POSSIBILITIES AND LIMITATIONS OF THE ČERENKOV RING-IMAGING TECHNIQUE AT VERY HIGH ENERGIES

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ABSTRACT

The ring-imaging technique for particle identification is discussed and the upper limits in particle energy at which the technique could be applied are estimated. It is concluded that provided that development work now in progress on position-sensitive detectors for photons of energies lower than 7.5 eV meets with success, it would be possible to construct a large-aperture storage ring detector with 1 atm argon gas as radiator for full π/K/p separation up to about 119 GeV, and a 13.4 m long fixed-target large-aperture forward detector with 1 atm helium as radiator for full π/K/p separation up to about 595 GeV.

1. INTRODUCTION

Čerenkov photons emitted by a particle emerging from the centre of curvature of a concave spherical mirror are focused to a circular image on a spherical focal surface situated half-way between the mirror surface and the centre of curvature (see Fig. 1). The radius of the circular image is determined by the angle θ at which the Čerenkov light is emitted:

$$R = f \cdot \tan\theta \,, \tag{1}$$

where f = focal length of the mirror. If the circular image is recorded by a position-sensitive photon detector with good spatial resolution, the velocity $\beta = p/E$ of the particle can be determined from the Čerenkov angle and the refractive index n of the dielectric medium in which the particle travels, using the relation

$$\beta = \frac{1}{n\cos\theta} = \frac{1}{n\sqrt{1-(R^2/f^2)}} \,. \tag{2}$$

Furthermore, the direction of the particle trajectory may be determined from the position of the circle.

These are the basic principles underlying the Čerenkov ring-imaging technique[1-5]. The virtue of this technique is that, combined with a relatively crude momentum determination, it may also be used to identify all charged particles in events of high multiplicity in the case where the particles emerge in tightly collimated jets distributed over a wide solid angle.

The limit of applicability of the method when going to higher energies is set by the requirement on the resolution in the Čerenkov angle needed to separate π's, K's, and p's, and the requirement on the minimum number of detected photons needed for the pattern recognition of a ring image.

A circle may be reconstructed from the positions of three detected photons. Each additional detected photon gives a constraint in the reconstruction, thus increasing the precision in the determination of the radius and position of the circle, and also increasing the

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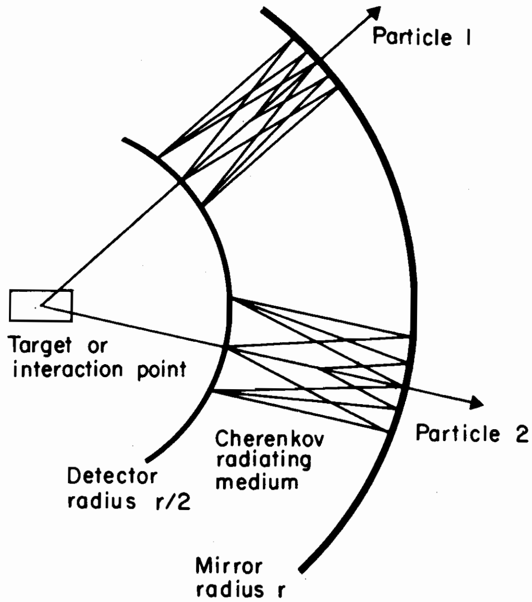

Fig. 1 Schematic drawing illustrating the optics of a Čerenkov ring-imaging detector (from Ref. 1).

reliability of the pattern recognition of the circular image. More than one such constraint is necessary in practical cases in which there may be several overlapping circle images as a result of several closely travelling particles, and also some background points. In the discussion below, it has somewhat arbitrarily been assumed that on the average a minimum of eight detected photons are needed to assure a reliable pattern recognition.

2. THE RESOLUTION IN THE ČERENKOV ANGLE

The resolution in the measurement of the Čerenkov angle θ is limited by several phenomena: chromatic dispersion of the Čerenkov light, due to variations of the radiator refractive index n with photon energy; errors in the measured position of the photons, due to the limited space resolution of the photon detector; multiple Coulomb scattering and energy loss (slowing) of the particle in the radiator medium; optical aberrations and diffraction. Of these, however, only the first two are of practical importance when using gaseous radiators at high energies.

