THE NUCLEON STRUCTURE FUNCTIONS AS DERIVED FROM LEPTON-NUCLEON SCATTERING

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Contents

1. Introduction

My duties are defined by the title of this talk. I have benefitted from discussions with many people about their data and I happily thank them for their help. I have done things with the data that they may not approve of and what I shall show will not represent the official experimenters' view. I apologize in advance if I misrepresent any data. There is one other point: I am a member of the European Muon Collaboration (EMC) and our spokesman, Hans Steir, has already presented our data. However, anything that I say about that data is my own responsibility and does not represent the official collaboration position. Let me acknowledge my indebtedness to my EM collaborators - in particular to the younger members who are actually doing the real hard work of analyzing the data.

STRUCTURE FUNCTION MEASUREMENTS

Group	Beam	Target
Electromagnetic		
SLAC	e^-	H
MIT/SLAC	"	H,D
CHIO	μ^+	H,D
BFP	"	Fe
MSU-F	"	"
BDCMS	"	C
EMC	"	H,Fe
Weak		
BEBC/GGM	$\nu,\bar{\nu}$	Freon H/Ne
CDHS	"	Fe

Fig.1 List of experiments, with their acronyms or initials, which are discussed in this review.

Figure 1 shows the sources of experimental data: the left hand column contains the initials and acronyms for various groups, institutions, or apparatuses. Let me run through these: SLAC[1] and MIT/SLAC[2] groups are so well known they need no introduction. CHIO stands for the Chicago, Harvard, Illinois, Oxford collaboration which studied muon scattering at Fermilab[3]. The European Muon Collaboration (EMC) are studying muon scattering with a large spectrometer at CERN, as described at this symposium by Steir[4]. The Bologna, Dubna, CERN, Munich, Saclay (BDCMS) groups are studying muon scattering with a large integrated target plus iron toroid at CERN as described by Benvenuti[9]. The Michigan State-Fermilab (MSU-F) group studied muon scattering in an instrumented iron target and a toroidal iron spectrometer[5]. The Berkeley-Fermilab-Princeton (BFP) group studied muon scattering in an integrated iron target and magnet, as described by Strovink[6].

BEBC/GGM stands for the group who have used the Big European Bubble Chamber and the Gargamelle bubble chamber for the study of neutrino scattering[7]. The CERN, Dortmund, Heidelburg, Saclay (CDHS) group have studied neutrino scattering in an iron toroid[8].

I have divided the experiments very roughly into two groups: "high-Q^2" and "low-Q^2". The latter group for reasons of limited luminosity or of limited energy are effectively restricted to a maximum Q^2 of about 30 GeV^2. The "high-Q^2" experiments have higher ambitions and although they may have sensitivity well below 30 GeV^2, they are also aimed at Q^2 values as high as 200 GeV^2.

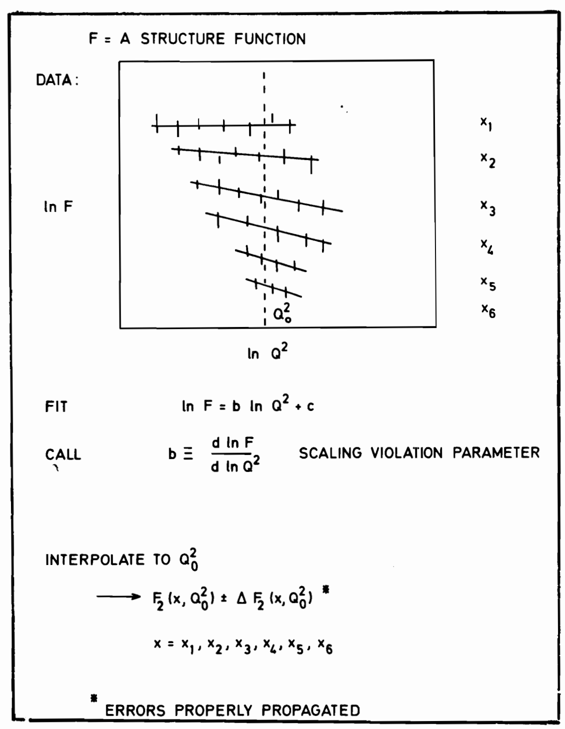


Fig.2 Diagram showing the method of interpolation and defining the scaling violation parameter.

Clearly there is interest in the consistency of experiments and the extraction of essential data on scaling violations. I have wanted to avoid comparison by the method of laying transparency upon transparency upon transparency; I find that leads to a density of information that frequently becomes too great to be useful. Instead I shall extract comparison numbers in a way I shall now describe.

Data on a structure function, F, from any one experiment are normally given over a range of Q^2 values for a set of x ($\equiv Q^2/2M\nu$) values. I fit a line

$$b \ \ell n Q^2 + c$$

to $\ell n F$. See figure 2. I can interpolate using this fit to obtain $F(x, Q_o^2)$ where Q_o^2 can be a common Q^2 or one which allows comparison with an overlapping data set from another experiment. In this way I can compare data even though it is not quoted at the same Q^2. I cannot compare at the same x unless I build in another prejudice, this time about how $F(x, Q^2)$ varies with x. I do not do this. Fortunately many experiments are giving data at the center of the x bins used by CDHS. I hope that this will continue and in addition that we shall see experimenters giving results at the same Q^2 values.

There may be no physics justification for using a fit which has $d \ \ell n F / d \ \ell n Q^2$ constant. However, this is a convenient parameter for presenting scaling violations and, since the fits are in almost all cases very good, it allows reliable interpolation. Strovink used a similar interpolation assuming $\ell n F$ is linearly dependent on $\ell n \ \ell n(Q^2/\Lambda^2)$. This may be better but it is Λ dependent and since I had done much of the fitting before coming to Fermilab, I decided to continue using the one I have described. If the fit has $\chi^2 >$ NDF, I increase errors by $\sqrt{\chi^2/\mathrm{NDF}}$.

2. Hydrogen and deuterium results

Let us now look at the "low-Q^2" electromagnetic data. This is familiar data, but we can see how the fitting method works and allows comparison. In the future I have

$$b = \frac{d \ \ell n F}{d \ \ell n Q^2}$$

with superscripts γ, ν for charged lepton and neutrino beam, respectively, and p, d, for proton or deuterium targets. No target superscript means a complex nuclear target (e.g. C or Fe).

Figure 3 shows this scaling violation parameter $b^{\gamma p}$ for $F_2^{\gamma p}$ (charged lepton scattering) as a function of the scaling variable x.

Now some comments:

1) There are three sets of data: from SLAC, MIT/SLAC, and CHIO.

2) Only data with $Q^2 > 2$ GeV^2 are used.

3) SLAC and MIT/SLAC have properly separated F_2 and xF_1. This $b^{\gamma p}$ is for F_2.

4) CHIO have measured the average $R^{\gamma p}$ ($\equiv \sigma_L/\sigma_T$) in their data having $x < 0.1$ to be 0.52 ± 0.35, and they use this value throughout. Decreasing $R^{\gamma p}$ by one standard deviation from

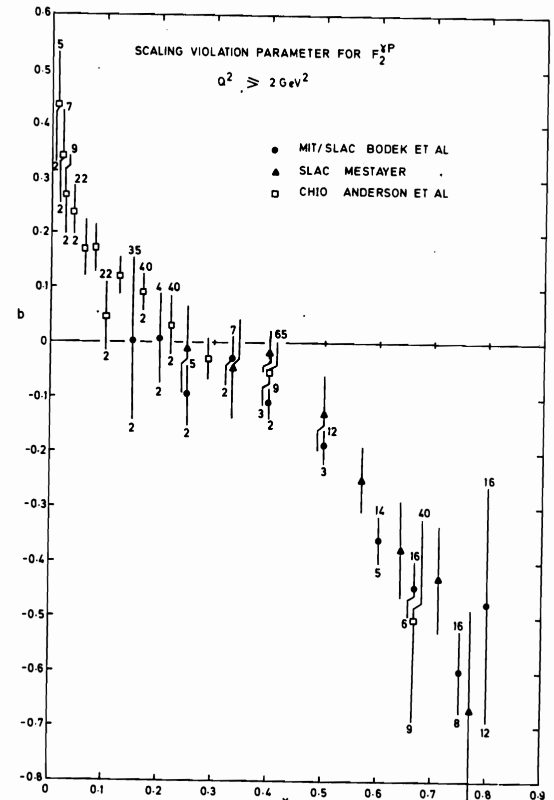


Fig.3 The scaling violation parameter for the proton structure function $F_2^{\gamma p}$ as a function of x for the "low-Q^2" data. The range of Q^2 is shown for some points by numbers at the bottom and top of their error bar.

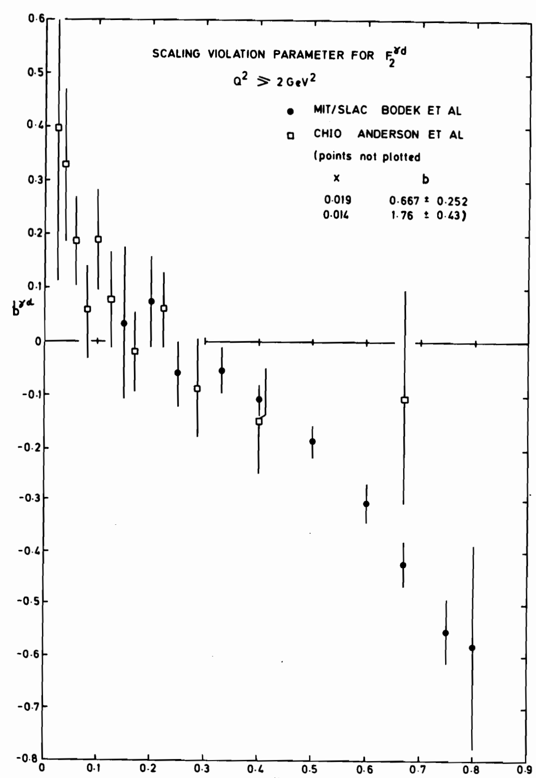


Fig.4 The scaling violation parameter for the deuteron structure function as a function of x for the "low-Q^2" data.

