Report of the Weak Interactions/ EM Final States Group

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ABSTRACT

In the reaction $e^+e^- \rightarrow \mu^+\mu^-$ the interference of weak and electromagnetic amplitudes is expected to produce measurable effects at the highest PEP energies in runs with integrated luminosities of order 10^{38} cm^{-2}. The prediction for $\mu^+\mu^-$ polar asymmetry and absolute rate, which give g_A^2 and g_V^2, respectively, are highly model dependent. The s-dependence of these effects is sensitive to the existence of an intermediate boson with mass in the range below 100 GeV. The use of transverse electron beam polarization should be helpful in sorting out higher order electromagnetic effects. It is of negligible statistical advantage in determining the weak-electromagnetic interference terms. More difficult experiments requiring longitudinal beam polarization or the measurement of muon helicity could determine the weak V-A interference terms and provide a test of μ-e universality.

PEP offers an unique opportunity for studying weak interactions in the absence of hadrons through the effects of interference between a possible weak neutral current and the electromagnetic current. At 15 GeV, the effects of the interference are expected to be large enough to allow a measurement of the weak interaction coupling constants involved in the reaction $e^+e^- \rightarrow \mu^+\mu^-$. In particular, this reaction will provide a sensitive test of the Weinberg model. In this report we consider measurements of the cross section, angular distributions, and polarization of muons produced by stored e^+e^- beams that are unpolarized, transversely polarized, or longitudinally polarized, with emphasis on how such experiments might affect the design of PEP. We conclude that measurements of the charge asymmetry and the integrated cross section of the reaction $e^+e^- \rightarrow \mu^+\mu^-$ will be important first-round experiments, whether the beams are transversely polarized or not (although the state of the polarization should be monitored). These experiments could profit from (1) the use of a high luminosity interaction region and (2) an accurate monitor of the transverse polarization of the stored beams. Other experiments such as the measurement of the polarization of the product muons or of the muon charge asymmetry produced by longitudinally polarized electrons and/or positrons are more difficult. They require the collection of a substantially larger number of ee $\rightarrow$ $\mu\mu$ events, particularly if the Weinberg model turns out to be correct. Still the effects may well be larger than predicted by this model and the experiments should be performed when high luminosity has been achieved. For this purpose, it will be desirable to have at PEP a facility for producing longitudinally polarized beams with sufficient space for detectors and monitoring apparatus.

1. BASICS

The differential cross section for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ is given by quantum electrodynamics to be:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\left[1 + \cos^2\theta - P_+ P_- \sin^2\theta \cos 2\phi\right]$$

This is under the assumption of single photon exchange for beam energies E much greater than the muon mass and for transversely polarized electron and positron beams (polarization magnitudes = P_- and P_+). θ and ϕ are the production angles of the muon with the azimuth (ϕ) measured from the polarization vector for the electrons.

If a neutral weak vector boson (Z°) exists which couples to electrons and muons, then we can get interference effects between photon exchange (Figure 1a) and Z° exchange (Figure 1b). For a Z° coupling to electrons and muons given by

$$\mathcal{L}_{INT} = i\,\bar{\ell}\,\gamma^\mu (g_V - g_A \gamma^5)\,\ell\, Z_\mu$$

with g_V and g_A the vector and axial vector coupling constants, we obtain the QED + weak cross section for $e^+e^- \to \mu^+\mu^-$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2}\left[\left(1 + \frac{2E^2 g_V^2}{\pi\alpha(4E^2 - M_Z^2)}\right)\left(1 + \cos^2\theta - P_+ P_- \sin^2\theta \cos 2\phi\right) + \frac{4E^2 g_A^2}{\pi\alpha(4E^2 - M_Z^2)}\cos\theta\right]$$

where M_Z is the mass of the Z°. Hence the weak vector term changes the absolute rate from the QED value, while the axial vector term introduces a charge asymmetry:

$$A = \frac{\sigma_- - \sigma_+}{\sigma_- + \sigma_+}$$

where σ_- (σ_+) is the differential cross section for $\mu-$ ($\mu+$) production at (θ,ϕ).

