COHERENT INSTABILITIES IN THE 20 TeV RING*)

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ABSTRACT

We discuss some of the limits on the average and peak current in the 20 TeV ring, produced by coherent instabilities. We also discuss some effects of synchrotron radiation.

1. INTRODUCTION

We investigate the limitations on the performance of the 20 TeV proton ring due to coherent instabilities. This investigation uses a simple model[1] to describe the stability limits, based on the assumption that all low frequency instabilities, $\omega < c/d$, where d is the vacuum tank dimension, can be cured by feedback. To stabilize high frequency effects one has to rely on Landau damping.

We discuss the limitations on the beam intensity and beam configuration obtained from this model. We also discuss some of the effects introduced by synchrotron radiation.

2. STABILITY CRITERIA

As discussed in Ref. (1) we use two stability conditions, one for longitudinal effects, the microwave instability, and the other for transverse effects. They can be written as

$$e\, I_p \left(\frac{Z}{n}\right)_{eff} \leq \alpha\, E \left(\frac{\Delta E}{E}\right)^2 \qquad (1)$$

$$\frac{e\, I_p\, Z_{T_{eff}}\, R}{2\pi\, \nu\, E} \leq \Delta\nu \qquad (2)$$

where $\Delta E/E$ is the beam energy spread, I_p is the peak current in a bunch, α is the momentum compaction factor, E is the beam energy, e is the electron charge, ν the betatron frequency, R the machine radius, $\Delta\nu$ the betatron frequency spread providing Landau damping, $(Z/n)_{eff}$ is the longitudinal coupling impedance of the bunch to the surrounding environment and

$$Z_{T_{eff}} = 2\,\frac{R}{d^2}\left(\frac{Z}{n}\right)_{eff} + Z_0\,\frac{R}{\gamma^2}\left(\frac{1}{a^2} - \frac{1}{d^2}\right) \qquad (3)$$

γ being the beam particle energy in rest energy units, a the beam radius, d the vacuum tank radius and Z_0 the vacuum impedance. The last term in (3) describes the space charge effects and can be neglected for our 20 TeV ring.

For a beam having B bunches, each of length L we introduce the total longitudinal emittance

$$\mathcal{E}_{\parallel} = B\ \Delta E\ L/C \tag{4}$$

the total average current

$$I_T = \frac{BL}{2\pi R} I_p \tag{5}$$

and a bunch factor

$$F_B = \frac{BL}{2\pi R} \ . \tag{6}$$

With the help of (4), (5) and (6) we can rewrite (1) and (2) as

$$e\ I_T \left(\frac{Z}{n}\right)_{\text{eff}} \leq \frac{\alpha c^2\ \mathcal{E}_{\parallel}}{(2\pi R)^2 F_B E} \tag{7}$$

$$e\ I_T \left(\frac{Z}{n}\right)_{\text{eff}} \leq \pi \left(\frac{d}{R}\right)^2 \nu\ E\ F_B\ \Delta\nu \ . \tag{8}$$

In (8) we neglected the space charge contribution to the transverse impedance.

One can see from (7), (8) that a small F_B is convenient to raise the longitudinal instability limit and a large F_B to raise the transverse limit. The optimum value is obtained when the two limits are equal or

$$F_B = F_B^* = \frac{c\ \mathcal{E}_{\parallel}}{(2\pi R)\ \frac{d}{R}\ E} \left(\frac{\alpha}{\pi\nu\ \Delta\nu}\right)^{1/2} \ . \tag{9}$$

For $F_B < F_B^*$ we are limited by the transverse instability and for $F_B > F_B^*$ by the longitudinal one. Of course if $F_B^* > 1$ the transverse effect is always the limiting one.

Using (9) we can write (7) and (8) as

$$e\ I_T \left(\frac{Z}{n}\right)_{\text{eff}} \leq \frac{c\ \mathcal{E}_{\parallel}\,(d/R)}{2\pi R} (\pi\alpha\nu\Delta\nu)^{1/2} \frac{F_B^*}{F_B} \tag{10}$$

$$e\ I_T \left(\frac{Z}{n}\right)_{\text{eff}} \leq \frac{c\ \mathcal{E}_{\parallel}\,(d/R)}{2\pi R} (\pi\alpha\nu\Delta\nu)^{1/2} \frac{F_B}{F_B^*} \ . \tag{11}$$

3. APPLICATION TO THE 20 TeV RING

We want to evaluate (10), (11) for the 20 TeV ring. Since $\alpha \approx 1/\nu^2$ there is only a weak dependence on ν and we will only evaluate (10), (11) for the case of the "weak focusing ring". We use the following values:

$$2\pi R = 5.7 \times 10^4 \text{ m}$$
$$\nu = 60$$
$$\alpha = 3.4 \times 10^{-4}$$
$$\mathcal{E}_{\parallel} = 5 \times 10^3 \text{ eV s}$$
$$I_T = 0.84 \text{ A}$$
$$R/d = 2.27 \times 10^5$$
$$\Delta\nu = 0.01$$

where the current corresponds to a total of 10^{15} circulating protons. We obtain:

$$F_B^* = 0.08 \qquad \text{at } E = 1 \text{ TeV} \tag{12}$$

and

$$F_B^* = 4 \times 10^{-3} \qquad \text{at } E = 20 \text{ TeV} . \tag{13}$$

Equations (10), (11) become

$$e\, I_T \left(\frac{Z}{n}\right)_{\text{eff}} \leq 2.93 \begin{cases} F_B^*/F_B \\ F_B^*/F_B \end{cases} \text{eV} . \tag{14}$$

