STUDY OF THE USE OF HADRON CALORIMETERS FOR THE MEASUREMENT OF THE TRANSVERSE MOMENTUM OF HADRON SHOWERS

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1. INTRODUCTION

The extension of the use of hadron calorimeters in many experiments raises the question whether it is possible to use these instruments not only to measure the shower energy but also to determine the direction of the primary particle.

Such additional information presumably should be derived from the transverse development of the cascade at different depths in the calorimeter.

The aim of the present work is to investigate this problem with a Monte Carlo simulation technique and to give preliminary answers about the best way of constructing these detectors.

2. DESCRIPTION OF THE METHOD

A Monte Carlo program which simulates the development of the nuclear-electromagnetic cascade initiated by a strongly interacting particle has been used. The program was originally written to study the energy resolution of iron calorimeters[1]. Nuclear interactions are simulated by using expressions which represent with good approximation the experimentally known longitudinal and transverse momentum distributions of the secondaries and of the leading particle. The multiplicity distribution of the secondaries was assumed to be of a poissonian type with average value increasing logarithmically with energy. The energy which is lost in nuclear excitation was computed by using an empirical formula[1]. The development of each electromagnetic shower initiated by the π^0 decay was also simulated by the Monte Carlo method. An analytical formula[2] was used to reproduce approximately the lateral development of the electromagnetic shower. In each interaction the produced secondaries were given a random momentum vector and the total energy conservation was achieved by attributing to the leading particle the part of the primary energy left by the secondaries; this process results in an appropriate shape of the leading part of the spectrum. The transverse momentum conservation is imposed by rearranging in a random way the transverse momenta of all particles.

3. METHOD OF CALCULATION

The calorimeter, assumed to be a cylinder with radius of 50 cm, consists of a sandwich of 50 equally thick absorber plates and of an equal number of position detectors. The position detectors are imagined to consist of 100 columns and 100 rows of elements, each being a string 1 cm wide and 1 m long. It is supposed that the development of the shower is not to be affected by the presence of the counters.

The first interaction of the primary hadron, travelling along the axis of the calorimeter, was forced to take place inside the first absorber plate.

The calculation was done for four different sorts of material: C, Al, Fe, Pb. For each material the length of the calorimeter was taken equal to approximately 10 absorption lengths.

For each simulated event the transverse spatial distributions observed in 50 position detectors were used to reconstruct the direction of the primary particle. The average angle of the shower was computed in two different ways, corresponding to different assumptions about the nature of the position detectors. In the first way called "the proportional mode", each element of the position detector is assumed to give a signal proportional to the number of charged particles hitting the element itself. In the second way the "yes-no mode", each element is only sensitive to the presence or absence of particles, but not to their number. For each shower the following quantity was constructed and interpreted as the best estimate of the angle of the primary particles:

$$\theta_{\text{proj.}} = \frac{1}{N}\left[\sum_{\substack{j \\ \text{all planes}}} \left(\sum_{\substack{i \\ \text{all columns} \\ \text{or rows}}} \frac{d_i}{R_j} \; W_{ij}\right)\right]$$

$$W_{ij} \begin{cases} = n_{ij} & \text{in the "proportional mode"} \\ = 0,1 & \text{in the "yes-no mode"} \end{cases}$$

n_{ij} is the number of particles in the column or row i in the plane j

d_i is the distance (positive or negative) of the column i from the axis of the calorimeter

R_j is the distance of the plane j from the front face of the calorimeter (which is close to the vertex of the interaction for the assumptions made)

$$N = \sum_i \sum_j W_{ij}$$

Also the transverse position, defined in the following way, was evaluated for each shower:

$$X = \frac{1}{N} \left[\sum_{\substack{j \\ \text{all planes}}} \left(\sum_{\substack{i \\ \text{all columns} \\ \text{or rows}}} d_i \ W_{ij} \right) \right]$$

where the symbols have the same meaning as before.