The angular error due to the limited resolution in the photon position determination can be decreased by increasing the focal length of the mirror. The chromatic aberration, however, is a property of the specific radiator medium itself, and there is no way of compensating optically for this aberration in cases where the angular acceptance is large. (When the acceptance is small, as in the case of the DISC counter and the spot-focusing Čerenkov counter, chromatically correcting lenses may be used to reduce this aberration considerably.) In the following discussion we will therefore assume that the chromatic aberration gives the dominant contribution to the error in the Čerenkov angle.

Introducing $\gamma = E/m = (1-\beta^2)^{-\frac{1}{2}}$ as the velocity parameter, relation (2) gives for small values of θ,

$$\theta = \sqrt{1 - \{\gamma^2/[n^2(\gamma^2-1)]\}} \quad (3)$$

When $\gamma \to \infty$, θ approaches asymptotically the maximum value

$$\theta_{max} = \sqrt{1 - (1/n^2)} \; , \tag{4}$$

which for $(n-1) \ll 1$ simplifies to

$$\theta_{max} \simeq \sqrt{2(n-1)} \; . \tag{5}$$

The Čerenkov radiation sets in above the velocity value γ_{th} at which $\theta = 0$, i.e.

$$\gamma_{th} = 1/\sqrt{1 - (1/n^2)} = 1/\theta_{max} \; , \tag{6}$$

which for $(n-1) \ll 1$ simplifies to

$$\gamma_{th} \simeq 1/\sqrt{2(n-1)} \; . \tag{7}$$

Figure 2 shows how the Čerenkov angle grows from zero at the threshold velocity up to the asymptotic value θ_{max} at high velocities. In this figure the angular scale of the ordinate has been divided by θ_{max} ($\theta = \theta_{max} \rightleftarrows \theta_n = 1$), and the velocity scale of the abscissa has been divided by γ_{th} ($\gamma = \gamma_{th} \rightleftarrows \gamma_n = 1$). In such a diagram the relation between the normalized quantities θ_n and γ_n is

$$\theta_n = 1/\sqrt{1 + \{(1/[\gamma_n^2-1)n^2]\}} \; , \tag{8}$$

which for $(n-1) \ll 1$ simplifies to

$$\theta_n \simeq \sqrt{1 - (1/\gamma_n^2)} \; . \tag{9}$$

Note that the shape of the curve is independent of n as long as $(n-1) \ll 1$.

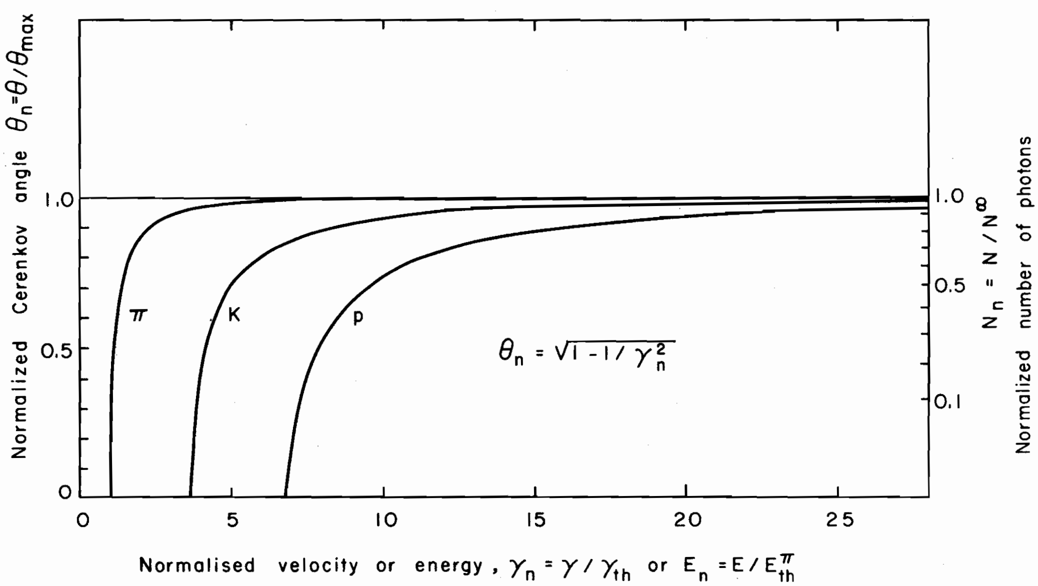

Fig. 2 In this linear plot the left curve shows how the normalized Čerenkov angle $\theta_n = \theta/\theta_{max}$ grows from zero at the normalized velocity $\gamma_n = 1$ up towards the maximum value 1 at large velocities. Taking the ordinate to be a relative energy scale normalized to the threshold energy for pions, the three curves correspond to π, K, and p as indicated. The ordinate on the right-hand side shows the normalized number of emitted Čerenkov photons ($N^\infty = 8$ in the text).

