LIMITS ON PROTON BUNCH LENGTH AND POPULATION

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ABSTRACT

Recently proton storage rings with bunched beams have become topical. The SPS proton-antiproton collider and DESY's proton-electron project, HERA, both require their circulating protons to be concentrated in a small number of short and dense bunches. In this paper we bring together experimental results from the SPS and current theories of bunch stability to discuss limits on proton bunch population for these and for future proton storage rings.

1. INTRODUCTION

In both $p\bar{p}$ and ep colliders one or other of the particles are in short supply and the circulating beams must be concentrated in a few intense bunches. These bunches must also be short. One of the reasons for this is that the waist in the beta function which has a minimum value, β^* at the interaction point only extends over a characteristic length equal to β^*. If the bunch length is larger than β^* particles meet outside the waist where the beams are less dense and there the probability of collision is reduced. If luminosity is not to be sacrificed the bunch length, $c\Delta t$, should not exceed β^*.

In the case of e-p colliders there is an additional reason to keep the bunch short. In most of the schemes proposed the beams must cross at a small but finite angle if they are to be easily separated after interaction. This angle is large enough for the effective width of the proton beam seen by an electron to be determined by the bunch length. Since this occurs in the denominator of the expression for the luminosity there is a strong incentive to minimise this length.

At first sight it might appear that these arguments simply demand as high an r.f. frequency as possible but there is an independent condition which, while reinforcing the argument for a short bunch, demands that the bunch length $c\Delta t$, be short compared with the bucket length $c\tau$. This stems from recent SPS experiments[1] where intense bunches have been accelerated and stored and where it has been found that if the bunch to bucket ratio is not small protons will diffuse out of the bucket. Even when considerable attention has been paid to minimising the noise in r.f. control loops which causes this diffusion, the lifetime of a larger bunch is much less than the 24 hours for which one might wish to sustain luminosity in such a collider.

The SPS experiments remind us that the line density of protons is ultimately limited by collective phenomena. Since recent advances in the theory of such phenomena[2],[3] now being applied to electron machine design[4] reveal a confluence of understanding which agrees with the new experimental evidence from proton machines, it seems an appropriate moment to summarise these limits. This we do in a way which is intended to aid the designer of a new collider in his choice of r.f. frequency and arrive at an optimum luminosity.

2. COUPLING IMPEDANCE

The dense bunches of the machines which we shall discuss are spaced well apart and we shall ignore inter-bunch coupling. Seen by one of these bunches, the various boxes, flanges and changes of vacuun chamber cross-section in a machine like the SPS or CPS add up randomly to a broad-band coupling impedance. The effect of this impedance is often approximated by the impedance of a broad band resonator in computer studies[5]. The vacuum chamber of a modern proton synchrotron has a cut-off frequency just over 1 GHz and the spectrum of a bunch of 50 cm length will mainly lie below this cut-off frequency where the resistive component of the impedance is monotonically rising and where the inductive component is comparable but peaked at about half the cut-off frequency (Fig. 1). Measurements of the inductive part of the impedance made in the ISR[6] give Z/n between 20 and 30 ohms and we obtain a good fit to SPS instability threshold with a similar value. We suppose the resistive part to be comparable.

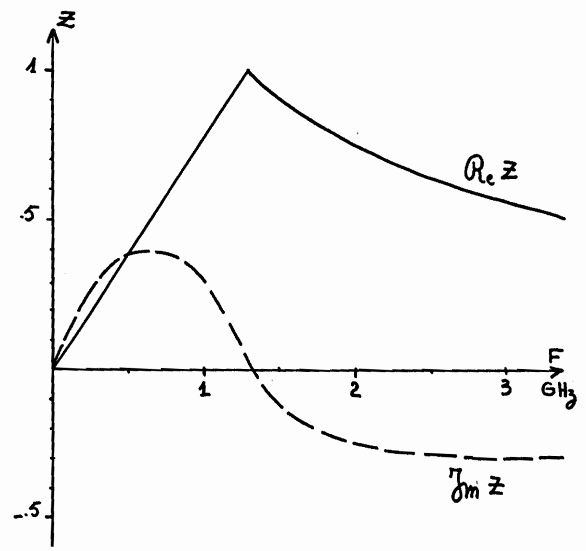

Fig. 1 : Real and Imaginary Part of Coupling Impedance

Future proton accelerators with aspirations to accelerate or store intense bunches would do well to adopt the measures taken in electron machines like PETRA and PEP to eliminate vacuum chamber discontinuities. In this way one might hope to lower Z/n by a factor 3 or even 10.