0.52 causes a small reduction ($\stackrel{<}{\sim}$ 0.1) in the values of $b^{\gamma p}$ for the very smallest x points. This reduction disappears as x increases and is negligible for x = 0.1 and beyond.

5) The Q^2 range from which these results are derived vary as x varies so that at x < 0.1 the average Q^2 might be 3 to 10 GeV^2, and at x > 0.6 it is 12 to 25 GeV^2. Therefore if $b^{\gamma p}$ varies with Q^2 in addition to its clear variation with x, that is mixed in there.

6) Agreement between different experiments is good.

7) The scaling violation as predicted by QCD exists.

Figure 4 shows the same scaling violation parameter for the deuteron, $b^{\gamma d}$. Some comments:

1) Only two data sets: MIT/SLAC and CHIO.

2) The comments 2) through 7) made for the proton data apply equally to the deuteron data.

3) A comparison of figures 3 and 4 indicates that these scaling violations for the deuteron are the same as for the proton, within errors.

The next step is to compare the actual values of $F_2^{\gamma p}(x, Q^2=3)$. This is done in figure 5. Remember these will be values interpolated from the fit.

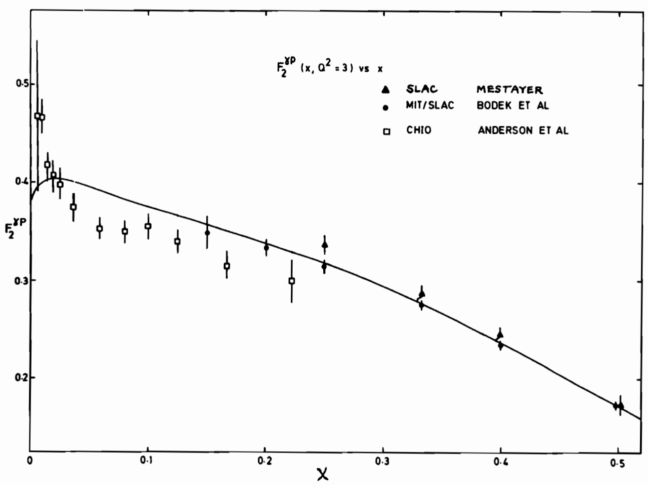


Fig.5 Values of the proton structure function, $F_2^{\gamma p}$, as a function of x at the interpolated value of Q^2 = 3 GeV^2. The solid line is the Buras and Gaemers fit[10].

Some comments:

1) Note that both scales are linear and that the F_2 axis has a suppressed zero.

2) The solid line is the fit of Buras and Gaemers[10] using the parameters given in their paper: Λ = 0.3 GeV and Q_o^2= 1.8 GeV^2.

3) In the overlap between CHIO and MIT/SLAC there is a discrepancy of about 7%.

4) At x = 0.25 the SLAC point is higher than the MIT/SLAC point.

5) CHIO are in poor agreement with Buras and Gaemers.

Figure 6 shows $F_2^{\gamma p}$ (x, Q^2=10). Note:

1) The vertical scale is logarithmic

2) The CHIO to MIT/SLAC overlap is limited but at x = 0.4 there is a 25% discrepancy.

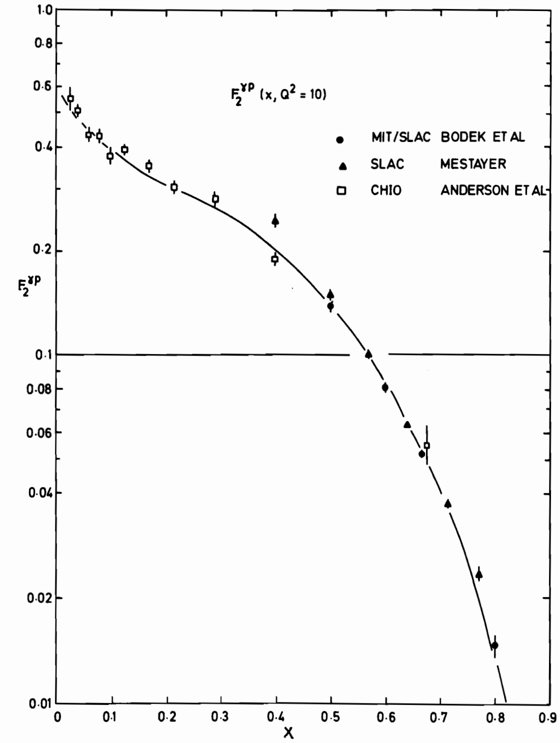


Fig.6 Values of the proton structure function, $F_2^{\gamma p}$, as a function of x at the interpolated value of Q^2 = 10 GeV^2.

3) If the MIT/SLAC data were extrapolated to Q^2 = 10 (a distance ΔQ^2=1) they would have a point

$F_2^{\gamma p}(x=0.4, Q^2=10)=0.206\pm0.006$

So in that region there is the discrepancy

SLAC > MIT/SLAC > CHIO

The numbers are:

0.244±0.009, 0.206±0.006, 0.188±0.008

respectively.

4) Note that the Buras and Gaemers' curve goes through the CHIO data rather well.

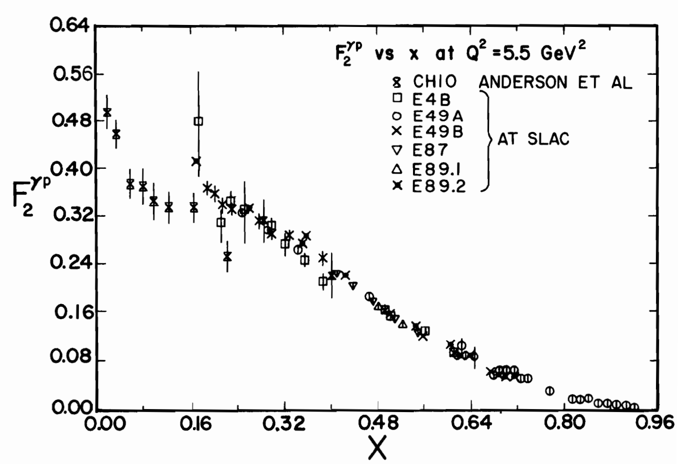


Fig.7 Values of the proton structure function $F_2^{\gamma p}$ at Q^2 = 5.5 GeV^2 versus x, derived from cross-section measurements in electron scattering made at SLAC (R = 0.21) and from muon scattering experiments at Fermilab.

The differences between the CHIO muon scattering data and the electron scattering data can be seen again in figure 7, which shows $F_2^{\gamma p}(x, Q^2=5.5)$. Here we are closer to the raw data: in this figure the electron scattering results are derived from cross-sections measured in all the experiments at SLAC, using R=0.21. Data from a narrow Q^2 bin has been empirically bin centered. It is not clear that the muon scattering data are joining smoothly to the electron scattering data.

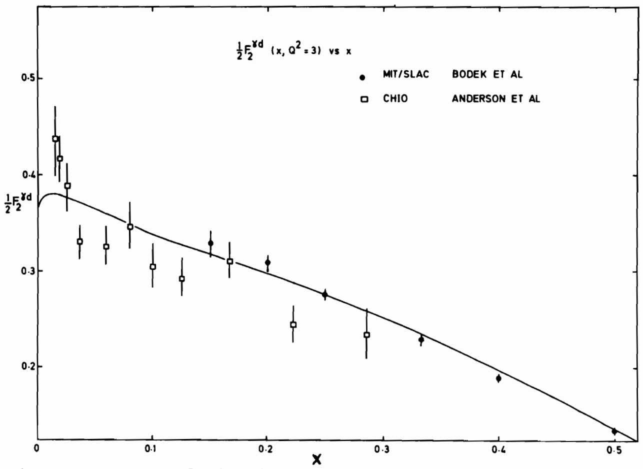


Fig.8 Values of the deuteron structure function per nucleon as a function of x at the interpolated value of Q^2 = 3 GeV^2.

Figure 8 shows the deuterium data interpolated to Q^2=3 GeV^2. Again we see the slight discrepancy between the MIT/SLAC and the CHIO data. Figure 9 shows the data interpolated to Q^2=10 GeV^2. Here the data sets join together smoothly.

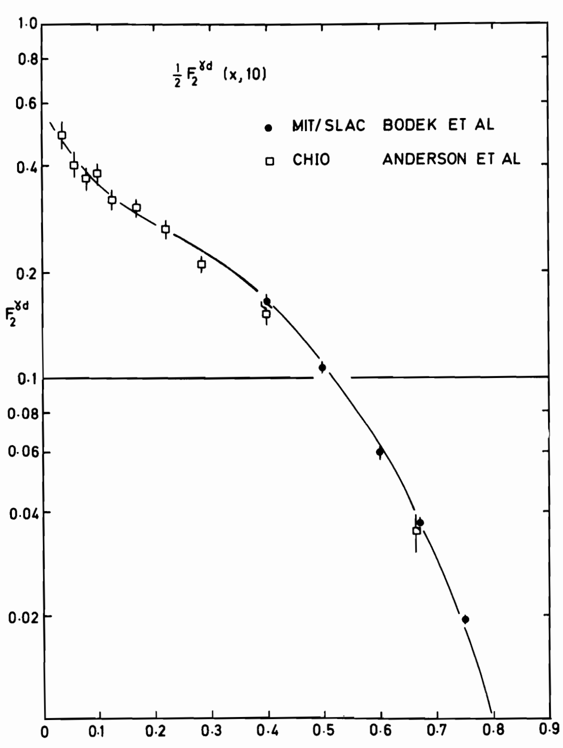


Fig.9 Values of the deuteron structure function per nucleon as a function of x at the interpolated value of Q^2 = 10 GeV^2.

