f. By careful design, combined with a measurement and tuning programme it may be possible to avoid harmonic resonances in the range of frequency covered by the Fourier transform of the bunch distribution.

Clearly if h.o. mode losses are avoided the energy distribution in a bunch of high intensity would be much reduced, being determined only by fundamental beam loading and the phasing with respect to the accelerating wave. This would make it possible to consider methods of particle recovery which become impracticable if there are appreciable h.o. mode losses.

In what follows the energy loss in a single bunch is first derived for a structure similar to the SLAC linac, using results of measurements at SLAC. This is followed by consideration of optimum energy spread compensation by phasing. A further section discusses the situation when the higher order modes are not damped. Some conclusions are drawn, together with suggestions for further study.

## 2. ENERGY LOSS

When an electron bunch passes through a cavity energy is radiated which eventually resolves itself into cavity modes. The effect on the bunch itself has been described in terms of a wake field. The field experienced by particles at one position in the bunch arises from the cumulative wake field from all preceding particles.

A wake field distribution appropriate to the SLAC linac has been computed<sup>2)</sup> using Keil's modal analysis<sup>3)</sup> together with Sessler's optical resonator model<sup>4)</sup>. This led to an average expected energy loss of 2.92 V/pC for a  $1^\circ \sigma$  bunch for each cell of the structure. Measurements on the SLAC linac<sup>5)</sup> produced a measured value of 3.68 V/pC for a bunch of about  $1^\circ \sigma$  ( $\sim 0.84^\circ$ ). The bunch was not quite Gaussian in shape but this was shown to make little difference.

The discrepancy is small and has been attributed to possible errors in current measurement together with the possible excitation of transverse modes by an off-axis beam, not allowed for in the theoretical computation. The measured value will be assumed here and this leads to the wake field from a delta function bunch as shown in Fig. 1.

