SOME REMARKS ON PLASMA INSTABILITIES IN ELECTRON AND POSITRON COLLIDING BEAMS

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ABSTRACT

The limitations on the intensity of colliding beams due to the development of plasma instabilities during the collision time are examined for a storage ring and a colliding linacs configuration. It is found that this effect can limit the luminosity for colliding linacs but not for storage rings.

The possibility that plasma instabilities, in particular the filamentation instability, might occur in a single crossing of an electron and positron bunch has been recently proposed by H.S. Uhm and C.S. Lin[1]. These instabilities would limit the electron and positron density and hence the luminosity of the two colliding beams. It is suggested in ref.[1] that this effect may be actually limiting the luminosity in Petra[2].

While Uhm and Lin have proposed this mechanism for electron and positron storage rings, the same effect might also be important for the case of colliding linac beams, which have been recently proposed as a way to obtain very high center of mass energies for electron-positron collisions[3,4].

In this note we want to establish a connection between the threshold for the filamentation instability and the parameters which are more commonly used to characterize the electron density limitation resulting from the beam-beam interaction. The parameters are: i) the beam-beam tune shift, ΔQ[5]; ii) the disruption parameter, D[4].

The beam-beam tune shift has been introduced to describe the requirement of beam stability over a number of revolutions of the order of 10^9-10^{10}, which is the typical beam life for particles in a storage ring. The stability condition can be written as [5,6].

$$\Delta Q \leqslant 0.05 \tag{1}$$

where ΔQ is expressed in terms of beam and storage ring parameters as

$$\Delta Q = \frac{r_e \beta^* N}{\pi a^2 \gamma} \qquad \text{(equal beams)} \tag{2}$$

where: N = number of electrons (positrons) per bunch; r_e = classical electron radius; a = electron (positron) bunch radius; γ = electron energy in rest energy units; β^* = storage ring parameter ($\beta^* \simeq .1$ - 1m) at the interaction point. In writing (2) we have assumed that the electron (positron) bunch is a cylinder of length L, radius a, with a uniform density distribution. This is a simplified description, which is, however, consistent with the model used in reference (1).

The disruption parameter can be written as[4]

$$D \equiv \frac{r_e L N}{2\gamma a^2} \tag{3}$$

In the design of colliding linacs it has been assumed up to now that[4)]

$$D \leqslant 1 \tag{4}$$

To establish a connection between ΔQ, D and the instability threshold it is convenient to introduce the plasma oscillation frequency, ω_p, for the system of electrons and positrons, calculated in the case of infinitely long bunches,

$$\omega_p^2 = \frac{4\pi\, r_e\, c^2\, n}{\gamma} \tag{5}$$

n being the electron (positron) density. In reference (1) it is shown that the maximum growth rate of the filamentation instability for the infinitely long bunches is given by

$$\frac{1}{\tau} \simeq \frac{\omega_p}{2} \tag{6}$$

The condition for stability for a bunch of length L is then written as

$$\frac{L}{2c\tau} \leq 1 \tag{7}$$

We now introduce a parameter P, measuring the number of plasma oscillations in a time, L/2c, corresponding to the bunch crossing time. This is

$$P \equiv \frac{L}{2c} \cdot \frac{\omega_p}{2\pi} = \left(\frac{L\, r_e\, N}{4\pi^2 a^2 \gamma}\right)^{\frac{1}{2}} \tag{8}$$

Using (6), (7), (8) we can now write the stability condition for the filamentation instability as

$$\pi P = \left(\frac{L\, r_e\, N}{4a^2 \gamma}\right)^{\frac{1}{2}} \leq 1 \tag{9}$$

Combining (2) and (3) with (8) we also have

$$\Delta Q = 4\pi \frac{\beta^*}{L} P^2 \tag{10}$$

and

$$D = 2\pi^2 P^2 \tag{11}$$

Since a storage ring is normally designed and operated with $L \leq \beta^*$ one has from (10) and (1) that in a storage ring the long term stability condition (1) implies that $P \leq 0.06$, so that (9) is satisfied and the filamentation instability has no time to develop in a single crossing. Whether the picture is significantly modified by multiple crossings of the beams remains to be seen.

From equations (9) and (11) it follows that

$$D \leq 2 \tag{12}$$

so that the disruption parameter can be as high as two before the filamentation instability can develop.

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