I do not claim that the method of comparison is revealing genuine discrepancies but in view of Strovink's claim of 18% discrepancy between electron scattering data and other lepton scattering data, the data sets should be re-examined.

These comparisons are in some sense unfair to the electron scattering data. The CHIO data are extracted assuming a fixed value of R. The electron scattering data is separated into F_2 and xF_1 so that the tables of these quantities in Bodek et al[2] and Mestayer's thesis[1] cover a kinematic range well inside that in which measurements have been done. If that measured data were to be processed with a fixed R, then electron F_2 data would be available over a larger kinematic range.

Changing the value of R in the CHIO data has a negligible effect in the overlap regions.

I want to turn now to a new hydrogen experiment done by the EMC. The data, which are preliminary, were shown by Steir. I show them again (Figure 10).

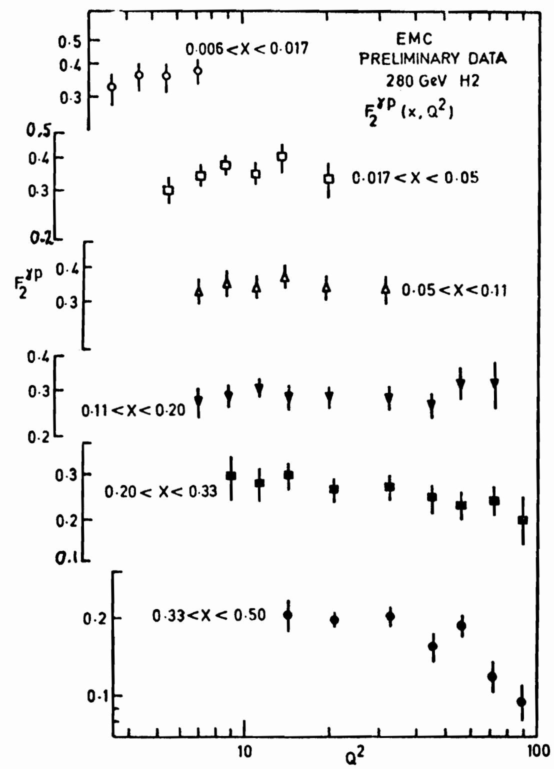


Fig.10 $F_2^{\gamma p}(x,Q^2)$ from new measurements by the European Muon Collaboration[4].

In general they extend to higher Q^2 values than the CHIO data. I have fitted these preliminary data in the usual way to find the scaling violation parameter $b^{\gamma p}$. Looking at the data we see that the scaling violations are of the usual kind. There is a danger that two points in the highest x bin and at the highest Q^2 will exaggerate the slope.

Figure 11 shows the $b^{\gamma p}$ values I have calculated. Comparing this with the previous $b^{\gamma p}$ results (Figure 3) we note that the trend is still of the same magnitude. There is a hint that near x = 0.4, the scaling violation is greater than in the previous data.

A word of warning. The EM Collaboration do not claim that their systematic errors are yet under

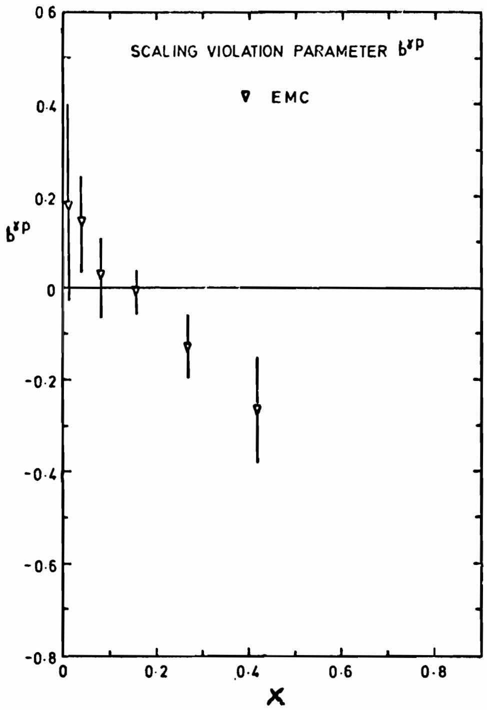


Fig.11 The scaling violation parameter b'' from new measurements by the EMC.

control. If, as is not unlikely, these errors contain some Q^2 dependence, the slopes $b^{\gamma p}$ will change. However, we should look forward to the official values of $b^{\gamma p}$ and note that for hydrogen the EMC data are hinting that, compared to the earlier data which do not extend to as high a Q^2, the scaling violations above $x \simeq 0.2$ are slightly greater.

3. High-Q^2 Heavy Target Results

There are four experiments I want to discuss involving μ^+ scattering in iron or carbon targets. See Figure 1. In addition I shall include some of the BEBC and CDHS neutrino scattering results in comparisons. Three muon experiments have first data and the MSU-F group have new data. That latter data on $F_2^{\gamma}(x, Q^2)$ is shown in Figure 12. This data causes me trouble if I want to fit straight lines on

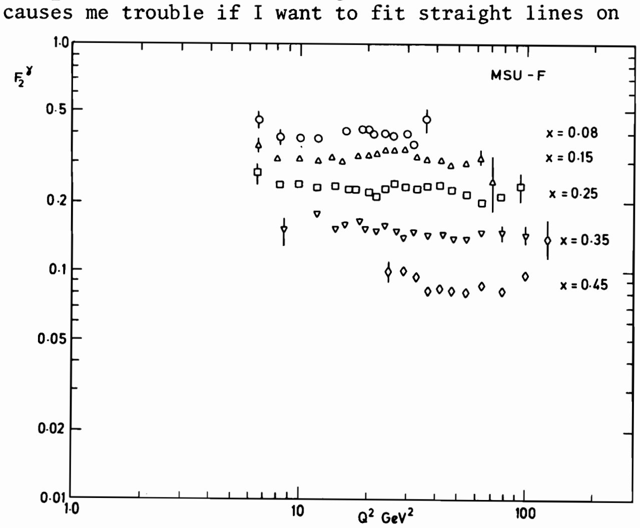


Fig.12 New measurements of F_2^{γ} by the MSU-F group.

a ℓnF_2 vs. ℓnQ^2 plot. The reason becomes apparent if the same data is plotted on a scale linear in F_2, Figure 13. This figure shows the results for three out of the five x bins. Note the suppressed zero. The feature of this data is a rise and fall in $F_2^{\gamma}(x, Q^2)$ as a function of x, well outside the statistical errors, in all these x bins. The MSU-F group claim there are no significant point-to-point systematic errors although there could be an overall

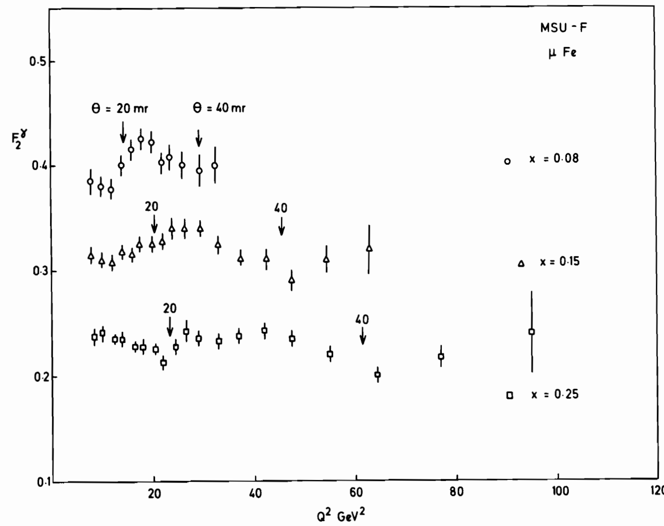


Fig.13 MSU-F group's measurements of F_2^{γ} at $x \lessapprox 0.25$. The laboratory scattering angle is θ.

normalization error. One feature of this data I would like to point out is the following: the acceptance for the scattered muon extends from about 11 mr to 100 mr in laboratory scattering angle and is smooth, as would be expected from a toroidal spectrometer. However, I have marked in Figure 13 the places where the laboratory scattering angle is 20 mr and 40 mr. You will notice that the data is increasing at 20 mr for these three x values and has a minimum near 40 mr for these three x values. The value of the recoil hadron mass is not the same at any one of these angles as x changes. It seems unlikely that there is a real physical effect of this angle-correlated kind which is independent of the apparatus.

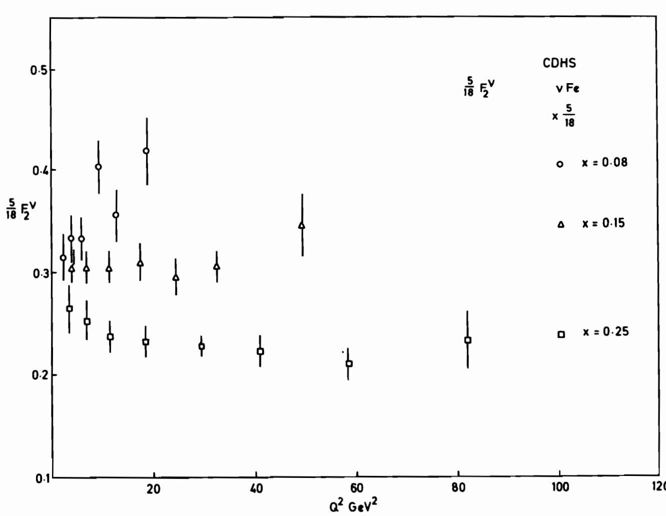


Fig.14 CDHS group's measurements of $\frac{5}{18}F_2^{\nu}$ at $x \lessapprox 0.25$.