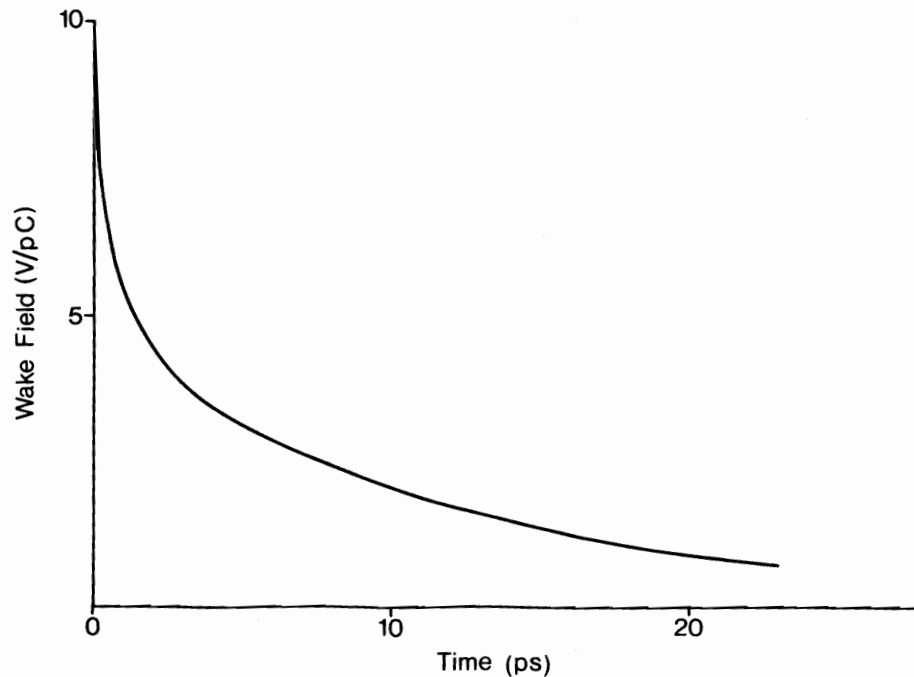


Fig. 1 Wake field function for SLAC Linac

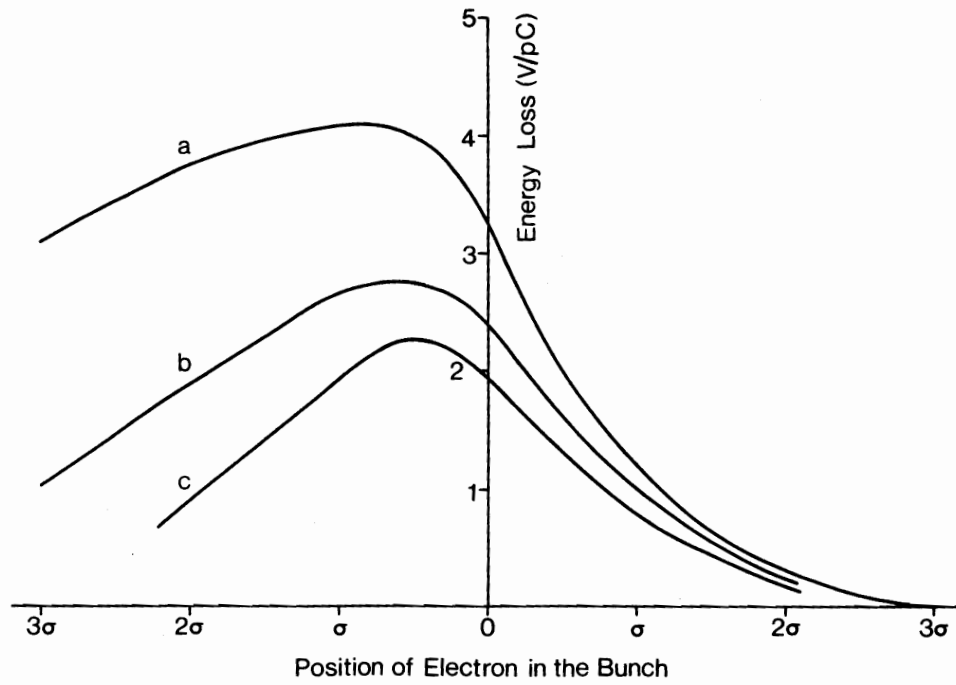


Fig. 2 Energy Loss distribution.