We shall throughout this report use two models to give us estimates of the weak effects. In the "simple V-A model" we set

$$g_V = g_A = g; \quad g^2/M_Z^2 = G/\sqrt{2}$$

where G is the Fermi coupling constant.

The Weinberg Model gives:

$$e = g \sin\theta_W$$
$$g_V = \frac{1}{4} g \cos\theta_W \left(3\tan^2\theta_W - 1\right)$$
$$g_A = \frac{1}{4} g \sec\theta_W$$
$$M_Z = \left(g \sec\theta_W\right) / \left(4\sqrt{2}\, G\right)^{\frac{1}{2}}$$

where Θ_W is the Weinberg angle.

2. CROSS SECTION $\sigma(e^+e^- \to \mu^+\mu^-) \equiv \sigma_{\mu\mu}$

The total cross section $\sigma_{\mu\mu}$ for the process $e^+e^- \to \mu^+\mu^-$ is, according to pure QED, $\sigma_{\mu\mu} = \pi\alpha^2/3E^2$. For E = 15 GeV, an integrated luminosity of 10^{38} cm^{-2} will produce 10^4 $e^+e^- \to \mu^+\mu^-$ events. This is sufficient statistically for a 1% measurement of $\sigma_{\mu\mu}$. However, the accuracy of the experiment will probably be limited to $\Delta\sigma/\sigma \sim 2\%$ due to uncertainties in the radiative corrections (including hadronic vacuum polarization effects) and in the absolute measurement of luminosity. This is to be compared to the change in the cross section due to interference between the weak and electromagnetic currents, which is given by:

$$\frac{\Delta\sigma}{\sigma} = \frac{-g_V^2}{2M_Z^2}\,\frac{s}{\pi\alpha}$$

where $s \doteq 4E^2$ is the square of the center of mass energy and we assume that $M_Z^2 \gg s$. In the simple V-A model, $\Delta\sigma/\sigma$ = -16% at E = 15 GeV. In the Weinberg model, $\Delta\sigma/\sigma$ is dependent on the Weinberg angle Θ_W and is proportional to $(4\sin^2\Theta_W - 1)^2$. The effect therefore disappears where $\sin^2\Theta_W = 1/4$ (Θ_W = 30° - there is no vector term for this Weinberg angle) and gives only $\Delta\sigma/\sigma$ = -1% for $\sin^2\Theta_W$ = 0.33. The value of $\Delta\sigma/\sigma$ as a function of $\sin^2\Theta_W$ is shown in Figure 2.

3. CHARGE ASYMMETRY (Beams unpolarized or transversely polarized)

Two mechanisms cause the angular distribution for μ^+ to differ from that of the μ^-. One, as we have noted, is due to interference effects between the weak and electromagnetic interactions. The other is purely electromagnetic in origin and comes from higher order electromagnetic processes. At PEP these charge asymmetries will be comparable in magnitude in certain angular regions, while in other regions one or the other mechanism will dominate.

a) Weak-electromagnetic interference charge asymmetry.

The charge asymmetry which arises from the interference between weak and electromagnetic interactions we write as:

$$A^{wem} = \frac{-2 a_3 \cos\theta}{1 + \cos^2\theta - P_+ P_- \sin^2\theta \cos 2\phi}$$

where $$a_3 = \frac{g_A^2\, s}{2\pi\alpha M_z^2}$$

and where we have taken $M_z^2 >> s$ and neglected terms of order $g_A^2\ g_V^2$. In the simple V-A model, a_3 has the value

$$a_3 = \frac{\sqrt{2}\, G s}{4\pi\alpha} = 0.16 \quad \text{at} \quad E = 15 \text{ GeV.}$$

The Weinberg model predicts:

$$a_3 = \frac{\sqrt{2}\, G s}{8\pi\alpha} = 0.08 \quad \text{at} \quad E = 15 \text{ GeV.}$$

and this is independent of the Weinberg angle.