To examine one step further I have plotted on the same scale results from four other experiments involving iron targets.

Figure 14 shows the CDHS results for $5/18\ F_2^{\nu}$ for these three bins of x. The lowest, x = 0.08, does not tell us anything but there is no support for a bump in F_2 from the two other bins.

Figure 15 shows the EMC results. In the two lowest x bins their results are well below MSU-F and give no support to the existence of the bump. The errors are too large in the EMC x = 0.25 bin to make any statement.

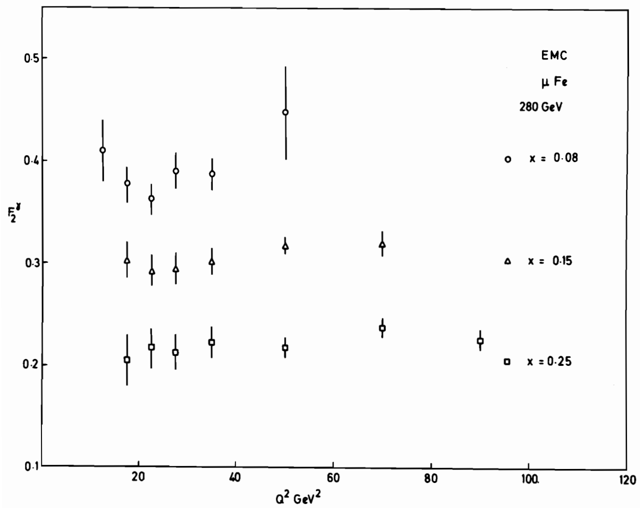


Fig.15 EMC measurements of F_2^{γ} at $x \leqslant 0.25$.

Figure 16 shows the BFP 209 GeV data. The bins with x = 0.08 and x = 0.15 show the BFP data clearly disagreeing with the increase in the MSU-F data. Figure 17 shows the BFP 90 GeV data. In this case, in the x = 0.15 bin, BFP, with large errors, follow MSU-F. In the bin x = 0.25, BFP goes below MSU-F with large errors.

The conclusions from this study are that:

1) There is no substantial support for the structure seen in the MSU-F data from other experiments.

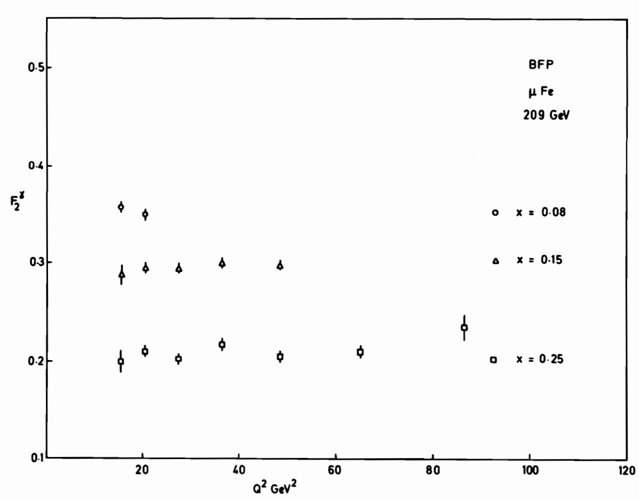


Fig.16 BFP group's measurement of F_2^{γ} at $x \leqslant 0.25$ for 209 GeV muons.

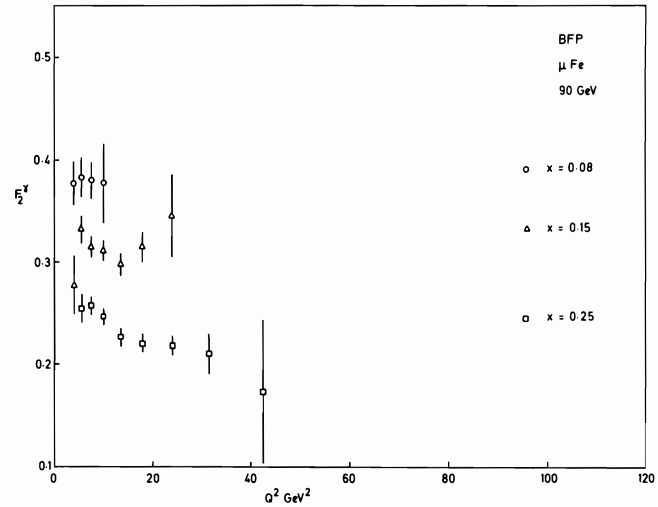


Fig.17 BFP group's measurements of F_2^{γ} at $x \leqslant 0.25$ for 90 GeV muons.

2) There could be systematic effect correlated with laboratory scattering angle.

3) I decided not to use the MSU-F data for x < 0.25 in any comparisons.

So, excluding the three low x MSU-F data, the question is "are these experiments consistent?"

I have looked for values of Q^2 at which I can compare data using my fits, as a function of x. Two experiments have data through all x from 0.08 to 0.65, EMC and CDHS, and I chose to normalize to

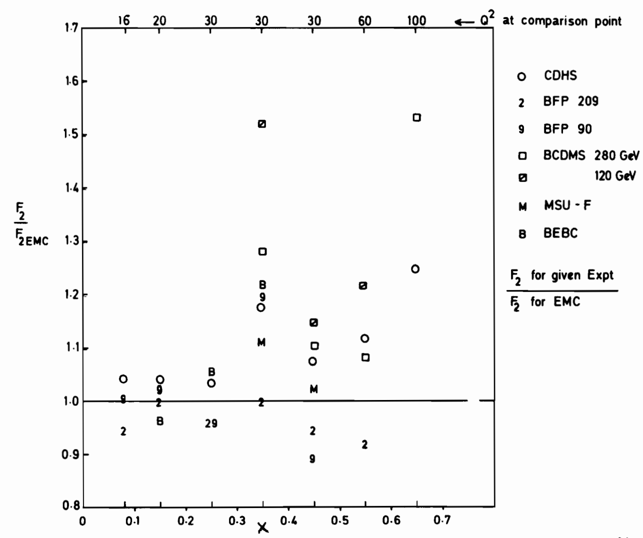


Fig.18 Normalisation of various measurements of F_2^{γ} and $\frac{5}{18}F_2^{\nu}$ in iron or carbon by various groups, relative to EMC measurements.

the former. Figure 18 shows the results. The horizontal scale is x, the vertical scale is $F_2/F_2(EMC)$. The values of Q^2 at which the comparison is made for each x are shown along the top.

Error bars have not been put on because:

1) They confuse the plot.

2) They overlap with 1.0 except in a few cases.

3) I am interested in systematic deviations in relative normalization.

The rough conclusions about relative values (and remember these are with respect to EMC) are:

CDHS	+5 to + 25%
BFP 90 GeV	0 to + 20%
209 GeV	-7 to 0%
MSU-F (two points)	2 and 16%
BEBC	-3 to 22%
BDCMS 120 GeV	15 to 52%
280 GeV	10 to 50%

The last two are in some cases beyond the errors and seem to indicate a systematic effort leading to high results for BDCMS even if EMC is low. CDHS comes out consistently high, BFP (90) low.

Conclusions are that:

1) there exist relative normalization errors, a fact which no-one disputes.

2) one experiment, BDCMS, comes out consistently high, 10 to 50%, and outside errors.

Strovink remarked on an 18% discrepancy between electron-deuteron scattering results from the SLAC spectrometers and the BFP and CDHS results. (Notice that this is a statement about comparing results from high resolution vacuum-path spectrometers to those from an iron cored spectrometer with a 7 to 10% resolution on momentum.) To examine this we first look at Figure 9 and note that the Buras and Gaemers' result is a good representation of the MIT/SLAC data for $\frac{1}{2}F_2^{\gamma d}(x, Q^2{=}10)$. Now look at Figure 19. This is a plot of $5/18\ F_2^{\nu}(x, Q^2{=}10)$ against x from the CDHS and BEBC groups and of $F_2^{\gamma}(x, Q^2{=}10)$ for the BFP 90 GeV results. The Buras and Gaemers' curve for $\frac{5}{18}F_2^{\nu}$, which is on this figure, is indistinguishable from that for $\frac{1}{2}F_2^{\gamma d}$ on Figure 9 for $x > 0.2$ and can serve as a relative normalizer. We note that the BFP(90) and CDHS results are consistent and for $x \gtrsim 0.35$ are together below the MIT/SLAC results by around 10%. I think this may be the same discrepancy.