The left curve in Fig. 2 has its threshold at $\gamma_{rel} = 1$. The two other curves in this figure have their velocity threshold values such that if the abscissa is taken to be a relative total energy scale ($E = m\cdot\gamma$), and if the left curve is assumed to represent pions, then the other two curves represent kaons and protons, respectively. Their relative threshold energies are

$$E^K_{th} = \frac{K_m}{m_\pi} E^\pi_{th} = 3.54\, E^\pi_{th} \quad \text{and} \quad E^p_{th} = \frac{m_p}{m_K} E^K_{th} = 1.90\, E^K_{th} = 6.72\, E^\pi_{th} \ .$$

Because of the limited angular resolution, the kaon curve will merge into the pion curve from below at a certain energy E^K_{max}, and the proton curve will merge into the kaon curve at some higher energy E^p_{max}. In order to show more in detail how the curves approach their asymptotic value, $(1-\theta_n)$ has been plotted versus the relative energy E_{rel} in a log-log diagram in Fig. 3.

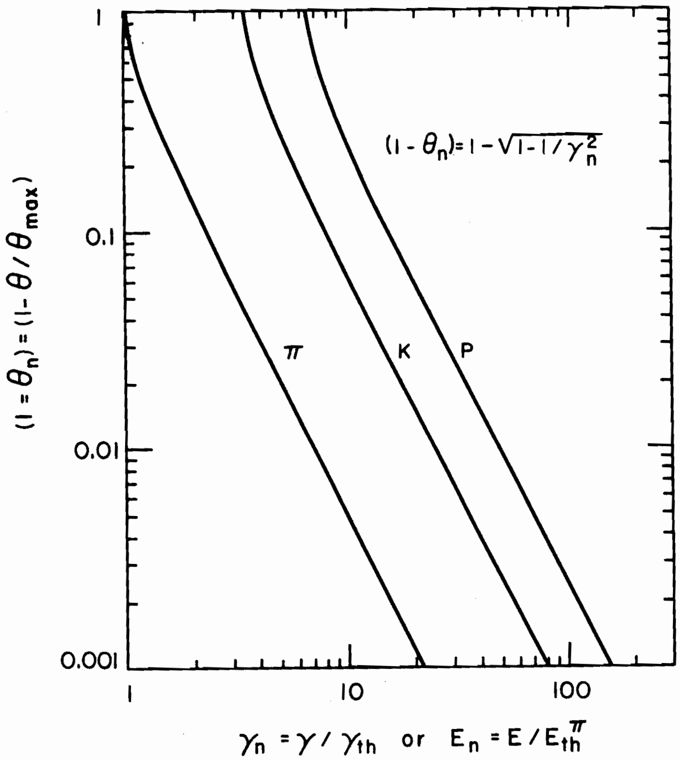

Fig. 3 Log-log plot of $(1-\theta_n)$ versus E_n showing in detail how the three curves in Fig. 2 approach their asymptote.

If the momentum of the particles is measured using a magnetic spectrometer with a $\Delta p/p$ not worse than of the order of 10%, then full separation between pions, kaons, and protons can in principle be made for energies between E^K_{th} and E^K_{max}. In the region E^π_{th} to E^K_{th} and E^K_{max} to E^p_{max}, only pions respectively protons can be uniquely defined.

The energies E^K_{max} and E^p_{max} are set by the uncertainty in θ. As discussed earlier, we will assume here that this uncertainty is essentially due to the chromatic dispersion of the Čerenkov light in the radiator. By differentiation of Eq. (3) and using (6) to introduce γ_{th}, we obtain

$$\Delta\theta = \frac{\gamma_{th}}{n^2} \frac{1}{\sqrt{1-(\gamma_{th}/\gamma)^2}} \frac{\Delta n}{\sqrt{12}} \ . \qquad (10)$$

Here Δn is the full variation in the radiator refractive index $n(E)$ over the bandwidth in Čerenkov photon energy (from E_1 to E_2) for which the photon detector is sensitive [i.e. $\Delta n = n(E_1) - n(E_2)$], and $\Delta\theta$ is the standard error in θ (this explains the factor $1/\sqrt{12}$). For $(n-1) \ll 1$ and $\gamma \gg \gamma_{th}$, using Eq. (7), Eq. (10) reduces to

$$\Delta\theta \simeq \gamma_{th}(\Delta n/\sqrt{12}) \simeq \Delta n/\sqrt{24(n-1)} \ . \tag{11}$$