The maximum magnitude of the charge asymmetry occurs at $\phi = 0$ or π, $\cos^2\theta = (1 - P_+P_-)/(1+P_+P_-)$ where

$$A^{WEM} = -a_3 \Big/ \sqrt{1-(P_+P_-)^2}$$

$= -1.9\ a_3$ for maximal beam polarization from synchrotron radiation ($P_+P_- = 0.85$). Although A can thus be as large as 15% in the Weinberg Model at E = 15 GeV with polarized beams, it will have this value only over a restricted angular region. At the PEP design luminosity it is impossible to obtain an accurate measurement of the charge asymmetry in such a small angular region with running periods measured in months. It is therefore necessary to use detectors covering a large fraction of 4π sr. solid angle and to use the data on A from the entire angular region in order to extract the value of a_3.

It is instructive to note that the interference term a_3 (and thus g^2_A/M^2_Z) is determined by a measurement of the difference in cross sections for μ^- and μ^+:

$$\sigma_- - \sigma_+ = \frac{-\pi^2}{s}\ a_3 \cos\theta$$

which is independent of the transverse beam polarization and decreases only very slowly with increasing Θ. If the beams are transversely polarized an asymmetry in ϕ is introduced in $\sigma_- + \sigma_+$ but $\sigma_{\mu\mu} = 1/2\int(\sigma_- + \sigma_+)\, d\Omega$ is unchanged. These facts are illustrated in Figure 3. Assuming that all systematic errors in the experiment and in the radiative corrections calculations can be kept sufficiently low, the statistical significance of a measurement of a_3 (and thus g^2_A/M^2_Z) is primarily determined by the total number of events detected and is nearly independent of the polarization.

The statistical error in a_3 for 10^4 $\mu^+\mu^-$ events produced is shown in Figure 4. The error is given as a function of the minimum production angle subtended by the detector, for unpolarized beams and for beams with maximal transverse polarization. For a different number N of events produced, this error should be scaled as $1/\sqrt{N}$. As expected from the previous discussion, the presence of even maximal transverse beam polarization has only a negligible effect on the statistical error. However, it is desirable that the polarization be well measured to check for internal consistency of the data.

It is worthwhile to note that the measurement of a_3 greatly benefits from a high value of s. Since $\Delta a_3 \sim 1/\sqrt{N}$ and N = luminosity (L) x time (T) x $\sigma_{\mu\mu}$ we have $\Delta a_3 \sim (s/LT)^{1/2}$. Then the fractional error (since $a_3 \sim s$ for $M_z^2 >> s$) is given by:

$$\Delta a_3/a_3 \sim (sLT)^{-\frac{1}{2}}$$

At PEP, where we expect $L \sim s$ up to 15 GeV, we get $\Delta a_3/a_3 \sim 1/s$ for a fixed amount of running time. In addition, the charge asymmetry due to radiative corrections (see below) becomes relatively less important as s increases.

b) Electromagnetic charge asymmetry.

The interference between one photon and two photon exchange diagrams, as well as between muon and electron bremsstrahlung diagrams, can also produce a charge asymmetry (A^{EM}).[1] These radiative corrections contain terms which depend on the energy resolution of the detector, ΔE, as well as terms which are independent of ΔE. The dependence is of the form $\ln \Delta E/E$. The asymmetry A^{EM} due to the two terms is plotted as a function of Θ for $\Delta E/E = 0.1$ in Figure 5. Also shown in this figure is A^{WEM} for unpolarized beams. At all Θ, A^{EM} must be taken into account in a measurement of A^{WEM}. For $\Theta < 20°$, A^{EM} will dominate the measured asymmetry. This provides a region where the calculation of the radiative corrections can be checked. Since A^{WEM} depends linearly on s while A^{EM} has a much slower dependence, a second check of the calculation may be made by studying the charge asymmetry as a function of machine energy.

When requirements of energy and collinearity are applied to the data, the background due to processes such as $e^+e^- \to e^+e^- \mu^+\mu^-$ should be small. However, it is undesirable to make $\Delta E/E$ very small (e.g., 1%), since A^{EM} then becomes much larger.