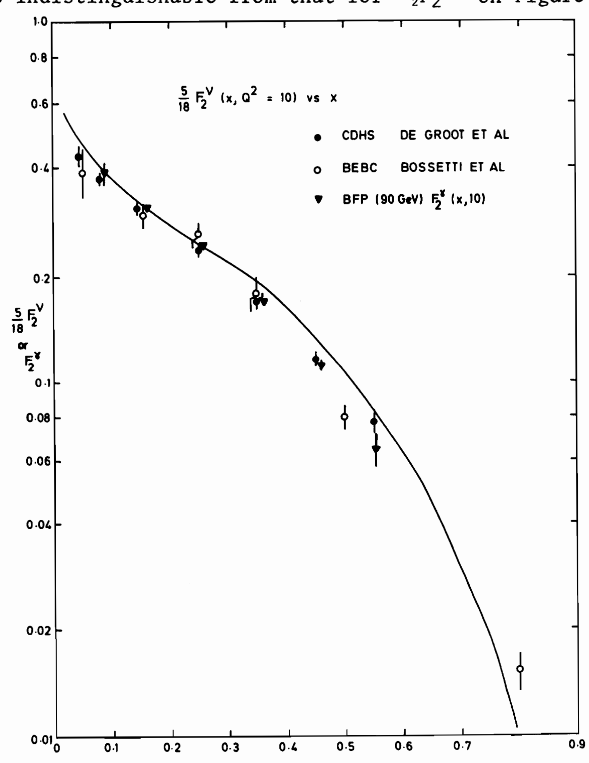


Fig.19 CDHS and BEBC groups' measurements of $\frac{5}{18}F_2^{\nu}(x, Q^2 = 10)$ and BFP's of $F_2^{\gamma}(x, Q^2 = 10)$.

A more systematic investigation of the possibility of discrepancies would be very interesting.

The next question is: What about the scaling violations in these high-Q^2 complex nuclear target experiments? Once again I use

$$b^{\gamma} = \frac{d\ \ell n F_2^{\gamma}}{d\ \ell n Q^2}$$

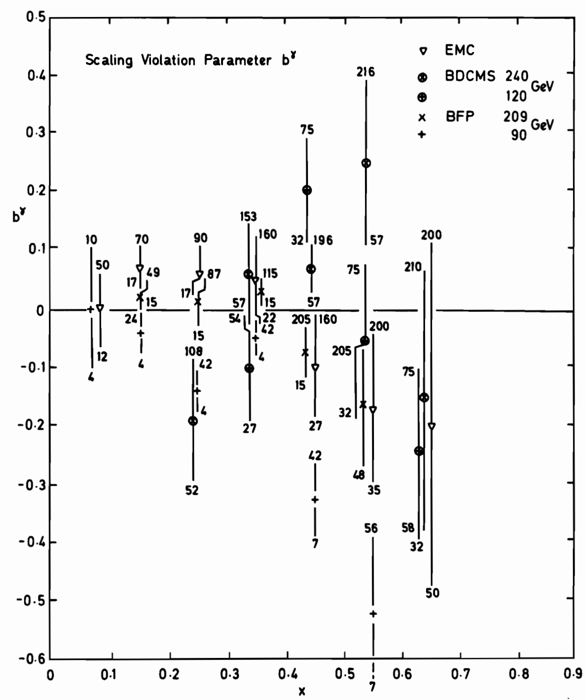


Fig.20 Scaling violation parameter b^{γ} from new measurements of muon scattering in iron or carbon. The Q^2 range is shown by numbers at the bottom and top of each error bar.

Figure 20 shows the results for these experiments. Now this figure is unfair on two counts:

1) A b^{γ} of 0.1 represents a 10% change of F_2 for a change of Q^2 by a factor of 2.72. Therefore systematic errors, particularly those depending on Q^2, even weakly, can radically alter the value of b^{γ}. We know that these preliminary results do not claim the absence of systematic errors.

2) The ranges of Q^2 in the various experiments are not the same and, since we expect to have b^{γ} varying with Q^2, some care must be exercised in comparing b^{γ} values from different experiments. The Q^2 ranges are shown on the figure.

Neglecting (unjustifiably) the question of systematics, what conclusions can we draw? If the density of points draws the eye to some kind of average, then it looks as if b^{γ} is smaller in magnitude than at lower Q^2, Figure 4.

It is interesting at this point to look at scaling violations in the neutrino experiments.

Figure 21 shows $b_2^\nu \equiv d\ \ell n F_2^\nu / d\ \ell n Q^2$ and Figure 22 shows $b_3^\nu \equiv d\ \ell n x F_3^\nu / d\ \ell n Q^2$ both from the CDHS and BEBC groups. Note that the Q^2 range spanned by the data is very large:

BEBC: $2.5 < Q^2 < 60$ GeV^2

CDHS: $1 < Q^2 < 180$ GeV^2

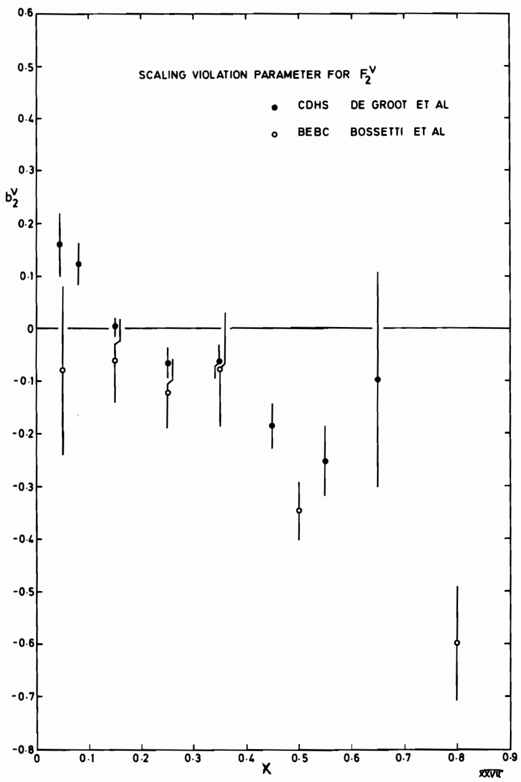


Fig.21 Scaling violation parameter b_2^ν for F_2^ν.

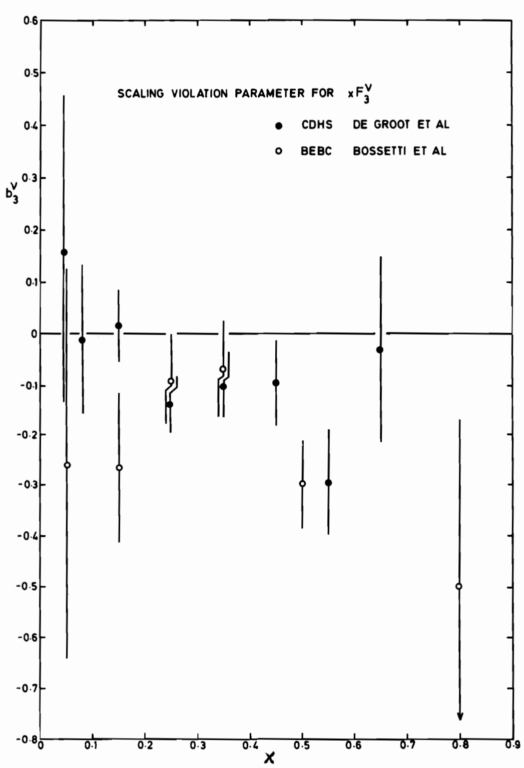


Fig.22 Scaling violation parameter b_3^ν for xF_3^ν.

In spite of this, the fits made to obtain b^ν are good in all but one case. A comparison with Figure 4 indicates that the scaling violations in the neutrino data are the same as for the electromagnetic deuteron structure functions. However, the errors are large and there could be hint from the CDHS data at very high Q^2 that b^ν is less than at lower Q^2. The points concerned are at $x \simeq 0.65$ in Figures 21 and 22 which have $30 < Q^2 < 150$ GeV^2.

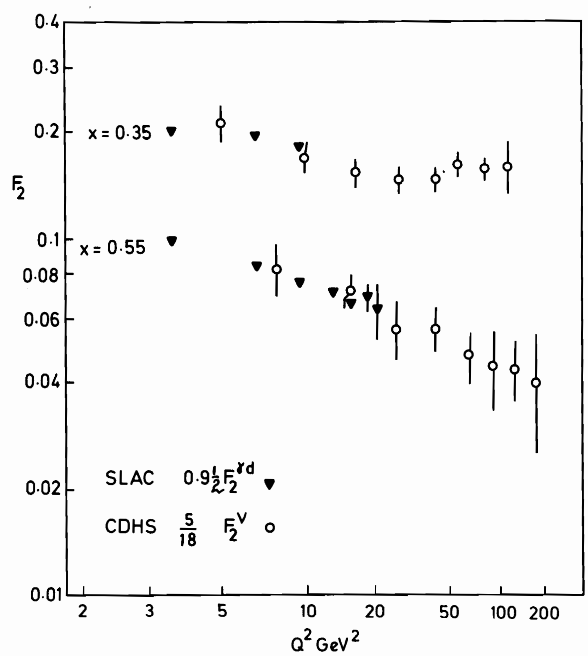
A very tentative conclusion about the high-Q^2 complex nuclear target data is that the scaling violations expressed as $d\ \ell n F_2 / d\ \ell n Q^2$ may be less than in the lower Q^2 data (for $x \gtrsim 0.3$).

4. Bridging from Low to High Q^2

What I would like to do is to bridge from the low Q^2 to the high Q^2 data. Strovink in his talk suggested that QCD as exhibited in the Buras and Gaemers' parametrisation, would have

$$\ell n\ F \simeq b\ \ell n\ \ell n(Q^2/\Lambda^2) + c, \quad \Lambda = 0.5\ \text{GeV}.$$

Figure 23 shows some rough evidence that this could be true in the data. The plot is linear in $\ell n F_2$ and in $\ell n\ \ell n(Q^2/\Lambda^2)$. I have taken points from one of Strovink's transparencies for x = 0.35 and x = 0.55 of SLAC $\frac{1}{2}F_2^{\gamma d}$ data and of CDHS $^5/_{18}\ F_2^\nu$ data. I have arbitrarily multiplied the SLAC data by 0.9. We see that it is perhaps possible that $\ell n F \propto \ell n \ell n(Q^2/\Lambda^2)$ over a large Q^2 range.