Using Eq. (5) the relative change in θ is obtained as

$$\Delta\theta_n = \Delta\theta/\theta_{max} = \Delta n/[\sqrt{48}\ (n-1)] \simeq 0.144[\Delta n/(n-1)] \ . \tag{12}$$

This is the standard error in θ that would be obtained from one detected photon if the centre of the circle were known. As three points are needed to reconstruct this centre, we will assume that the standard error θ in the case of N detected points is

$$\Delta\theta_n \text{ (N points)} \simeq \Delta\theta_n \text{ (one point)}/\sqrt{N-3} \ . \tag{13}$$

For the case $N = 8$, requiring that θ should differ from θ_{max} by three standard deviations for a reliable particle separation, E^K_{max} and E^p_{max} can be obtained for any given radiator (i.e. for any given values of n and Δn, provided $n-1 \ll 1$) by evaluating the K and p energies for which

$$1 - \theta_n = (3/\sqrt{5})\ \Delta\theta_n = 0.194[\Delta n/(n-1)] \ . \tag{14}$$

Using Eq. (9) and the fact that $(1-\theta_n) \ll 1$, we obtain

$$E^i_{max} = E^i_{th}/\sqrt{2(1-\theta_n)} = E^i_{th}/\sqrt{0.387[\Delta n/(n-1)]} \ , \quad \text{where } i = K \text{ or } p \ . \tag{15}$$

3. THE NUMBER OF DETECTED PHOTONS

The energy distribution of the emitted Čerenkov photons is flat, and the number of photons per 1 eV photon energy bin is equal to

$$N = (\alpha/\hbar c)\ L \sin^2\theta \simeq 370\ L\ \theta^2 \ , \tag{16}$$

where L = length of the radiator in centimetres. When the velocity of the particle grows, N tends towards a constant value N^∞ (see Fig. 2 right-hand-side ordinate axis):

$$N^\infty \simeq 370\ L\ 2(n-1) \ . \tag{17}$$

If the efficiency of the photon detector for a photon of energy E (measured in eV) is $\varepsilon(E)$, then the total number of detected photons is

$$N_d \simeq N_0\ L\ \theta^2 \ , \tag{18}$$

where

$$N_0 = 370 \int_{E_1}^{E_2} \varepsilon(E)\ dE = 370\ \bar{\varepsilon}\ \Delta E \ . \tag{19}$$

For the estimates given below, we will somewhat arbitrarily assume N_0 to be equal to 80 cm^{-1}. This value would be obtained with, for example, a quantum efficiency of the photon electron of 30%, a reflectivity of the mirror of 90%, an optical transmission through the radiator and the detector window of 80%, and an energy bandwidth of $E_2 - E_1 = 1$ eV. The value 80 cm^{-1} may seem somewhat optimistic with respect to the values so far obtained in practice, which do not exceed 30 cm^{-1} [5]. However, the authors of Ref. 5 argue that a value of N_0 = 88 cm^{-1} could be obtained by assembling detector components that have already been tested separately to stated specifications. It may also be noted that N_0 = 80 is a value currently obtained with threshold counters using photomultipliers for the photon detection.

4. THE CHOICE OF RADIATOR

In order to have a high velocity threshold, the refractive index should be as small as possible [see Eq. (7)]. This requirement is, however, in conflict with the condition that the number of emitted photons should not be too low [see Eq. (17)]. Furthermore, for $\Delta\theta_n$ to be small, the relative chromatic dispersion $\Delta n/(n-1)$ should be small [see Eqs. (12) and (15)].

The values for n and Δn for a radiator medium depend on the energy band of the detected photons. In Refs. 3, 4, and 5, thriethylamine (TEA) is used as the gaseous photoionizing component mixed into the operation gas of the multiwire avalanche chamber employed for the photon detection. TEA has a photoionization threshold of 7.5 eV, which at present is the lowest known threshold for a compound that is in liquid phase and has a reasonable vapour pressure ($\sim$ 20 mm Hg) at room temperature.