The charge difference $(\sigma_- - \sigma_+)$ arising from radiative corrections has a similar ϕ dependence as $(\sigma_- + \sigma_+)$ when the beams are polarized. This, in principle, permits an experimental separation of $\sigma_- - \sigma_+$ (weak), which is ϕ independent, and $\sigma_- - \sigma_+$ (electromagnetic), which is ϕ dependent.

c) Effect of $M_z \sim E$.

It is useful to examine the charge asymmetry due to weak-electromagnetic interference for M_z of the same order as E. In this case, the asymmetry calculated for $M_z^2 >> 4E^2$ is multiplied by the factor $\frac{1}{1-4E^2/M_z^2}$. A plot of A^{WEM} and the statistical error in A^{WEM} for a 10^4 event experiment as a function of s, for several values of M_z, is shown in Fig. 6. If M_z is as low as 50 GeV the asymmetry at $s = 900\ GeV^2$ will be increased by more than three standard deviations compared with the asymmetry expected for $M_z^2 >> 4E^2 = 900\ GeV^2$.

d) Experimentaat PEP

From Figure 4 we see that a run at PEP yielding 10^4 $\mu^+\mu^-$ events (44 days of running at E = 15 GeV with $\mathcal{L} = 2.5 \times 10^{31}\ cm^{-2}\ sec^{-1}$) will give

$\Delta a_3 \approx \pm 0.013$ in a close-to-4π detector, provided that the contribution of the higher order electromagnetic effects (and other systematic effects) can be understood. Hence, at $s = 900\ GeV^2$, we have $\Delta a_3/a_3 = \pm 8\%$ for the simple V-A model and $\Delta a_3/a_3 = \pm 16\%$ for the Weinberg Model. This is clearly an exciting experiment. However, a convincing demonstration of this effect will require runs at two or preferably three separate energies in order to show that a_3 is proportional to s and to confirm the radiative correction calculations.

We estimate that installation, debugging, data taking, and performing the necessary cross-checks might be expected to last for a period of order one year, provided that the average luminosity is greater than $2 \times 10^{31}\ cm^{-2}\ sec^{-1}$.

e) Detectors.

The characteristics for a mu-pair detector for PEP may be briefly summarized as follows: The detector should cover as much of 4π sr. solid angle as possible, and should go to small Θ. It is advantageous if it fits into a high luminosity region. The detector must discriminate positive from negative muons. (This may be accomplished by forcing the muons to traverse $\sim$1 m of iron magnetized to $\gtrsim$ 12 kG.) The detector must be able to impose energy and collinearity requirements to aid in suppressing backgrounds. Time-of-flight information will aid in discriminating against cosmic rays. It would be an additional advantage if a detector were able to measure $\sigma(ee \rightarrow ee)$. Some possible detectors are described in the appendix. These detectors can measure other processes in parallel. Provided that systematic precision is not compromised this is an important consideration since the running time required for a measurement of $\sigma_{\mu\mu}$ and a_3 is appreciable. Furthermore, detection of a relatively well defined process such as $ee \rightarrow ee$ can provide a necessary luminosity monitor.

4. MUON POLARIZATION

a) Experiment

Both Paschos and Berman have written down the expression for the muon longitudinal polarization P_μ due to weak-electromagnetic interference effects, for unpolarized or transversely polarized beams. This muon polarization is:

$$P_\mu = B\left(1 + \frac{2\cos\theta}{1+\cos^2\theta - P_+P_-\sin^2\theta\cos 2\phi}\right)$$

where $B = \frac{g_V g_A}{2\pi\alpha}\frac{s}{M_Z^2}$

and assuming $M_Z^2 \gg s$. A measurement of P_μ thus yields information on the product of the weak vector and axial vector coupling constants. In addition, a measurement of $P_\mu \neq 0$ with unpolarized beams would be evidence for a parity violating effect which could not be produced electromagnetically.