LINEAR IN LOG LOG (Q^2/Λ^2), $\Lambda = 0.5$

Fig.23 $\frac{1}{2}F_2^{\gamma d}$ for deuterium (SLAC) and $\frac{5}{18}F_2^\nu$ (CDHS) for iron at x = 0.35 and x = 0.55 from Q^2= 3 to 200 GeV^2. The abscissa is linear in $\ln\ln(Q^2/\Lambda^2)$ with Λ = 0.5 GeV. The SLAC data have been arbitrarily multiplied by 0.9.

I have not had time since Strovink's talk to refit all the data with this form and because I have used $b = d\ \ell n F / d\ \ell n Q^2$ I will show what Buras and Gaemers predict for this parameter. I have tables for $F_2^{\gamma d}$ for Λ = 0.3 GeV and Q_o^2 = 1.8 GeV^2.

I extract d $\ln F_2^{\gamma d}$/d $\ln Q^2$ at Q^2 = 10 and 100 GeV^2 and plot against x. This is shown in Figure 24 and we see that Buras and Gaemers' violations expressed by this parameter are less at higher Q^2.

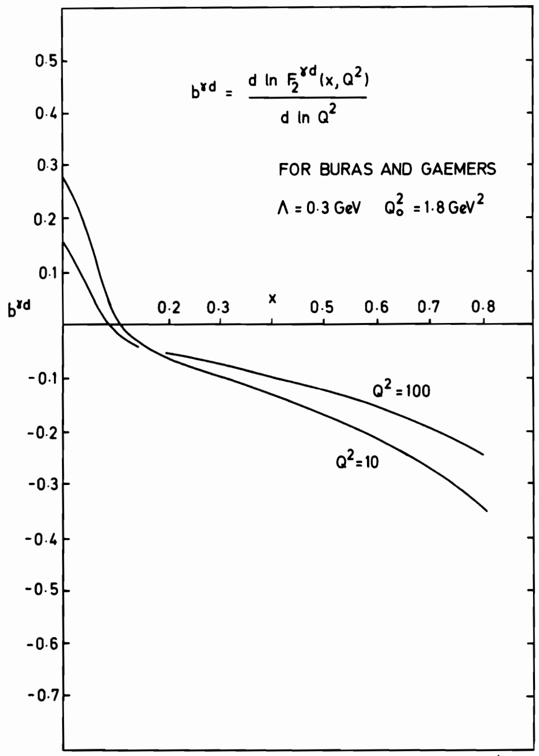


Fig.24 Value of $b^{\gamma d}$ given by Buras and Gaemers' fit[10].

The conclusions seem to be

1) The scaling violations seen in the new high-Q^2 data may be of the type having d $\ln F$/d $\ln\ln(Q^2/\Lambda^2)$ = constant.

2) This moderation of the violations could be what is expected from QCD.

Figure 25 shows the Q^2 ranges of data on F_2 (iron and carbon) published or available at this conference from some of the experiments I have mentioned. It does not include data that may have been obtained but have not yet been analysed to give F_2. I have not included BEBC/GGM data which extends to

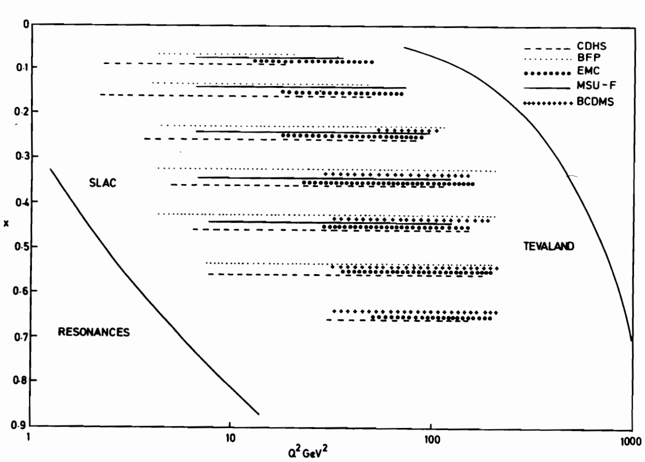


Fig.25 Span in x and Q^2 of recent heavy target measurements of F_2.

low-Q^2. Data taken at SLAC covers the low-Q^2 region. The main point to emphasise is that we could do with more data in the mid-Q^2 ranges 10 → 50 GeV^2 in the high x region, and in very low x region. We also look forward to being able to extend the coverage to Q^2 approaching 1000 GeV^2 with the Tevatron.

5. Moments

The moments of the structure functions are very important: leading order QCD can make testable predictions about the evolution of the moments with Q^2. Figure 26 will remind people about the use of moments. Unless otherwise mentioned I will take it that it is the Nachtmann moments that are used. I will discuss the moments of F_2, but the non-singlet moments are particularly interesting as they are expected to develop with Q^2 in a simple way which can be tested using any of the three equivalent formulae (Figure 26). And, of course, what we hope to determine is Λ, which is related to the QCD coupling constant.

MOMENTS

$$M(n,Q^2) = \int_0^1 x^{n-2}\, F(x,Q^2)\,dx \qquad \text{CORNWALL - NORTON}$$

$$M(n,Q^2) = \int_0^{\xi_{max}} \xi^{n-2} \ldots\ldots F(\xi,Q^2)\,d\xi \qquad \text{NACHTMANN}$$

(a function of ξ, x, n, M, Q^2)

$$\xi = 2x/(1+\sqrt{1+4M^2x^2/Q^2})$$

$F = F_2$

$F = xF_3^{\nu}$ NON - SINGLET M_{NS}^{ν}

$F = F_2^{\gamma p} - F_2^{\gamma n}$ NON - SINGLET M_{NS}^{γ}

QCD LEADING ORDER

I) $$M_{NS}(n,Q^2) = M_{NS}(n,Q_0^2)\left[\frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}}\right]^{-d_{NS}^{(n)}}$$

OR

II) $$\left[M_{NS}(n,Q^2)\right]^{-\frac{1}{d_{NS}^{(n)}}} \propto \ln Q^2 - \ln \Lambda^2$$

OR

III) $$\frac{d \ln M_{NS}(n,Q^2)}{d \ln M_{NS}(m,Q^2)} = \frac{d_{NS}^{(n)}}{d_{NS}^{(m)}}$$

Fig.26 Formulae relevant to the study of structure function moments.

First the n = 1 Nachtmann moment of xF_3^{ν}. The Gross - Llewellyn-Smith sum rule says it should be 3 with a QCD correction. Figure 27 shows a re-analysis of the BEBC/GGM data of the moment against Q^2, including an elastic and a Δ production contribution[16]. A comparison with QCD predictions would lead the naive person to say the data suggests Λ is <0.2 GeV. However, the QCD calculations are not applicable where the elastics contribute and no conclusion can be drawn. However it is a nice result. The CDHS group have a similar result[8].

The n = 2 moments of the proton and deuteron F_2^{γ} structure functions are shown in Figure 28. This moment is the energy-momentum sum and as $Q^2 \to \infty$ the value for the neutron and that for the proton

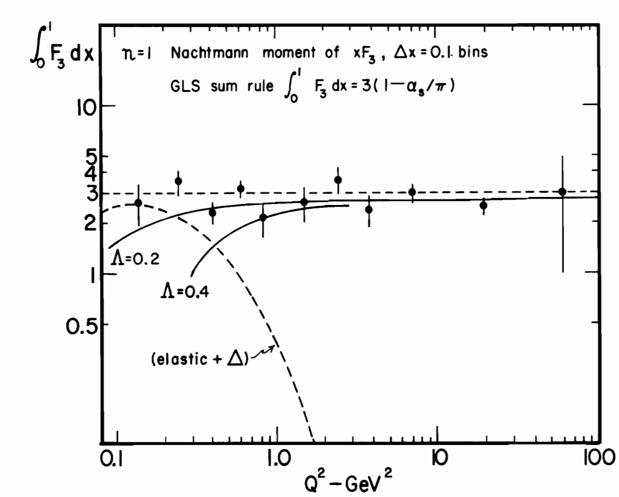


Fig.27 Values of n = 1 moment of xF_3^{ν} (Gross-Llewellyn-Smith sum rule).

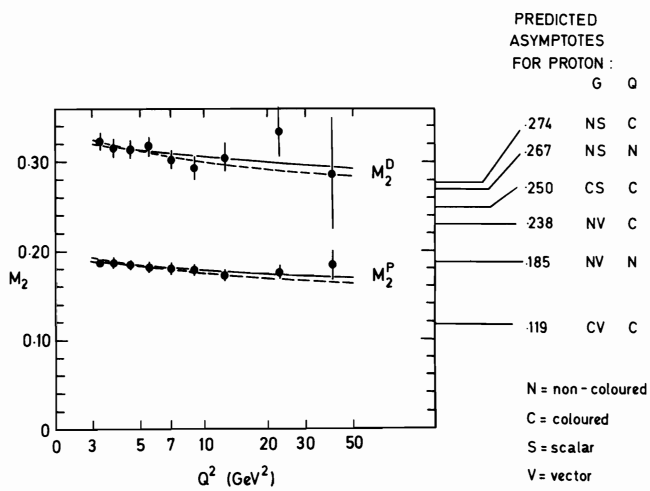


Fig.28 Moments n = 2 of F_2^{γ} for deuteron and proton.

should become equal and, of course, that for the deuteron twice that for the proton. QCD for 4 flavors, 3 colors and vector gluons predicts an asymptotic value of 0.119 for the nucleons. Other possibilites such as scalar gluons, or no color, predict asymptotic values that are greater than the observed values at non-asymptotic Q^2. Since these moments are decreasing with Q^2, it has been claimed that these experimental results are pointing unequivocally at QCD (colored vector gluons) as being the correct theory. Incidentally, more flavors pushes the asymptotic values up. For example QCD with 6 flavors, 3 colors and vector gluons has an asymptotic value of 0.147 for the n = 2 nucleon moment.