The small values of F_B^* mean that we are mainly limited by the longitudinal instability. In fact for a coasting beam at 1 TeV and for the assumed $\mathcal{E}_{\parallel}$ the energy spread is very small $\Delta E/E \approx 2.6 \times 10^{-5}$. For $F_B = F_B^* = 0.08$ the energy spread is increased to 3.3×10^{-4}.

For a total circulating current of 0.84 A we have from (14) that the effective longitudinal coupling impedance must satisfy the condition

$$\left(\frac{Z}{n}\right)_{\text{eff}} \leq 3.5\ \Omega . \tag{15}$$

This condition need only be satisfied in the frequency range $\omega \geq c/d$ where feedback cannot be used. An impedance of the order of a few ohm has been obtained in the electron-positron storage ring, PETRA.

4. SYNCHROTRON RADIATION EFFECTS

The synchrotron radiation energy loss per turn U_0, can be obtained from[2)]

$$U_0 = \frac{4}{3}\pi \frac{r_0}{\rho} \gamma^4 \ (mc^2) \tag{16}$$

where ρ is the bending radius, r_0 the classical proton radius and mc^2 the proton rest energy. For an energy of 20 TeV, $\gamma = 2.13 \times 10^4$, $\rho = 6.68 \times 10^3$ m we have

$$U_0 = 1.82 \times 10^5 \text{ eV} . \tag{17}$$

For a total current $I_T = 0.84$ A the radiated power is

$$W = 152 \text{ kW} . \tag{18}$$

To evaluate the damping times we assume that the radial and vertical betatron damping times are equal to twice the synchrotron damping time, τ_s. This is then given by[2)]

$$\tau_s = \frac{2\pi R}{c}\frac{E}{U_0} = 2.09 \times 10^4 \text{ s} = 5.8 \text{ hrs} . \tag{19}$$

The equilibrium energy spread produced by radiation damping and quantum fluctuations is given by

$$\frac{\Delta E}{E} = \left\{\frac{55}{96\sqrt{3}}\frac{\tau_s c}{2\pi R}\frac{R}{\rho}\frac{U_0}{E}\frac{E_c}{E}\right\}^{1/2} \tag{20}$$

where

$$E_c = \frac{3}{2}\hbar c \frac{\gamma^3}{\rho} . \tag{21}$$

At 20 TeV we obtain

$$E_c \simeq 415 \text{ eV} \tag{22}$$

$$\frac{\Delta E}{E} = 3.05 \times 10^{-6} . \tag{23}$$

The energy spread is smaller than the value determined by the longitudinal emittance $\mathcal{E}_\parallel$, which is 2.6×10^{-5} for a coasting beam and 6.5×10^{-3} for a beam with a bunching factor $F_B = 4 \times 10^{-3}$ and at 20 TeV. Hence, according to our present understanding of the microwave instability, as soon as the energy spread starts to decrease the bunch should become unstable and the energy spread should remain at the value corresponding to the threshold of the instability.

The radial betatron emittance as determined by radiation alone is given by[2)]

$$E_x = \pi\gamma \frac{2R}{\nu^3}\left(\frac{\Delta E}{E}\right)^2 = 5.2 \times 10^{-8} \text{ m rad}$$

and is again much smaller than the emittance at injection, $E_x \sim 2\pi \times 10^{-5}$ m rad. The reduction in transverse emittance produced by radiation might become important in a p-p̄ configuration, where the reduction in beam transverse size would increase the beam-beam tune shift, $\Delta\nu$, and the luminosity by a factor equal to the emittance ratio or about 1200. Again we expect that the beam-beam interaction will start to blow up the beams long before such a large increase is reached. However we can assume that the p-p̄ system will adjust itself to the optimum value of $\Delta\nu$. Also the radiation damping might be helpful in counteracting slow diffusion processes.

5. CONCLUSIONS

The results obtained in Section 3 show that coherent instabilities should not prevent us from reaching the design intensity of 10^{15} circulating protons. However to reach this goal one will have to use feedback to control the low frequency instabilities and to reduce the longitudinal coupling impedance to a few ohm. Also when these conditions are satisfied one requires a very small bunching factor to avoid a blow up of the longitudinal phase space. Of course, a much more detailed analysis will be needed when the ring is designed.

Synchrotron radiation effects are not negligible and can influence the beam properties in the storage configuration after a time of the order of ten hours.

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REFERENCES

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