3. THE MICROWAVE INSTABILITY

This is a fast growing instability which can afflict accelerators as well as storage rings and if present will preempt the slow coherent mode instability described later.

It has been shown elsewhere[7] by an extension of the Keil-Schnell criterion that microwave instability depends on the local current I and the local momentum spread $\Delta p/p$ as we pass along the bunch and demands for stability that :

$$\frac{Z}{n} < F' \frac{E_0}{e} \frac{\eta\gamma}{I} \left(\frac{\Delta p}{p}\right)^2 \qquad (1)$$

where γ is the Lorentz factor

F' is a form factor which we approximate to 1

and $\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$.

For parabolic distributions and provided the bunch is not comparable to the length of the bucket we may rewrite (1) in terms of bunch parameters as :

$$\frac{Z}{n} < \frac{c\ V\ f}{6\ eNR}\ \Delta t^3 \qquad (2)$$

where V is the accelerating voltage amplitude

f is the accelerating radio frequency

$c\Delta t$ is the bunch length ($\sim 4\sigma_s$)

N is the number of particles in the bunch

R is the machine radius

Finally recasting this in a form which reveals the dependence on design parameters, we have the stability criterion :

$$\left(\frac{\Delta t}{\tau}\right) \Delta t^2 > \frac{5.1 \times 10^{-28}\ N}{(V/2\pi R)} \cdot \frac{Z}{n} \qquad . \qquad (3)$$

4. THE COHERENT MODES INSTABILITY

In the recent SPS experiments we found that short intense bunches with line densities approaching 10^{11}/m became unstable. Coherent modes grew very slowly, over minutes, developed into higher modes and subsided after as long as half an hour leaving the bunch longer and prone to r.f. diffusion loss. The frequency shift due to the inductive wall, $\Delta\Omega_{sc}$, is thought to exceed the spread in synchrotron frequencies, S, within the bunch and thus prevent Landau damping which otherwise would not allow such slowly growing modes to appear. Sacherer[8] expresses a criterion for stability against such phenomena as :

$$\frac{S}{\Omega_0} > k_m\ \frac{\Delta\Omega_{sc}}{\Omega_0} \qquad (4)$$

where k_m is a coefficient which depends on the mode number

Ω_0 is the small amplitude synchrotron frequency.

We can approximate S very well by :

$$\frac{S}{\Omega_0} = \frac{2\pi}{64} \left(\frac{\Delta t}{\tau}\right)^2 \qquad (5)$$

and find accurate prediction of k_m for the various modes in Besnier's thesis[9].

Table 1

Stability Coefficients

m	1	2	3	4
k_m	3.4	1.6	0.9	0.65

The right hand-side of the stability condition (4) can be expressed for a parabolic bunch as :

$$\frac{\Delta\Omega_{sc}}{\Omega_0} = \frac{3}{2\pi^2}\ \frac{I_0}{hV} \left(\frac{2\pi R}{c\Delta t}\right)^3\ \frac{Z}{n} \qquad (6)$$

where I_0 is here the d.c. component of a single bunch

h is the harmonic number.

We rewrite (4) with the help of (5) (6) and (7) as :

$$\left(\frac{\Delta t}{\tau}\right)^3 \Delta t^2 > \frac{1.3\times10^{-28}\ k_m N}{(V/2\pi R)} \cdot \frac{Z}{n} \qquad (7)$$

The units in this as in previous expressions are MKS and the almost astronomical constant is merely $24e/c\pi^4$.