Photons with $\hbar\nu \geq 7.5$ eV are in the far ultraviolet region (λ < 1650 Å); in this region quartz is opaque, and there are only certain crystalline salts such as CaF_2 or MgF_2 that are transparent. The window which is needed to separate the radiator gas from the detector gas must therefore be made from one of these salts. This constitutes a fatal technical complication when a large-solid-angle acceptance and thus a large-surface window is needed.

In order to be able to work in the photon energy region where quartz is transparent, a photoionizing compound must be used which has a lower threshold than that of TEA. All such compounds that are known, however, are solid at room temperature. To explore the possibility of using these compounds for photon detection, we may proceed along two lines. One is to deposit the solid compound (e.g. CsI, threshold 6.5 eV) on the cathode mesh of the multiwire proportional chamber (MWPC). A technical complication in this case is that the photocathode must not be facing the region where gas multiplication -- and thus photon emission -- occurs. This line is at present being worked on at CERN by the authors of Ref. 5. The other line is to use compounds with a low melting point [such as the electron-rich olefines[6]) or the metallocenes[7]) with melting points of the order of 50-100 °C and thresholds between 6 and 6.5 eV], and to heat the whole detector to a temperature where the compound is liquid and has sufficient vapour pressure (a few mm Hg) to produce total absorption of the photons over 10-20 mm. (This implies that the photon-electron conversion gap in the MWPC must be increased to this width for such an application.) This line is at present also being taken up at CERN.

For the calculations below, it has been assumed that either or both of these methods will work, and the values used for n and Δn for the different gases correspond to a photon energy band of 6.5-7.5 eV.

For applications at high energies, both (n-1) and Δn should be small. This condition is best satisfied by the light noble gases. The values for (n-1) at 7.0 eV and Δn = n(7.5 eV) - - n(6.5 eV) for the gases in Table 1 have been calculated at NTP (1 atm, 0 °C) by extrapolation of data at lower photon energies using the Cauchy dispersion relations as given by Langhoff and Karplus[8]). From these values have then been calculated the radiator length L(8) needed at atmospheric pressure to obtain, on the average, eight detected photons, and the lower and upper energy limits E^K_{min} and E^K_{max} for full π/K/p separation at the level of three standard deviations for each of the lighter noble gases. As lower limit, E^K_{min}, is given not the threshold energy, E^K_{th}, but the energy at which the number of detected photons [see Eq. (18)] is 80% of its maximum value, i.e. on the average 6.4 photons. This limit is situated at about $E^K_{min} = 2.24\ E^K_{th}$.

Table 1

Gas (1 atm at 0°C)	(n-1) (10^{-5})	Δn (10^{-5})	L(8) (cm)	γ_{th}	E_{th} (GeV)	E_{min} (GeV)	$\left(0.194 \frac{\Delta n}{n-1}\right)$ (10^{-2})	E_{max} (GeV)
He	3.74	0.0882	1340	115	57	128	0.46	595
Ne	7.22	0.180	690	83	41	92	0.48	416
Ar	34	2.19	147	38.4	18.8	42	1.25	119
Kr	55	4.88	91	30.1	14.8	33	1.72	80

5. A COLLIDING-RING DETECTOR

As already noted, one of the advantages of the ring-imaging technique is that it may be used to cover large acceptance angles. This is particularly valuable at storage rings where the method could be used to cover up to the full solid angle (4π) around an intersection region. In such a case, however, the volume of the detector grows as L^3, and one is therefore strongly restricted in the choice of the radiator length L. From Fig. 1 it can be seen that the diametre of a spherical detector is equal to 4L. In addition to the problems caused by the rapidly growing amount of construction material and space needed for the detector, the mechanical rigidity of the spherical mirror and the photon detector will become a technical problem at large L.

If one were free to choose the length L, then helium or neon would be the best choice for the radiator as they have the lowest ratio of Δn to (n-1) and thus the smallest chromatic dispersion $\Delta\theta_n$. However, the low values of (n-1) for helium and neon at atmospheric pressure implies unpractically long radiator lengths L (for neon the detector diametre would be 27.6 m). To reduce this length, one could in principle pressurize the gas. Given the large volumes involved and the requirements on mechanical rigidity, the pressurization will, however, also present technological problems.

The required radiator length in argon at atmospheric pressure is about 1.5 m, implying a detector diameter of 6 m. This may be technically feasible, in particular as the volume need not be pressurized. The price to be paid in comparison with helium and neon is an increase in the relative chromatic dispersion Δn/(n-1) by nearly a factor of 3.