In the simple V-A model

$$B = \frac{\sqrt{2}\,G s}{4\pi\alpha} = 0.16 \quad \text{at} \quad E = 15 \text{ GeV.}$$

The Weinberg Model gives:

$$B = \frac{\sqrt{2}\,G s}{8\pi\alpha}\left(4\sin^2\theta_W - 1\right)$$

$$= 0.027 \quad \text{with} \quad \sin^2\theta_W = 1/3 \quad \text{and} \quad E = 15 \text{ GeV.}$$

The maximum value of P_μ is 2B for unpolarized beams and 2.9B for beams with maximal polarization (Beware!--P_μ will be this large only over a limited angular region). A measurement of P_μ to ±1% to see Weinberg Model effects would require an integrated luminosity of about 10^{40} cm^{-2} which is impractical at presently envisioned luminosities. A less accurate measurement seems still worthwhile, however, since the effect may be much larger than the Weinberg Model prediction. (a factor of 6 larger, for example, in the simple V-A model).

b) Detector.

A polarimeter detector must stop 15 GeV muons, detect the decay positrons (E_{max} = 53 MeV) over an energy interval determined by the muon energy straggling (∿5% of 15 GeV) and obtain sufficient statistics to overcome the low analyzing power of the muon decay asymmetry and the dilution effects due to production of unpolarized muon pairs. A 4π detector which appears somewhat larger than practical and a more modest detector covering Θ near 0° are discussed in a following report.

5. LONGITUDINAL BEAM POLARIZATION

a) Experiments

Two different measurements can be devised with longitudinally polarized e^+ and/or e^- beams to detect weak-electromagnetic interference effects. Each depends in a different way on the coupling constants. At PEP, the most likely circumstance is that one beam will be longitudinally polarized and the other beam unpolarized.

The first measurement is the charge asymmetry

$$A^{WEM} = \frac{g_A^2\, s}{2\pi\alpha\, M_Z^2}\left[1 + \frac{g_V}{g_A}\,\frac{\lambda^+ + \lambda^-}{1+\lambda^+\lambda^-}\right]\frac{2\cos\theta}{1+\cos^2\theta}$$

where λ is the longitudinal polarization of the respective beams. The first term in the brackets represents the asymmetry obtained if the beams are unpolarized. The second term represents the change in asymmetry due to the longitudinal polarization. In the simple V-A model, $g_V/g_A = 1$. In the Weinberg Model $g_V/g_A = 4\sin^2\Theta_W - 1$ which = 1/3 for $\sin^2\Theta_W = 1/3$.

The ratio g_V/g_A can be obtained by measuring the charge asymmetry with different values of longitudinal polarization for the single polarized beam. For example:

$$g_V/g_A = \frac{A^{WEM}(\lambda=+1) - A^{WEM}(\lambda=-1)}{A^{WEM}(\lambda=+1) + A^{WEM}(\lambda=-1)}$$

and the absolute error on g_V/g_A will equal the corresponding relative error $\Delta a_3/a_3$. To measure g_V/g_A (with $\sin^2\Theta_W = 1/3$) in this way to 25% would require a total of 4×10^4 $\mu^+\mu^-$ events at E = 15 GeV. This looks hard but of course the effects may be much larger than the Weinberg prediction. (factor of 6 in the simple V-A model).

The second measurement is a comparison of the total cross section for two values of the longitudinal polarization. For example:

$$\frac{g_V g_A s}{2\pi\alpha M_Z^2} = \frac{\sigma_{\mu\mu}(\lambda=+1) - \sigma_{\mu\mu}(\lambda=-1)}{\sigma_{\mu\mu}(\lambda=+1) + \sigma_{\mu\mu}(\lambda=-1)}$$

which equals, at E = 15 GeV, 16% in the V-A model but only 3% in the Weinberg Model ($\sin^2\Theta_W = 1/3$). It should be feasible to measure effects somewhat larger than those predicted by the Weinberg Model.

b) Muon-electron Universality

Considered in detail the V-A interference terms identified above involve mixtures of the muon and electron coupling constants. From the internal consistency of the g_A^2, g_V^2, $g_A g_V$ and g_V/g_A measurements it is in principle possible to detect differences in e.g. $g_A^\mu\, g_V^e$ and $g_A^e\, g_V^\mu$. This provides a test of muon-electron universality.

c) Detectors

Until now the only promising method found for achieving longitudinal polarization is that described in PEP-87 by Schwitters and Richter. In this system, unfortunately, the synchrontron radiation levels are high, and considerable length in the intersection region is required for the bending magnets. This not only limits the possibilities for obtaining high luminosity, but creates background from synchrotron radiation and also leaves only a few meters free along the beam line for the detector. Several examples given in the following reports show how muon detectors can be installed under such limitations. For the study of $\mu\mu$ final states alone it is not impossible that the detector could completely surround the beam elements with no free length at all.