5. DISCUSSION

Comparing the stability conditions for microwave instabilities (3) and for the inductive wall shift which allows coherent modes to develop (7), we find that they have the same form. A more accurate calculation of the microwave limit which does not contain the short bunch approximation is given in Fig. 2. It makes use of the local elliptic energy distribution[11] which gives the same microwave instability threshold all along the bunch. This distribution, while giving about the same results as (2) for short bunches, allows more accurate calculations for full buckets. The microwave instability is very fast and cannot be fought by means of a feed-back system. It therefore must be avoided and it determines the maximum intensity which can be stored in a bunch given the bunch length ($c\Delta t \sim \beta^*$) and other machine parameters (V, R and Z/n).

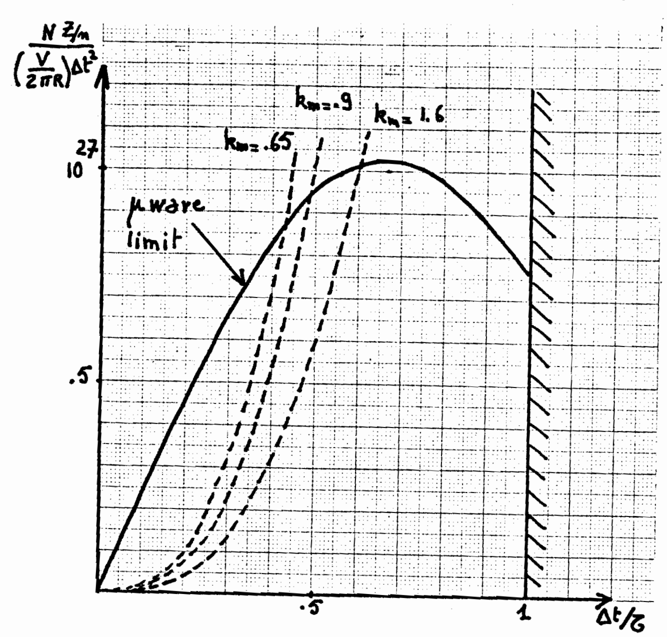

Fig. 2

On the same graph we have plotted the slow instability threshold for various values of k_m, corresponding to more and more sophisticated feed-back systems (k_m = 1.6 corresponds to a feed-back working on the dipole mode only, k_m = 0.65 to dipole, quadrupole and sextupole). The maximum intensity is little affected by the complexity of the feed-back system for $\Delta t/\tau$ larger than, say 0.5.

However, the bunch life-time decreases very rapidly as the bunch is made long. Theory[6] suggests a dependence of the type :

$$1/T \sim P_N \left(\frac{\Delta t}{\tau}\right)^4 V\ h$$

T being the bunch life-time and P_N the noise power in the r.f. system. SPS experiments have rather shown a $P_N\left(\frac{\Delta t}{\tau}\right)^4$ dependence within a limited range of r.f. voltages (1 to 5). They have shown that without sophisticated electronics one can reach 24 hours life-time with $(\Delta t/\tau) \simeq 0.3$. In order to obtain the maximum possible intensity which corresponds to $(\Delta t/\tau) \simeq 0.5$ without sacrificing the life-time one would have to reduce P_N by an order of magnitude (10 dB in power, that is a factor 3 in the r.m.s. noise level). However the other noise sources which are not taken into account in this simple picture (e.g. amplitude noise,

noise at $n \times f_{rev} \pm f_s$) may become the limiting factor. In any case a situation in which $\Delta t/\tau$ is larger than 0.5 is not interesting as is shown by Fig. 2. Given a $\Delta t/\tau$ which is related to the noise properties of the system (in practice $0.3 < \Delta t/\tau < 0.5$) the choice of the other parameters becomes obvious. The r.f. frequency is chosen as low as possible such that $c\Delta t \simeq \beta^*$ and the r.f. voltage as high as possible in order to get the maximum intensity.

Following this philosophy and assuming that not only is the vacuum chamber of the machine smooth, but that feed-back systems suppress the first few coherent modes, a re-design of the SPS might have the following parameters, staying at the safe side from the point of view of r.f. noise :

$$\Delta t/\tau = 0.4$$
$$\beta^* = 0.6 \text{ m}$$
$$\Delta t = 2 \text{ ns}$$
$$f = 200 \text{ MHz}$$
$$Z/n = 3\ \Omega$$
$$V/2\pi R = 1 \text{ MV/km}$$
$$N = 10^{12} \text{ particles/bunch}$$

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