Taking into consideration all the aspects discussed here, argon at atmospheric pressure seems to be the best choice for the radiator in a storage ring detector at high energy. The maximum Čerenkov angle is 26 mrad, which corresponds to a ring image radius of 39 mm using a mirror focal length (= radiator length) of 1.5 m. The relative chromatic aberration of about 7% corresponds to a spatial dispersion of about 0.4 mm of the photons around the circular circumference. The spatial resolution of the photon detector should thus not be worse than of the order of 0.15 mm (standard error). The rigidity and uniformity of the mirror and the photon detector should also be compatible with this figure. It should be noted that this condition puts strong requirements on the mechanical construction of the detector. It should also be noted that it would be difficult to profit fully from a reduction by a factor of 3 in chromatic dispersion even if it were possible to obtain this.

Under the assumptions made above, it would thus be possible to construct a ring-imaging detector with a 1.5 m argon radiator, capable of fully separating π's, K's, and p's in the energy interval 42 to 119 GeV, and of separating p's from π's and K's up to 226 GeV. With the transition radiation technique discussed at this Workshop in Group 8, it seems possible to cover the region down to about $\gamma = 500$, i.e. π's of energy of about 70 GeV and K's of energy of about 247 GeV. It thus appears that the ring-imaging technique can be made to cover the energy region approximately up to where the transition radiation can take over.

6. A FIXED-TARGET DETECTOR

At fixed-target machines, only a limited solid angle around the forward direction needs to be covered in most experiments, and longer radiators may therefore be foreseen. With a 13.4 m long helium radiator at atmospheric pressure, full π/K/p separation can in principle be made up to about E^K_{max} = 595 GeV. The Čerenkov angle is 8.7 mrad, the circle radius is 115 mm, the dispersion due to chromatic aberration of the points around the circular line is 0.4 mm, and the required spatial resolution of the photon detector is thus again of the order of 0.15 mm (standard error).

To increase E^K_{max} further would require a reduction in the radiator gas pressure proportional to $(1/E^K_{max})^2$ in order to increase E^K_{th}, and an extension in the radiator length proportional to $(E^K_{max})^2$ in order to keep the average number of detected photons at eight. The requirements on the spatial resolution of the photon detector can be relaxed when increasing E^K_{max}, as the radius of the Čerenkov ring increases as E^K_{max}. The ultimate limit in this extension to higher energies will be set by the requirements on the rigidity and regularity of the mirror, the photon detector and the detector support, and on the homogeneity and stability of the radiator density. These requirements become increasingly difficult to meet when the detector grows in length. To set an upper limit to E^K_{max} for fixed-target detector, a detailed technical feasibility study of these requirements -- specifically those concerning the spherical mirror -- would be needed. Furthermore, the rapidly growing radiator length would eventually become a practical problem also for fixed-targed experiments.

It is important to keep in mind that if the angular acceptance needed is less than the order of a few degrees, chromatically correcting lenses may be used to radically reduce the effects of chromatic aberration[9]. In addition, the spot-focusing technique[10] may be used to considerably relax the requirements on the spatial resolution of the photon detector, making possible the use of an array of small photomultipliers instead of a photosensitive MWPC.

Thus it is only when a larger aperture should be covered that the ring-imaging technique is needed in fixed-target experiments, e.g. measurements of deep inelastic scattering involving large momentum transfers and large effective masses[11]; whereas measurements on diffractive dissociation, for example, would be better made with the spot-focusing technique.

7. CONCLUSIONS

Provided that photon detectors with high spatial resolution ($\sigma_x \sim 0.15$ mm) and high efficiency (quantum efficiency 30%) for photons in an energy region where quartz is transparent ($\hbar\nu < 7.5$ eV, $\lambda > 1650$ Å) will become available, the Čerenkov ring-imaging technique will provide an interesting possibility for particle identification at high energies. The inherent characteristic of good two-track separation over a large acceptance aperture makes the technique particularly well suited to storage ring experiments. A spherical counter with a 1.5 m long argon radiator at atmospheric pressure could be used for full π/K/p separation up to about 119 GeV and proton identification up to about 226 GeV to analyse jet-like events over an acceptance solid angle of up to 4π. In fixed-target experiments the technique will be useful when a large aperture is needed in the forward direction, as in experiments on deep inelastic scattering. With a 13.4 m helium radiator, full π/K/p separation can be made up to about 595 GeV. At higher energies, very long helium radiators at low pressure could possibly be used, provided that mirrors of sufficient quality can be produced.

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