The analysis of moments to obtain a value of Λ has been done by several groups. The BEBC group[7] and CDHS group[11] have each fitted their own xF_3^{ν} moments using formula II to obtain values of Λ for some of the moments. The results are shown in Figure 29. There is a big discrepancy. Para, at this symposium, has discussed this and shown that there are many differences in the analyses which could together explain this discrepancy[17].

Two groups have analysed the electromagnetic non-singlet moments. Quirk et al at Oxford[12] and Duke and Roberts[13] at the Rutherford Laboratory. The data used is that from all the SLAC and MIT/SLAC

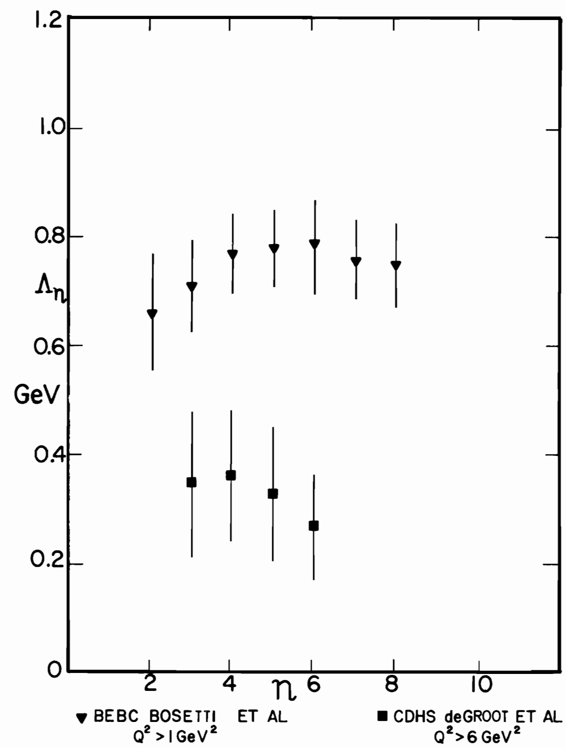


Fig.29 Values of Λ_n versus n for moments of xF_3^{ν}.

electron scattering and from the CHIO muon scattering measurements. Fitting the simple formula (I, Figure 26) gives a value of Λ which increases with n (Figure 30).

Let me show the plot of the log of one moment against the log of another, Figure 31. According to formula III in Figure 26 we see that these points should lie on a straight line with slope which is the ratio of the relevant anomalous dimensions.

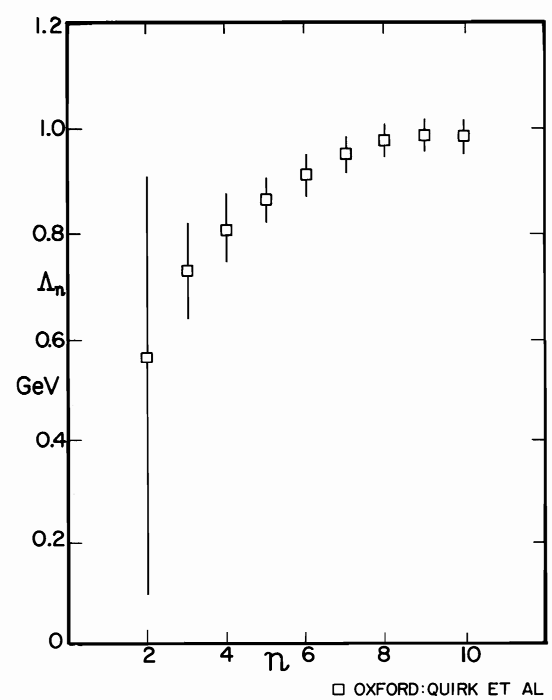


Fig.30 Values of Λ_n versus n for M_{NS}^{γ} leading order fit [12].

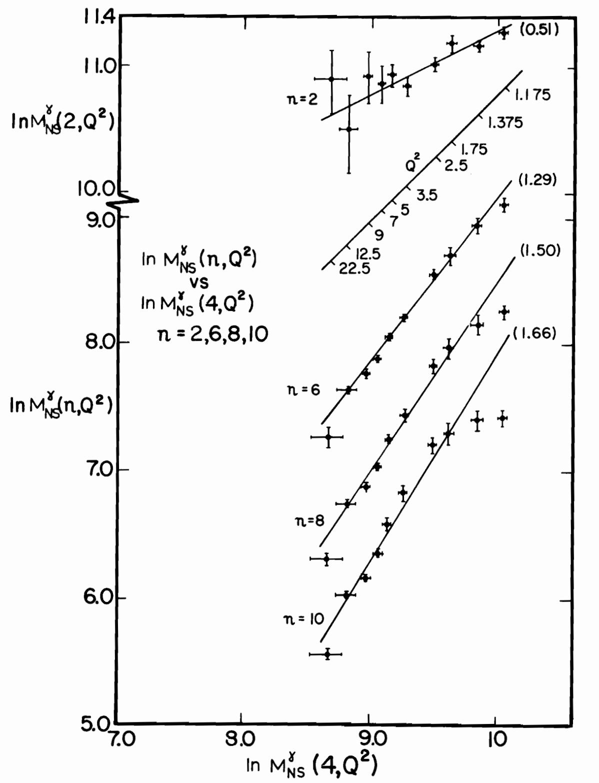


Fig.31 Plot of log $M_{NS}^{\gamma}(n,Q^2)$ against log$M_{NS}^{\gamma}(4,Q^2)$ for n = 2,6,8,10. (Moments of $F_2^{\gamma p} - F_2^{\gamma n}$). The straight lines show the slopes expected from leading order QCD.

Figure 31 has log $M_{NS}^{\gamma}(n,Q^2)$ against log $M_{NS}^{\gamma}(4, Q^2)$ for n = 2,6,8,10. The straight lines give the expected slopes but are not fits. For n > 2, Q^2 2.5 GeV^2 the points tend to lie along straight lines of slightly greater slope than predicted by leading order QCD. (Remember that the error bars on the points are deceptive: there is a very strong positive correlation between the errors on each point).

Similar plots for neutrino data[7,11] lead to an unjustified euphoria about QCD.

The increasing Λ with n and the slightly incorrect slopes of Figure 31 prompt an attempt to fit with second order QCD formulae. Figure 32 shows the Oxford and Rutherford Laboratory second order results for Λ for different n. The increase of Λ with n persists. In addition, the two groups differ even though they used the same data; and it was not just the same data from published papers but the same numbers on a shared magnetic disc file at the Rutherford Laboratory computer. What differences and similarities do the two analyses have?

1) The lower cuts were the same at $Q^2 = 2$ GeV^2.

2) The upper cuts were different but the bin between has almost zero statistical weight and has no appreciable effect.

3) Oxford used three flavors, Duke and Roberts used four. If Duke and Roberts had used three, their results for Λ would have been increased by about 60 MeV.

4) Oxford centered all data in each Q^2 bin using an empirical formula. Duke and Roberts used different binning and calculated an average Q^2 for that bin (effects of order 70 MeV on Λ).

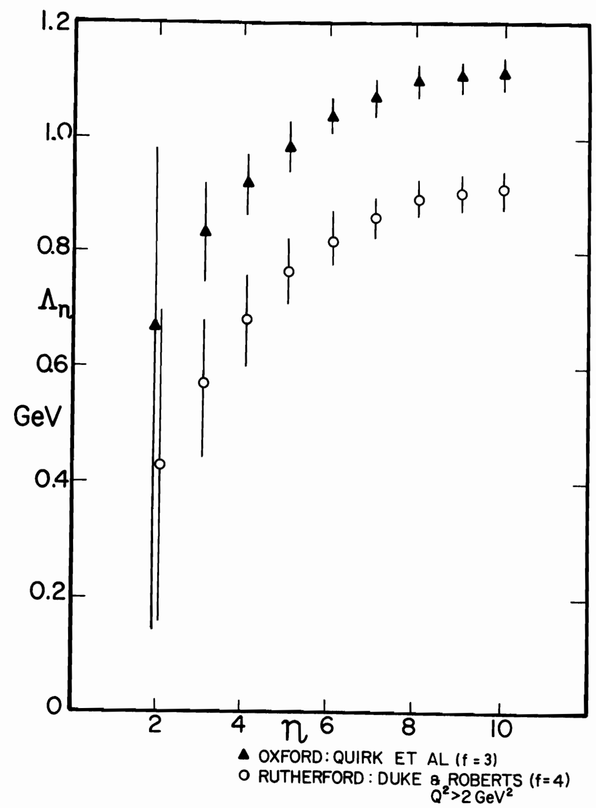


Fig.32 Values of Λ_n versus n for $M_{NS}{}'$ from second order fits[12,13].

5) The results are sensitive to the method of unsmearing the Fermi motion. Both groups believe they used the same method (A.Bodek's unsmearing numbers in his MIT thesis[14]).

6) $R = \sigma_L/\sigma_T$ is the same for both analyses.

It is clear that the details of the calculation of moments can have a great effect on the value of Λ. In addition the following remarks are relevant.

1) The value of R has an effect. Changing by ± one standard deviation has an effect of ~20-, 80 MeV on Λ.

2) The lower Q^2 cut can have a profound effect. The value of Λ decreases as the cut is pushed down.

3) It is essential to include contributions from resonance production and elastic scattering in the moments calculation, in order to obtain moments falling in anything like the way expected from QCD. Excluding these contributions leads to a small Λ. In fact these contributions are large and we need to go to a Q^2 greater than 20 GeV^2 to reduce them to less 5% in the n = 8 moment, for example. Yet QCD is only supposed to work at Q^2 where these contributions are negligible.

