# NOTE ON LONGITUDINAL BEAM POLARIZATION 

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#### Abstract

Longitudinal beam polarization achieved through g-2 precession is examined for the dependence of the power emitted as synchrotron radiation on the free length at the intersection. Using only dipoles for dispersion, the power doubles for every three meters of free length. The power can be reduced by using existing focusing elements to increase the dispersion, but this procedure also limits the luminosity because of chromatic aberration.


In PEP-87, Schwitters and Richter have proposed using the g-2 precession to produce longitudinal electron beam polarization. The net deflecting field integral for a $\pi / 2$ rotation is $23.05 \mathrm{kG}-\mathrm{m}$, independent of beam energy. At 15 GeV , therefore, the orbit at the intersection point makes an angle of 0.0461 with the median plane.

## I. Dipoles for Variable Free Length

In general, dipoles in the $\pm 10 \mathrm{~m}$ length between quadrupoles can be used to satisfy the boundary conditions for an arbitrarily selected free length, f. Figure la shows a typical configuration. The magnet lengths and/or strengths must be adjusted to give:

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at z = f: y = f foo; 0= 说 = 23.05 kG-m / B\rho;
at z = d = 10m: y = 0; 0 = 0.
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Note that both angles and displacements are proportional to the angle at the intersection point. Hence recombination recurs independently of momentum.

For a given beam momentum, the radiated power is related to the curvature.


If two uniform field dipoles are used, this gives

$$
P \propto \frac{\left(\Delta \theta_{1}\right)^{2}}{l_{1}}+\frac{\left(\Delta \theta_{2}\right)^{2}}{l_{2}}
$$

where $\Delta \theta_{1}, \Delta \theta_{2}$ are the deflections in the two magnets of lengths $\ell_{1}$ and $\ell_{2}$, respectively. Figure 2 shows the relative power for a number of cases. The highest solid curve corresponds to equal field strengths in the two dipoles when the available space is completely filled with magnets (we neglect end effects). The particular case considered in PEP -87 is indicated in Figure 2. This corresponds to $\approx 8 \mathrm{kG}$ in the magnets and a radiated power of 300 kW per beam.

With two uniform dipoles filling the available space, the power is minimized for $\ell_{1}=\ell_{2}$ (middle solid curve). We can reduce the power further by searating the dipoles somewhat. The dashed curves show the results for differant sets of equal length dipoles.

Further improvement is possible with a tapered field which is strong at the ends of the available space and goes through zero near the center. The lowest solid curve in Figure 2 is for a linearly tapered field. (The radiated power for a sinusoidally tapered field is about 1 per cent larger).

We note that the power loss is a sensitive function of free length, approxmately doubling for every three meters.

## II. Use of Quadrupoles.

To reduce the power or increase the free length, we might consider moving the quadrupole further away from the intersection point. This modification would sacrifice luminosity. An alternative is to provide vertical deflection between or outside of the quadrupole elements. Two examples are shown in

Figure 3. In 3a, dipoles alternating with quadrupole elements add the necessary dispersion.

A conceptually simple solution is shown in 3b. Because the quadrupoles must focus essentially parallel rays to a point, they will provide the necessary dispersion for spin rotation if the beam beyond the quadrupoles is made parallel to the axis and offset by

$$
y_{0}=\theta_{0} F_{V}
$$

where $F_{v}$ is the vertical focal length of the doublet. If the long external magnet-free space can be used to provide this offset, the power in the offsetting magnets can be made negligible.

The lower dashed curves in Figure 2 show the power needed for the configuration of Figure 3b for several sets of equal length doublets. We have assumed that the half-space available for quadrupoles plus free length is 16 m . For free lengths of more than a few meters, this solution needs much less power than do the dipoles.

One effect of moving the quadrupole elements closer to the intersection point is to increase the luminosity. An estimate of this may be obtained from the (inverse) product of the horizontal and vertical focal lengths. This is shown in Figure 4, normalized to the corresponding product for the normal quadrupole configuration (see Figure 1).

Unfortunately, there are problems associated with using the configuration of Figure 3b:

1. The vertical aperture must be large (see Figure 4). The quadrupole fields are not too large, however, and can probably be achieved with a Panofsky-type (current sheet) quadrupole. The (damped) beam is not large in cross-section so that most aberrations can probably be corrected. The possibilities for displacement of the beam in the rf accelerating section would certainly have to be examined.
2. A very severe problem is that chromatic aberration in the highly dispersed orbit is significant. At the intersection, an off-momentum particle has a displacement normal to the beam given by

$$
\begin{aligned}
& \Delta y \approx \theta_{0} \Delta F_{v} \approx 3 \theta_{0} F_{v} \Delta p / p \\
& \text { using } \theta_{0}=0.0461, F_{v}=3.3 \mathrm{~m}, \Delta p / p=10^{-3} \\
& \text { we find: } \Delta y \approx 0.5 \mathrm{~mm} .
\end{aligned}
$$

This is about an order of magnitude larger than the normal vertical beam size at the intersection. Hence the use of quadrupoles in such configurations would lead to a significant decrease in luminosity. More moderate combinations of focusing and dipole elements, such as that indicated in Figure 3a have not been examined.
FIGURE 1 - DIPOLE CONFIGURATIONS




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