# LIMITATIONS ON p̄-p LUMINOSITY WITH DIRECT INJECTION AND STACKING OF ANTIPROTONS

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ABSTRACT

If protons of very high energy impinge on a target, a large part of the resulting antiprotons are sufficiently collimated to be injectable into a stacking and accelerating ring. They can then be stacked and injected into the main proton accelerator so as to produce p̄-p collisions without low energy antiproton cooling. A scheme is presented for the VBA, where 20 TeV protons produce $9 \times 10^{-4}$ antiprotons per proton at 100 GeV, which are then stacked, accelerated to 1 TeV, and injected into the main ring. With 16 proton pulses of $10^{15}$ protons, one obtains a luminosity of the order of $10^{32}$ $cm^{-2}$ $sec^{-1}$ with a beam-beam tune shift of $10^{-3}$ per interaction region. The beams are bunched into 1000 bunches; the orbits are separated by means of relatively modest electostatic electrodes.

To stack antiprotons at low energy one generally depends on some cooling scheme to reduce the phase space or increase the phase space density. However, both electron cooling and stochastic cooling become ineffective for p̄ energies above some 10 GeV if cooling times of a few seconds or shorter are desired. On the other hand, at incident energies in the TeV region the p̄ are produced in such a narrow forward cone that the phase space density is naturally quite high and useful luminosity can be obtained for p̄-p collisions even without cooling. We examine here the limitations on luminosity obtainable with direct production and stacking of the p̄ beam at high energies.

## A. Production of Antiprotons

We assume the antiprotons are produced by a proton beam at TeV energy focused on a small spot on a heavy target. The invariant cross section

$$\sigma' = E\,\frac{d^2\sigma}{dp^3}$$

for inclusive p̄ production has an asymptotic value of about 3 mb/GeV$^2$ for incident proton energies above 1 TeV and for p̄ produced at rest in the center-of-mass frame of the incident p-nucleon system. The p̄ energy $\bar{E}$ is, therefore, given by

$$\bar{E} = \sqrt{ME/2}$$

where E and M are the energy and rest energy of the incident proton. For E = 20 TeV this gives $\bar{E}$ 100 GeV. The integrated production cross section is

$$\sigma_{prod} = \int \sigma' \frac{d^3p}{E} = \int \sigma' p^2 d\Omega\, \frac{dp}{E} = \sigma' M^2 \frac{\epsilon_t^2}{\pi a^2} \frac{\Delta p}{p}$$

where

$a$ = radius of p spot on target

$\epsilon_t$ = normalized transverse emittance of $\bar{p}$ beam

and where the replacement of integration by a product is valid for a production semi-cone angle

$$\epsilon_t/\pi a < 0.4 \text{ GeV}/\bar{E} \simeq 4 \text{ mrad}$$

and momentum spread $\Delta p/p < 20\%$.

The longitudinal emittance normalized in the same manner as the transverse emittance is given by

$$\epsilon_\ell = \frac{\Delta p}{M}\ell$$

where $\Delta p$ is the full momentum spread and $\ell$ is the length of the $\bar{p}$ beam (equal to the length of the proton beam). Thus we have

$$\sigma_{prod} = \sigma' \frac{M^3}{p} \frac{N}{\pi a^2 \ell} = 6.2 \times 10^{-4} \frac{N}{\pi a^2 \ell}$$

where we have put in the values $\sigma' = 3$ mb, $\sigma_{abs} = 40$ mb, $p = 100$ GeV corresponding to $\bar{p}$ production by 20 TeV protons.

We see that the six-dimensional phase space density of antiprotons is proportional to the physical density of the proton beam on the target. To increase it we have to reduce the volume $\pi a^2 \ell$ of the proton beam. Hence we must compress or bunch the p beam longitudinally to reduce $\ell$ and focus the beam to a small spot on the target to reduce $\pi a^2$.

B. Numerical Example

We assume the following parameters for a 20 TeV main synchrotron (VBA):

| | |
|---|---|
| Injection energy | 1 TeV |
| Final energy | 20 TeV |
| Circumference C | 57 km |
| Protons per pulse | $10^{15}$ |
| Pulse rate | $(100 \text{ sec})^{-1}$ |

To get the maximum density of protons on the target we assume that the $10^{15}$ protons can be compressed by RF manipulation to a tenth of the circumference ($\ell$ = 5700 m) before extraction and that the extracted p beam can be focused to a spot of radius a = 0.15 mm. We also assume a target efficiency of 50%. This gives for each main synchrotron pulse

$$\frac{\bar{N}}{\epsilon_t^2 \epsilon_\ell} = 7.7 \times 10^{14} \text{ m}^{-3}$$