4) The effects of higher twist operators are neglected in the QCD predictions. I will return to this point shortly.

The moments other than the non-singlet ones vary in a more complicated way. However, in spite of the possible difficulties of the kind just discussed, Anderson et al[3] have fitted these moments for the proton and deuteron (with Fermi motion unfolded), to obtain Λ and the moments of the up, down and gluon momentum distributions in the proton.

This analysis was done in both leading and second order but the results for these parton distributions were almost identical in the two cases. Figure 33 shows these moments derived from M_2^γ (n, Q_o^2) at Q_o^2 = 10 GeV^2. The ordinate is $\langle p \rangle_n$ and the abscissa is n in

$$\langle p \rangle_n = \int_0^1 x^{n-1} p(x) dx$$

where p(x) is the fractional momentum distribution of parton p. Thus the n = 2 points show that at Q^2 = 10 GeV^2, the gluons are carrying about 45% of

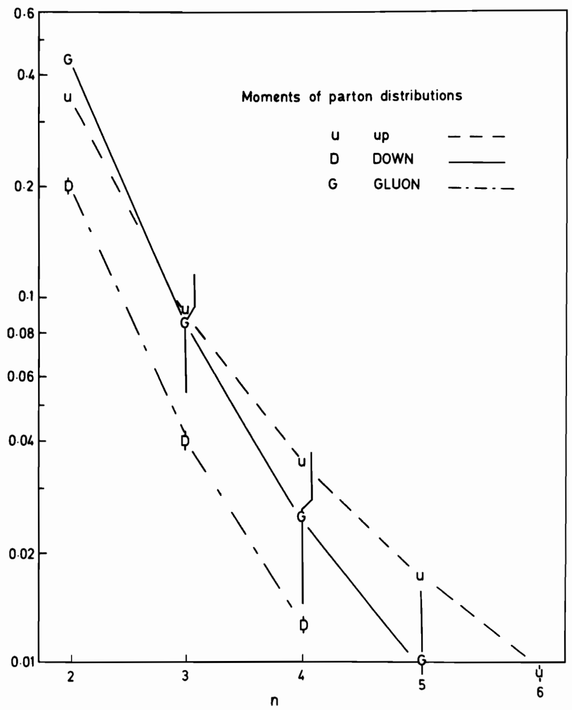


Fig.33 Moments of parton distributions from fits to $F_2^{\gamma p}$ and $F_2^{\gamma d}$ by Anderson et al.[3].

of the momentum. The higher moments indicate that the gluons and down quarks have momentum distributions which are probably very similar, but that the up quarks have a harder momentum distribution.

Let's look briefly at the possible effects of the higher twist operators. These are expected to be the appearance of factors like $(1 + \frac{b_n}{Q^2})$ in the formulae (I, Figure 26) which give the Q^2 development of the moments. Abbot and Barnett[15] have pointed out that the precision of the observed Q^2 development is not sufficient to detect unambiguously the presence of QCD logarithmic terms if higher twist terms are present. This point is dramatically illustrated by plot of $M_{NS}^\gamma(7,Q^2)$ against $^1/Q^2$ (Figure 34). Over the range of $0.7 < Q^2 < 40.0$ GeV^2 the points would fit a + b/Q^2 with no difficulty whilst the QCD fit with $\Lambda^2 = 0.50$ GeV^2 could also fit moderately well for $Q^2 > 1$ GeV^2. Thus the data can be fitted with a higher twist factor alone, or any combination of the two terms.

Perkins[16] has fit the moments M_{NS}^γ calculated by Quirk et al at Oxford and the moments M_{NS}^γ from the combined CDHS/BEBC/GGM data with a form

$$M_{NS}(n,Q^2) = C_n(1 + b_n/Q^2),\ n = 2,\ldots 7.$$

The results for b_n are shown in Figure 35. The

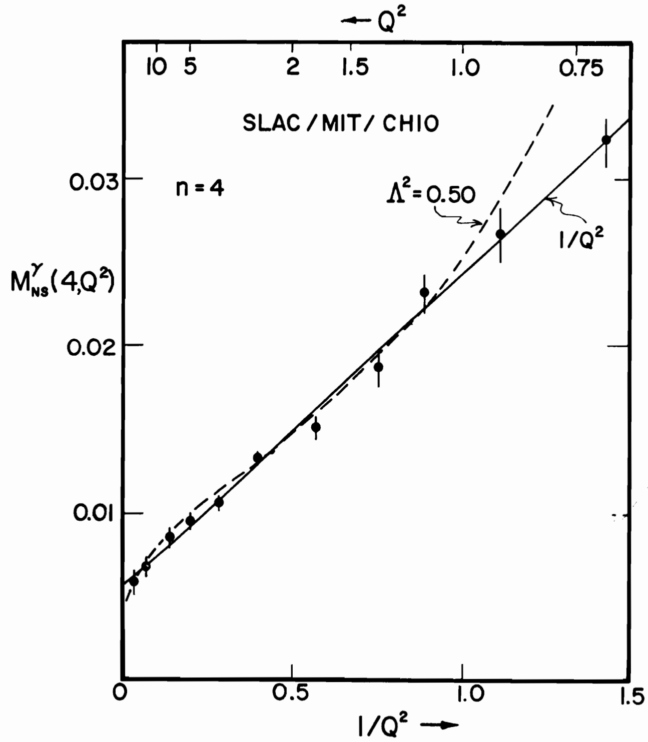


Fig.34 M_{NS}^γ $(4,Q^2)$ versus $\frac{1}{Q^2}$.

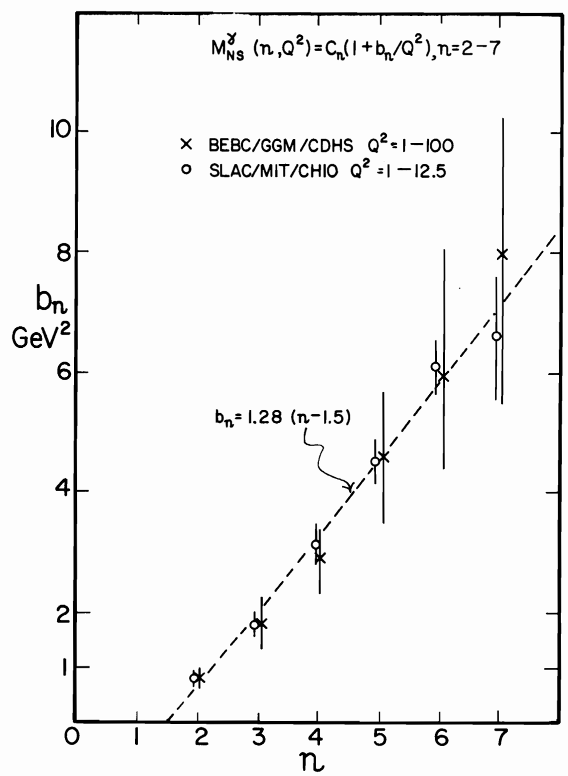


Fig.35 The values of b_n found from fits, which are linear in $1/Q^2$, to non-singlet moments.

parameter b_n is well represented by

$$b_n = 1.28(n - 1.5)\ \text{GeV}^2$$

Clearly to distinguish the relative importance of higher twist terms requires moments data of much greater precision than exists at present or (probably and) data at much greater $Q^2(10^3 \rightarrow 10^4 \text{ GeV}^2)$.

However, as I have tried to convey, there is no agreement as to how the moments should be calculated from measured structure functions, and how the calculation is done affects the values of the moments. In these circumstances it is difficult to see how strong conclusions can be made about the validity of QCD in deep inelastic scattering.

5. Conclusions

1. There are indications that there are some discrepancies between established experiments.
2. The scaling violations are established below $Q^2 \simeq 25 \text{ GeV}^2$.
3. Scaling violations in the "new" experiments above $Q^2 > 30$ GeV may be there but may be less (as expected) than at low Q^2. However, systematic errors exist in the preliminary data from these new experiments. Until these are under control it is really impossible to make firm statements about scaling violations.
4. The moments analyses are very sensitive to cuts, assumptions, method etc. So, beware! In addition, comparison with QCD predictions is not giving very consistent results and is haunted by the uncertainty of the magnitude of higher order and higher twist effects.

6. Acknowledgements

I am grateful to many experimenters for data and discussion about data, to Joe Feller my scientific secretary at the symposium for assistance, to Elsa at Oxford and Emilie at Fermilab for drawing excellent figures.

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DISCUSSION

Q. (M. Strovink, University of California, Berkeley).

You compared F_2 for five experiments at fixed x and interpolated Q^2. Could you say for each experiment whether the values refer to (a) x bin center, (b) unweighted average over the x range, or (c) average over the x range weighted by whatever data fall in that x range.

A. I think the answers are (a) for MSU-F and BFP, (b) for CDHS and EMC and (c) for BCDMS. You should confirm that with each group.

Q. (H. Steir, University of Freiburg, Germany).

In the comparison of Q^2 range of the different muon experiments you have only considered the 280 GeV heavy target data of the European Muon Collaboration. I would like to stress that this is only one sixth part of the iron data we have taken until now. The forthcoming analysis will essentially increase the Q^2 range, also towards lower values of Q^2. The same is true for the hydrogen and deuterium data we have already on tape for different energies.

A. The data span on that transparency refers only to experiments with carbon or iron targets which have been published or have submitted data to this Symposium. The EMC, as you say, have very much more data to come.

Theoretical Developments I

J. D. Bjorken, SLAC and Fermilab
Session Organizer