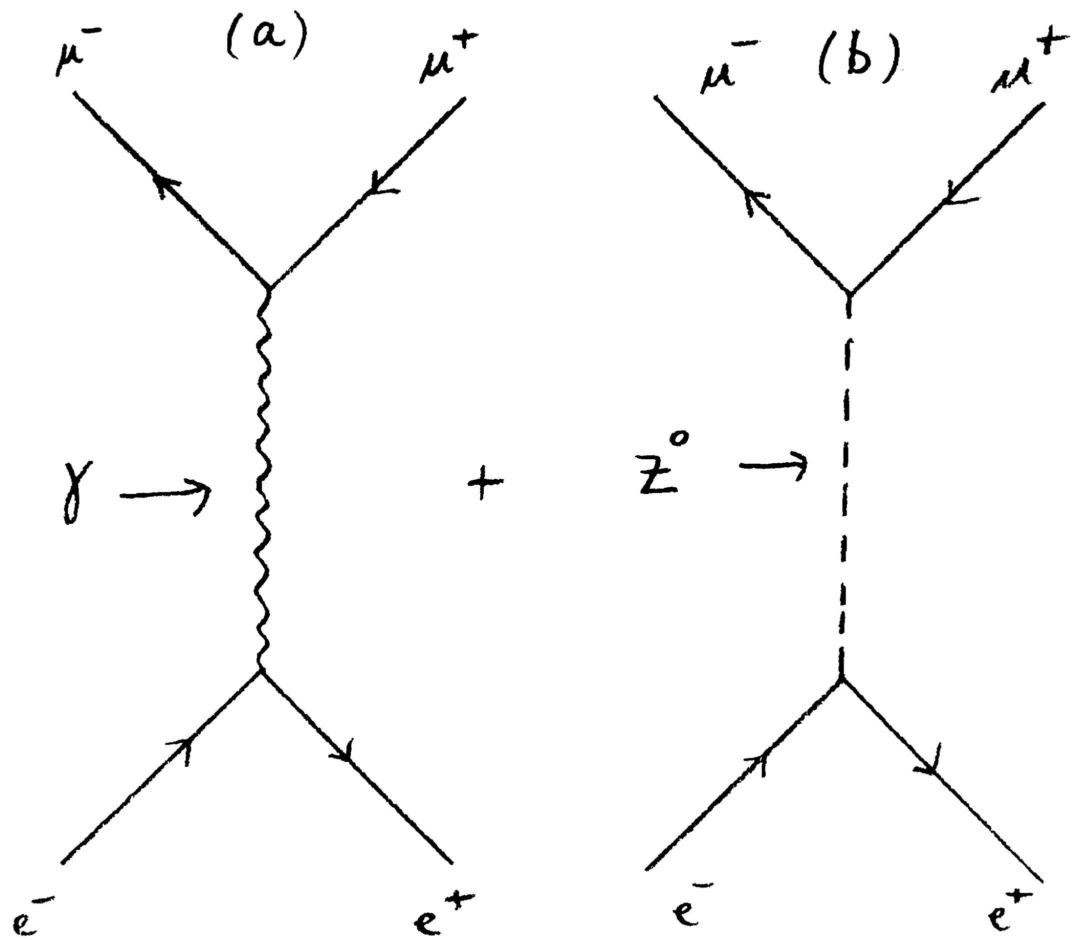
However, normalization of runs made under different conditions of beam polarization requires access to the intersection region for a luminosity monitor which presumably implies the detection of secondaries other than muons.