The phase space volume is limited first by what is available coming from the target, and second by the aperture of the main synchrotron at injection, as well as by the aperture of the ring the antiprotons are first injected into. As stated in section A above, the semi-cone angle of the 100 GeV $\bar{p}$ beam is $\alpha = 4$ mrad and the momentum spread is 20%; however, we assume the stacking ring can only accommodate $\Delta p/p = 5\%$. Thus

$$\epsilon_t = \pi a \alpha \gamma = 2 \times 10^{-4} \text{ m}$$

$$\epsilon_\ell = \gamma\ell\Delta p/p = 3 \times 10^4 \text{ m}$$

To get an estimate of the emittance limitation arising from the main synchrotron aperture at injection we assume

$\beta_{max}$ 400 m

$\eta_{max}$ 5 m

Max. betatron width $\sigma$ = 5 mm

Max. momentum width $\delta$ = 5 mm ($\Delta p/p$ = .001)

This gives transverse and longitudinal acceptance for the main synchrotron

$$\epsilon_t = 4\pi\sigma^2\gamma/\beta_{max} = 8.4 \times 10^{-4} \text{ m}$$

$$\epsilon_\ell = C\gamma\delta/\eta_{max} = \gamma C\Delta p/p = 6 \times 10^4 \text{ m.}$$

The longitudinal acceptance is half filled in one pulse, but to fill the transverse acceptance in both planes one needs about 16 pulses. In principle, then, the six-dimensional acceptance volume of the VBA can be filled in about 16 pulses or 25 minutes. But, of course, one has to take a close look at the practical filling procedure to determine the actual number of pulses needed. Here we assume that we can fill the main synchrotron with

$$\bar{N} = 4^2 \times (7.7 \times 10^{14} \text{ m}^{-3})(2 \times 10^{-4} \text{ m})^2(3 \times 10^4 \text{ m}) = 1.5 \times 10^{13}$$

antiprotons in a phase space volume given by

$$\epsilon_t = 2 \times 4 \times 2 \times 10^{-4} \text{ m} = 16 \times 10^{-4} \text{ m}$$

(assuming dilution by a factor of 2 and four-fold multiturn injection in each plane);

$$\epsilon = 3 \times 10^4 \text{m}$$

(assume the VBA can actually accept twice the emittance given above).

C. Tune Shift and Luminosity

For two identical bunched beams colliding head-on the tune shift $\Delta Q$ and the luminosity L are given by

$$\Delta Q = \frac{r_p N}{n\epsilon_t} \quad \text{and} \quad L = \frac{f\gamma N}{r_p\beta^*}\Delta Q$$

where N = number of particles in each beam = $1.5 \times 10^{13}$

n = number of bunches per beam (assume n = 1000)

f = revolution frequency = 5260 Hz

$\beta^*$ = $\beta$ at collision point = 1.5 m

$r_p$ = classical proton radius = $1.54 \times 10^{-18}$ m.

These values give

$$\Delta Q = 1.5 \times 10^{-5}$$

$$L = 1.0 \times 10^{30} \text{ cm}^{-2}\text{sec}^{-1}.$$

Since the tune shift is so small we can safely consider a much stronger proton beam. If we take the full beam of $10^{15}$ protons with the same emittance as the antiproton beam, the luminosity and tune shift are each increased by a factor $10^{15}/1.5\times10^{13} = 67$ giving $\Delta Q = 0.001$ and $L = 0.7\times 10^{32}$.

D. Antiproton Stacking Scenario

We present a scenario which may be the cheapest but not necessarily the simplest or easiest. We need one relatively small antiproton stacking and accelerating ring with approximately the following parameters:

| | |
|---|---|
| Circumference (1/10 of the VBA) | 5.7 km |
| Stacking energy | 100 GeV |
| Stacking dipole field | 0.5 T |
| Maximum energy (= VBA injection) | 1 TeV |
| Max. dipole field | 5 T |
| $\beta_{max}$ | 50 m |
| $\eta_{max}$ | 0.5 m |

The circumference just accommodates the length of the $\bar{p}$ beam produced by the proton beam which has been compressed in the main ring. The values of $\beta_{max}$ and $\eta_{max}$ indicate that this is a rather strong focusing ring. The $\bar{p}$ beams from 16 main synchrotron cycles, each with $\epsilon_t = 2\times10^{-4}$ m and $\epsilon_l = 3\times10^{4}$ m (corresponding to $\Delta p/p = \pm 2.5\%$ at production) are stacked in transverse phase space (4 times per plane) at 100 GeV. The stack contains $1.5\times10^{13}$ antiprotons and has emittance $\epsilon_t = 16\times10^{-4}$ m (assuming a factor of 2 dilution) and $\epsilon_l = 3\times10^{-4}$ m, with widths of $\pm$ 7.7 mm (betatron oscillations) and $\pm$ 12.5 mm (momentum spread). The ring needs a good field aperture (total) of about 70 mm.