- a)  $\sigma = 1.7$  ps
- b)  $\sigma = 6.8$  ps
- c)  $\sigma = 13.6$  ps

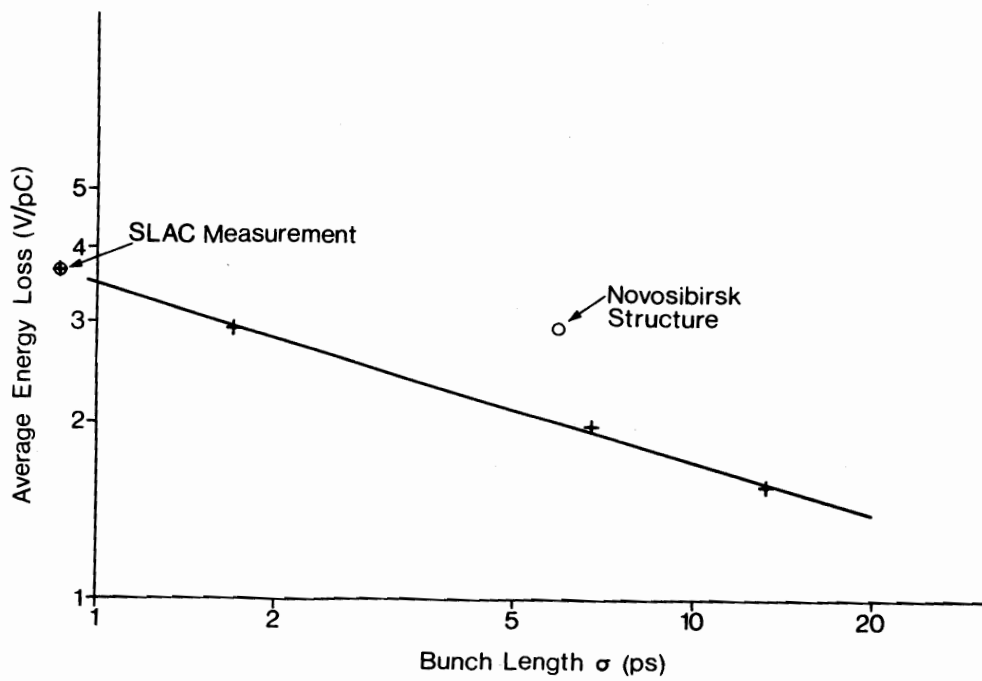


Fig. 3 Energy loss vs. bunch length

We need to derive the energy loss distribution due to this field, along the bunch. This can be derived by folding in a chosen Gaussian distribution with the wake field. This has been done for three cases, with  $\sigma = 1.7$  ps, 6.8 ps and 13.6 ps corresponding to  $1.65^\circ$ ,  $6.6^\circ$  and  $13.2^\circ$  respectively. The results are shown in Fig. 2. The average energy loss may be derived and is 2.90 V/pC, 1.97 V/pC and 1.56 V/pC respectively. Plotted on a log-log scale against bunch length in Fig. 3, it is apparent that the variation goes approximately as  $1/\sigma^{1/3}$ . This is at variance from the suggestion by Balakin<sup>6</sup>) that the loss varies as  $1/\sigma$ . The loss value computed for the Novosibirsk proposed linac structure<sup>7</sup>) is also shown for comparison in Fig. 3. The assumed bunch distribution in this case was  $\rho = \rho_0 \sin^2(\pi x/L)$  with  $L = 10$  mm, corresponding to  $\sigma \sim 6$  ps for an equivalent Gaussian distribution. Balakin<sup>6</sup>) has also suggested that loss is dependent on structure dimensions as  $\ln(d/a)$ , where  $d$  is the cavity pitch and  $a$  the aperture radius. This factor together with a modified wake field appropriate to their structure will probably account for the difference from the SLAC result.

### 3. ENERGY LOSS COMPENSATION

Consider the form of the energy loss distribution in Fig. 2. Clearly an approximate compensation is possible, in principle, by riding the centre of the bunch ahead of the crest of the accelerating wave. The best compensation is when the energy loss curve is nearest in shape to the crest of a sine wave.

The peak loss for the SLAC linac structure (at  $\sigma = 6.8$  ps) is  $2.77 \times 27 = 75$  V/pC per metre length. If the maximum peak field strength available is 20 MV/m then the intensity per bunch which gives best energy loss compensation is  $6.6 \times 10^{10}$ . The degree of compensation in this case is illustrated in Fig. 4. In this case, if the centre of the bunch is  $5^\circ$  ahead of the crest, 93% of the particles lie within an energy band of 0.08 MV/m or 0.4%. Clearly this percentage could be improved if the Gaussian distribution were clipped at  $2\sigma$ .

For a given bunch length the peak field and the number of particles in the bunch are strictly related. If the peak field achievable is only 10 MV/m then only  $3.3 \times 10^{10}$  particles per bunch can be compensated. Some improvement can be obtained by lengthening the bunch and by control of bunch shape, if this proves to be practicable. Thus the Novosibirsk studies claim to be able to obtain 90% of particles within a 1% energy spread with a peak field of 100 MV/m, a bunch population of  $10^{12}$  particles and a bunch length of 10 mm (sine squared distribution).

It should be noted that the maximum acceleration rate is less than the peak field because of the need for compensation. For example in the case illustrated in Fig. 4 the acceleration rate would be 19.2 MV/m.

### 4. REMARKS ON HIGHER ORDER MODE DAMPING<sup>8,9,10</sup>

If, in a superconducting linac, higher order modes are not damped, then their decay times will be long, much longer than the bunch repetition period in the proposed arrangement. This means that a bunch will not only see its own wake field but also the field arising from the passage of previous bunches.