4. RESULTS OF THE CALCULATION

The values of the most important parameters used for the different materials in the present calculation are reported in Table 1. Table 2 contains the number of charged particles at the maximum of the shower and the position of the maximum itself for the different elements and for incident hadrons of 30 and 100 GeV, respectively. Table 3 contains the main features of the development of the nuclear-electromagnetic cascade in the various calorimeters; the results shown represent for each energy and for each element averages over 200 simulated showers. The meaning of the various entries in Table 3 is explained below.

a) (H + E) is the average total number of particles which crossed the 50 planes of the position detectors. The fractional independent contributions from charged hadrons (H%) and electrons (E%) are also given;

b) the fraction of the primary energy deposited inside the calorimeter and the contributions of the different processes, through which the energy is degraded inside the calorimeter (nuclear excitation, hadron ionization losses and electromagnetic showers) are shown;

c) the calculated energy resolution;

d) the r.m.s. deviation of the projected angle ($\sigma(\Theta_x)$, $\sigma(\Theta_y)$), distribution ($\sigma(x)$, H + E) are reported for the "proportional mode" and for the "yes-no mode" when both hadrons and electrons (H + E) are observed. In addition, for the "proportional mode" the results are shown only when hadrons (H) or electrons (E) are considered. The average values of the angle distributions were found to be equal to zero, within the statistical errors in agreement with the assumption that the primary particle was travelling along the axis of the calorimeter;

e) the r.m.s. deviation of the projected position distribution ($\sigma(x)$, $\sigma(y)$) are reported for the same conditions as in d).

The study of the table leads to the following considerations:

a) the angular dispersion and the spatial dispersion (expressed in g/cm^2) are linked together and they both increase as the atomic number of the material becomes larger. The increase of the angular and of the spatial dispersion with the atomic number is found to be much faster in the "yes-no mode" than in the "proportional mode". This is due to the fact that in heavy materials the electromagnetic showers are much more collimated than in the light materials and consequently a larger part of the information is lost;

b) the angular dispersions obtained with only electrons or hadrons are much larger than those computed with electrons and hadrons taken together. This may be explained by a negative correlation produced by transverse momentum conservation between the two separate contributions; this effect is less strong in the case of the spatial dispersions; in particular in Pb the position is better determined by looking at the electrons only;

c) the magnitude of the angular and spatial dispersion of the hadrons increases when the atomic number increases; this is likely due to the fact that also the absorption length becomes larger when Z increases. The angular and spatial dispersions of the electrons, on the other hand seem to have in general a minimum for iron. This behaviour could be explained by the fact that the development of the electromagnetic part of the cascade is determined at the same time by the radiation length and by the absorption length of the material; the former affects the lateral spread of the electrons around the direction of the primary π^0; the latter the lateral spread of the π^0 themselves. As the atomic number increases, the radiation length decreases and the absorption length increases, and what is seen is the combined effect of the opposite contributions;

d) the projected angular dispersion is typically 20 mrad for a 30 GeV shower, 10 mrad for a 100 GeV shower for all materials but for Pb. These results provide an answer to the question raised in the introduction. The total angular dispersion is of course $\sqrt{2}$ times larger.

In the calculation described above the calorimeter was supposed to be homogeneous (the perturbation produced by the insertion of the detectors was neglected). However it may be expected that other arrangements of the absorbers and detectors inside the calorimeter could provide an angular resolution better than those calculated above. It could be observed that $\sigma(x)$ expressed in centimetres decreases as Z increases. At 30 GeV and for C, Al, Fe, Pb, the results of Table 3 read, in the "proportional mode", $\sigma(x)$ = 34 mm, 24 mm, 12 mm and 11 mm, respectively. At 100 GeV, $\sigma(x)$ = 17 mm, 10 mm, 4 mm and 5 mm for C, Al, Fe, Pb, respectively. These numbers, as they are, could give an angular resolution of the order of 10 mrad if one thinks of building a calorimeter 1 m away from a target and proportionally better for a longer drift space. Better resolution in position could be obtained using heavy materials and suppressing at the same time, as much as possible, the contribution of hadrons. This leads to the idea of "short calorimeters" where the hadron cascade develops less than the electromagnetic cascade. In Table 4 the values obtained with a device of this kind are shown. The results were obtained using only 10 planes of position detectors. No appreciable differences were found using 25 and 50 planes of position detectors. The spatial dispersions are of the order of a few millimetres, suggesting that this way of using calorimeters could be profitable for determining the angle. The energy will have to be determined in the usual way by a following standard calorimeter.

The results reported were obtained with a Monte Carlo computation based on several approximations. In reality, the development of the cascade is much more complicated than the one we have imagined in this simplified model. Nevertheless the calculated spatial resolution (6 mm, Table 4), which was obtained at 30 GeV for an iron calorimeter, 19 cm long, checks reasonably well with the value of 7 mm measured by Atac et al.[3] with primary neutrons of 13-20 GeV, using an iron calorimeter 16 cm long and 5 proportional chambers with wires 1 cm apart.