At the top energy of 1 TeV the $\bar{p}$ beam is "unstacked" in momentum, i.e. extracted by momentum peeling in 10 turns. The momentum spread of $\pm$ 2.5% at 100 GeV will have been reduced to $\pm$ 0.25% by acceleration and is further reduced to $\pm$ 0.025% (full spread $5\times10^{-4}$) by the ten-turn momentum peeling (we may allow a dilution factor of 2 to get a full spread of $10^{-3}$). This beam now has the same length as the main synchrotron, and is injected into it. Then a normal pulse of $10^{15}$ protons is injected, and both counter-rotating beams are accelerated, with 1000 bunches per beam, to 20 TeV and used for colliding beam experiments. The orbits are kept separated by an electrostatic scheme outlined below.

In this scenario there are several unconventional procedures:

1. Transverse phase space stacking in both planes has never been done with any reasonable efficiency. One possible method is the inverse of resonant extraction: the beam enters along an incoming separatrix, and the central stable region is expanded in one turn by just the amount to accommodate the incoming beam. Because this process is untried we have allowed a dilution factor of 2 per plane.

2. Longitudinal phase-space unstacking is also an untried process. This could again be the inverse of momentum stacking. The RF is turned on abruptly to form longitudinal phase-space buckets in the middle of the stack. When the buckets are moved out of the stack by frequency modulation the particles in the buckets are "peeled" away from the beam stack. One should also allow for some dilution here, but if we assume zero dispersion at the collision points, this does not affect the luminosity.

3. It is necessary to have different orbits for the two beams in order to avoid beam-beam interactions other than at the desired interaction points. This may be done with the help of electric fields (static or RF). A possible electrostatic separation scheme is as

follows: If there is a field $E_y = E_o \sin n\theta$ in the main synchrotron, where n is the integer nearest to Q, the orbit equation is approximately

$$\frac{d^2y}{d\theta^2} + Q^2y = \pm \frac{eE_oR^2}{\gamma Mc^2} \sin n\theta$$

and the orbit is

$$y = \frac{\pm 1}{Q^2-n^2} \frac{eE_oR^2}{\gamma Mc^2} \sin n\theta$$

Take Q = 60.25 for the VBA, and n = 60. To get an orbit amplitude of 1 cm (separation between p and $\bar{p}$ of 2 cm at the peaks) we need

$$E_o = 0.73 \text{ kV/cm}$$

If we use 60 short dipoles, one per orbit wavelength, we find that each one must be of strength 73 kV/cm x 4.75 m, which appears feasible.

In this scheme the orbits intersect in 120 places, rather than just the six or eight envisioned for experiments. To avoid unwanted beam-beam interactions, we may use timing: If the beams are bunched in a number m of short bunches, there are just 2 m places where the two beams pass synchronously. If m is chosen so that the greatest common factor of m and n (the number of orbit wavelengths) is 4, then there will be just 8 of the 2n orbit intersection places where p and $\bar{p}$ bunches pass simultaneously. Thus, with n = 60, we choose m = 1004 bunches (or m = 999 for six crossings). However, there will still be a number of places where near misses occur: at two locations per octant the bunches miss by only 0.07 bunch spacings, i.e., 4 meters. The bunch length is likely to be larger than this, especially at injection. These spurious crossings will contribute to the beam-beam tune shift, but probably not too much to the nonlinear long-term instability (since they repeat periodically).

E. Alternate Scenario

The longitudinal energy unstacking can be avoided by adding a full-circumference stacking ring in the same tunnel as the main ring. One now transfers from the 100-1000 GeV $\bar{p}$ accelerator into one tenth of the main ring circumference, and repeats ten times, at the expense of a correspondingly long filling time. Here we may be somewhat more modest in filling the "small" stacking ring, say with a momentum spread of only 1% (total) and with an emittance smaller by a factor 2. This reduced the number of main ring pulses per stacking cycle in the $\bar{p}$ accelerator to 8. The filling time is increased by a factor of 5; the number of $\bar{p}$ is the same as before, but the luminosity is larger because of the smaller emittance. There is no debunching required in the transfers from $\bar{p}$ accelerator to stacking ring to main ring; therefore the longitudinal phase space efficiency is likely to be better, and the bunch length shorter (this reduces the beam-beam effects in the unwanted crossing points, as discussed above). This scheme is probably more expensive than the other one, but requires somewhat less extreme optimism in the capacity of the accelerator ring and the longitudinal stacking scheme; it also promises somewhat better luminosity.

A further variation is to use 5 TeV protons rather than 20 TeV to make antiprotons, thus reducing the basic cycle time. The antiprotons are now made at 50 GeV. The antiprotons may now be accelerated to 1 TeV after every production pulse using the same accelerator that is used in the main proton accelerating complex (thus eliminating the extra antiproton accelerator). Now all the stacking, transverse as well as longitudinal, would have to take place at 1 TeV.

## F. Summary

We have shown that there are several schemes for producing a proton-antiproton collision capability by direct high-energy production of antiprotons without cooling. The attainable luminosity appears to be around $10^{32}$ $cm^{-2}sec^{-1}$.