REFERENCES

1. F. A. Berends, K.J.F. Gaimers, R. Gastmans, Nuclear Physics B, B63, 381 (1973).

2. R. W. Brown, V. K. Cung, K. O. Mikaelian, E. A. Paschos; Physics Letters B, 43B, 403 (1973).

FIGURE 1



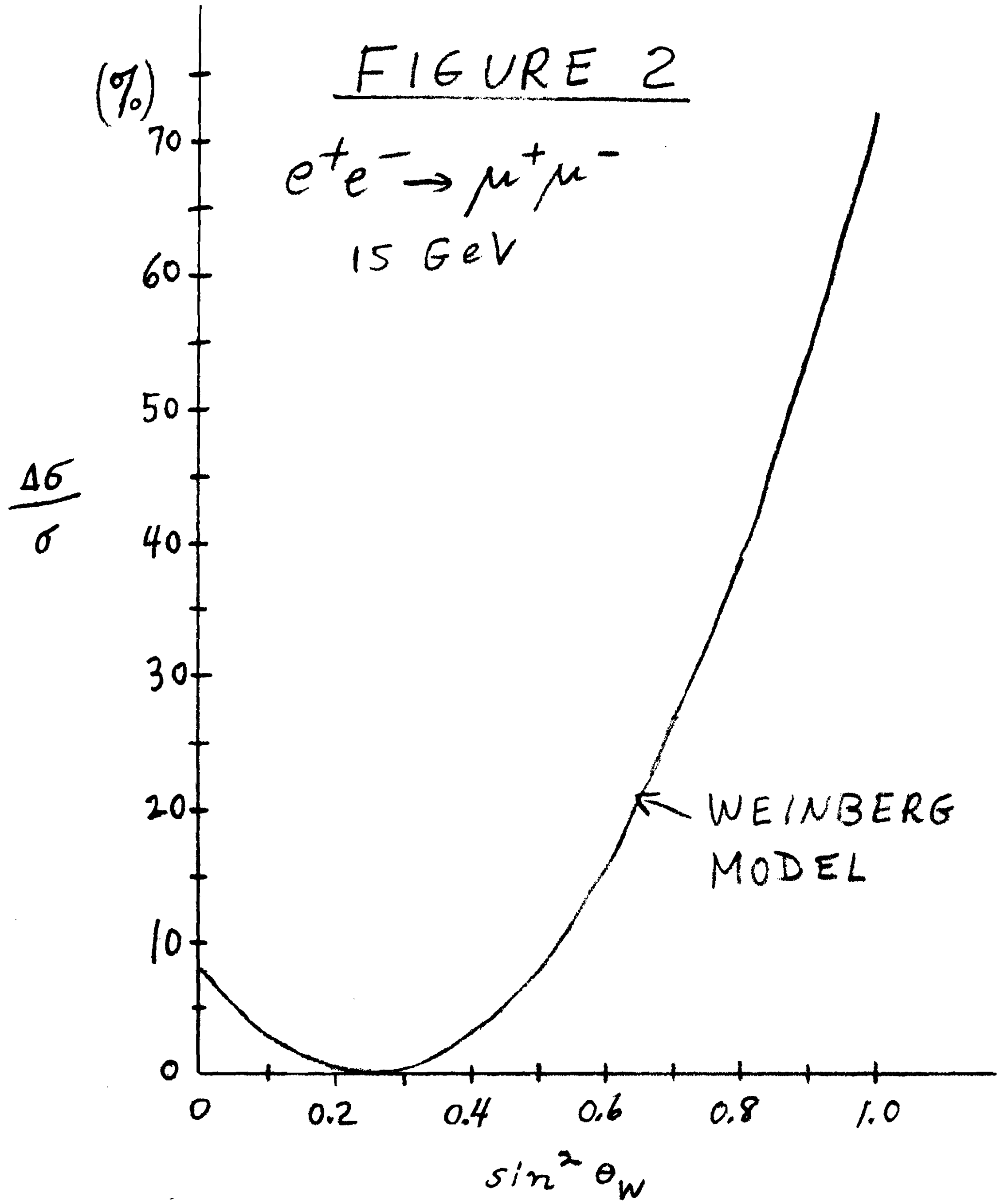


FIGURE 2
$e^+e^- \rightarrow \mu^+\mu^-$
15 GeV
(%)
70
60
50
40
30
20
10
0
$\frac{\Delta\sigma}{\sigma}$
WEINBERG MODEL
0
0.2
0.4
0.6
0.8
1.0
$\sin^2\theta_W$