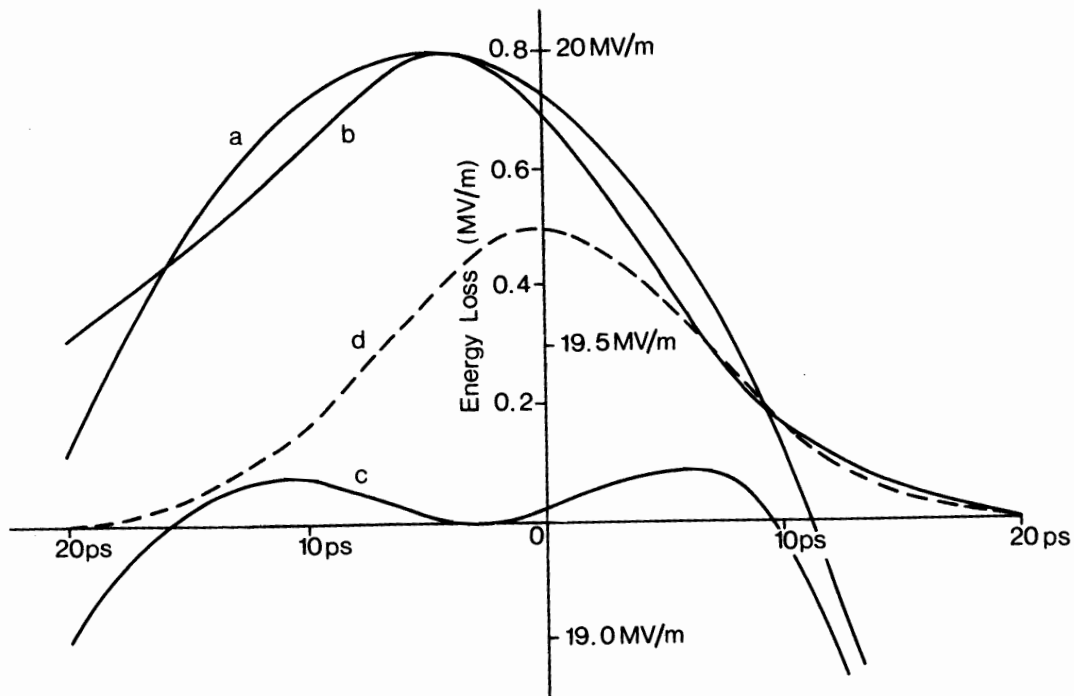


Fig. 4 Energy loss compensation ( $\sigma = 6.8$  ps (2 mm),  $N = 6.6 \times 10^{10}$ ,  $E = 20$  MV/m).

- a) Accelerating field
- b) Energy loss
- c) Net energy gain
- d) Bunch intensity distribution

The effective voltage will be

$V_n(t) = V_{on} e^{-\alpha_n t} \cos \omega_n t$  for each mode number,  $n$ , where  $V_{on}$  is the voltage induced in that mode by a single bunch,  $\alpha_n$  is the decay constant determined by the  $Q$ -value and  $\omega_n$  is the mode frequency. The steady state will be given by the sum of terms in which  $t = T, 2T, 3T$ , etc. where  $T$  is the time between bunches, and this may easily be evaluated for various values of decay constant and phase shift per orbit.

The energy loss in the steady state is

$$U_n = U_{on} f(\delta_n, \theta_n)$$

where  $\delta_n = \alpha_n T = \frac{\omega_n T}{2Q_n}$

and  $\theta_n = |\omega_n T - 2\pi j_n|$ ,  $j_n$  is an integer and  $U_{on}$  is the single bunch loss to the  $n$ th mode.

For the superconducting linac  $\delta$  will be typically about 0.001-0.002. Sands<sup>9)</sup> has estimated that the chance of hitting a resonance is of order  $\delta/2$ . This means that only modes very near synchronism build up, the rest having destructive interference which limits their growth. The question now arises whether by careful design together with a measure-

ment and adjustment programme it is practicable to avoid harmonic resonances.

However, we have so far only considered longitudinal modes. Transverse modes giving beam deflection do not need to be harmonically related and can build-up under certain conditions, as is well known from the experience at SLAC<sup>11</sup>) and elsewhere. The possibility of build-up of transverse modes can be minimised by a number of measures; namely, dimensional variation of the structure along the length, accurate alignment (because the modes are only excited by offset beams), strong focussing quadrupoles and the use of feedback. Whether such measures would be sufficient needs investigation.

## 5. CONCLUSIONS

5.1 For the superconducting linac with h.o. mode damping, assuming a peak field of 20 MV/m at S-band, the number of particles per bunch which should be used to give best energy compensation is  $6.6 \times 10^{10}$  assuming a bunch length  $\sigma = 6.8$  ps (2 mm). If more particles are required a longer bunch length is needed or a higher accelerating field.

5.2 Under these conditions the resulting energy spread need only be 0.4% for 93% of the particles. Clipping of the Gaussian tails would allow a higher percentage.

5.3 If particles are decelerated in a linac after interaction, then clearly no compensation is possible since the loss distribution and the field distribution will be additive. Thus particles would emerge from the decelerating structure in our typical example (Fig.4) with a spread of energy corresponding to 8% of the maximum energy. Quite clearly this makes any scheme of particle recovery quite impracticable.

5.4 It may be possible to avoid the use of h.o. mode damping in which case the energy spread in the bunch would be much reduced, though compensation by phasing would still be necessary.

## 6. TOPICS FOR FURTHER STUDY

The energy loss and compensation derived above is based on the SLAC linac structure, clearly not suitable for a superconducting linac. Further study should therefore be made to determine the energy loss parameter for structures which might be used and how this parameter might be minimised by optimum design.

Secondly, once the basic structure shape has been determined then calculations and measurements need to be carried out of the higher order modes occurring in the frequency range determined by the Fourier transform of the proposed bunch distribution. The aim should be to avoid harmonic resonances.

Thirdly, a theoretical investigation of beam break-up in the proposed linac needs to be undertaken to see whether damping of transverse modes can be avoided.

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