It may be concluded that for hadrons of 30 GeV to 100 GeV, in a homogeneous calorimeter, angular resolutions of the order of 20 mrad can be achieved. For higher precision the calorimeter has to be made of discrete elements with drift spaces between.

REFERENCES

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Table 1

	C	Al	Fe	Pb
Absorption length (g/cm^2)	78.	101.	135.	210.
Radiation length (g/cm^2)	43.3	24.3	13.9	6.4
Critical energy (MeV)	79.0	48.8	21.0	7.4
Density (g/cm^3)	1.6	2.7	7.9	11.4

Table 2

	30 GeV				100 GeV			
Element	C	Al	Fe	Pb	C	Al	Fe	Pb
Position of max. of shower (g/cm^2)								
Charged hadrons	175	200	240	270	210	240	300	295
electrons	200	120	70	40	230	160	110	55
Nr.of particles at max.position								
Charged hadrons	7.4	7.4	6.5	4.4	18.9	17.2	16.1	9.0
electrons	11.6	16.0	26.0	30.9	39.6	56.2	72.6	201.5

Table 3

		30 GeV				100 GeV			
Element		C	Al	Fe	Pb	C	Al	Fe	Pb
Length (g/cm^2)		1000	1000	1600	2000	1000	1000	1600	2000
Nr.of λ_{abs}		10	8	12	9.5	10	8	12	9.5
Nr.of particl.	H+E	341	397	255	157	1267	1428	966	650
	H%	37	40	42	37	29	32	30	25
	E%	63	60	58	63	71	68	70	75
percent. of energy deposited	Total	69.0	81.0	86.2	90.5	74.7	84.9	90.6	93.0
	Nuclfrag.	8.3	13.6	21.4	48.2	6.7	10.5	16.2	35.7
	h.ioniz.	17.6	20.2	20.2	10.3	15.1	17.	16.3	8.7
	em.shower	43.1	47.2	44.6	32.0	52.9	57.4	58.1	48.6
Energy resolution %		13.	9.2	11.	20.	9.7	11.2	6.4	19.
$\sigma(x)$ g(cm^2)	prop.mode H+E	5.5	6.5	9.3	12.	2.7	2.7	3.3	6.0
	prop.mode H	7.9	11.1	17.9	26.	5.0	6.5	11.5	22.2
	prop.mode E	14.7	12.3	9.7	7.	5.0	4.2	3.3	4.7
	yes-no " H+E	5.7	7.4	15.	20.	2.9	4.0	10.9	20.
$\sigma(\Theta_x)$ mr	prop.mode H+E	19.	18.	17.	24.	7.5	7.3	8.8	20.5
	prop.mode H	35.	46.5	51.	67.	19.5	18.	29.3	41.3
	prop.mode E	67.2	67.8	46.	72.	16.5	16.	17.6	41.0
	yes-no " H+E	20.	19.	24.	24.	9.	10.	20.8	23.5
Nr. of events		50	50	200	50	50	50	50	50

Table 4

	30 GeV		100 GeV	
	Fe	Pb	Fe	Pb
$\sigma(\Theta_x) = \sigma(\Theta_y)$ proportional mode	24.8 mrad	29.2 mrad	21.6 mrad	29.2 mrad
$\sigma(\Theta_x) = \sigma(\Theta_y)$ yes-no mode	26.2 mrad	31.3 mrad	28.5 mrad	26.8 mrad
$\sigma(\bar{x}) = \sigma(\bar{y})$ proportional mode	4.7 g/cm^2 = 0.59 cm	5.1 g/cm^2 = 0.45 cm	3.4 g/cm^2 = 0.42 cm	3.0 g/cm^2 = 0.26 cm
$\sigma(\bar{x}) = \sigma(\bar{y})$ yes-no mode	6.0 g/cm^2 = 0.76 cm	9.2 g/cm^2 = 0.81 cm	7.9 g/cm^2 = 1.cm	8.3 g/cm^2 = 0.73 cm
Length	150 g/cm^2 = 19 cm	200 g/cm^2 = 18 cm	200 g/cm^2 = 25cm	200 g/cm^2 = 18 cm