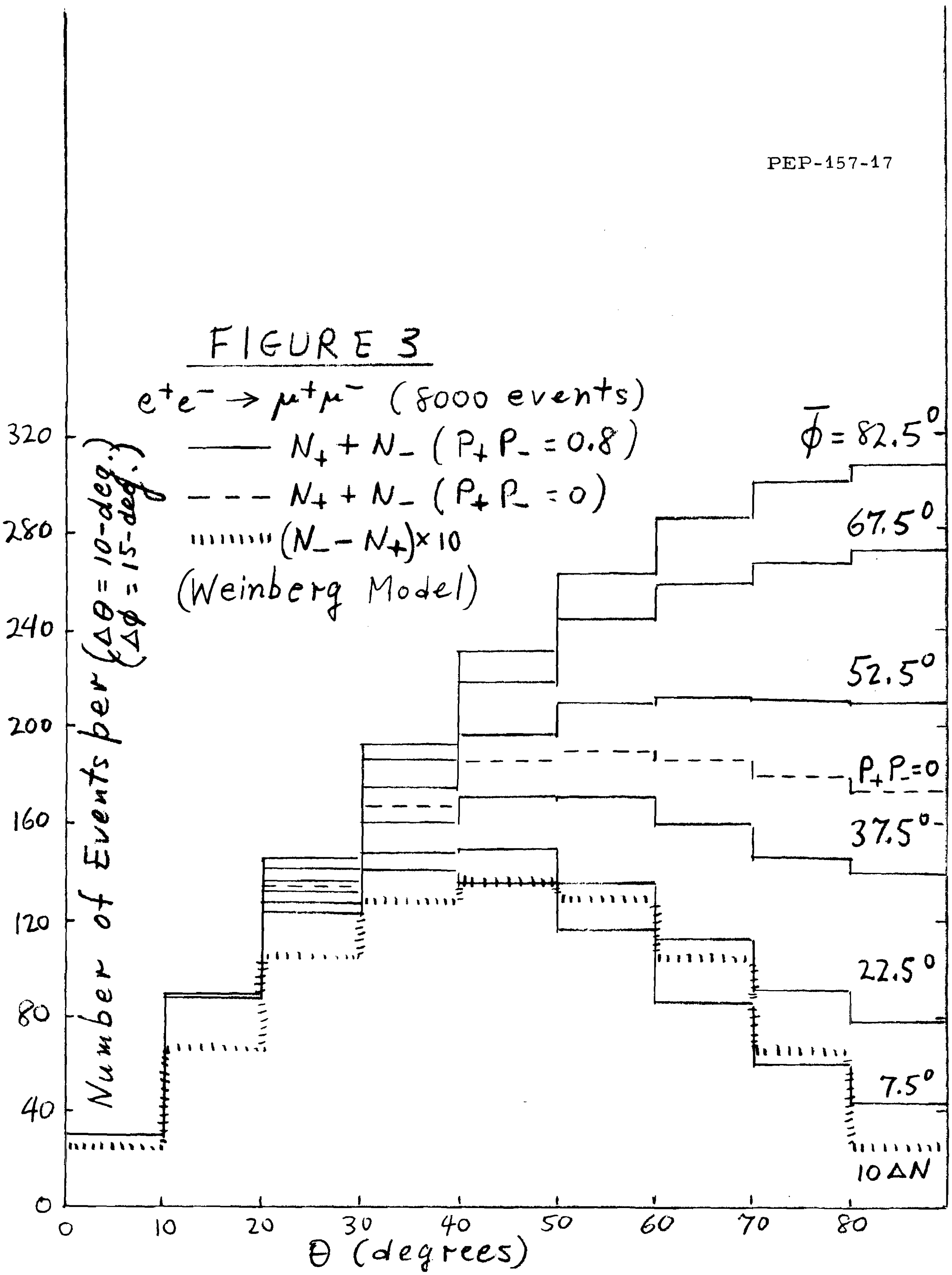
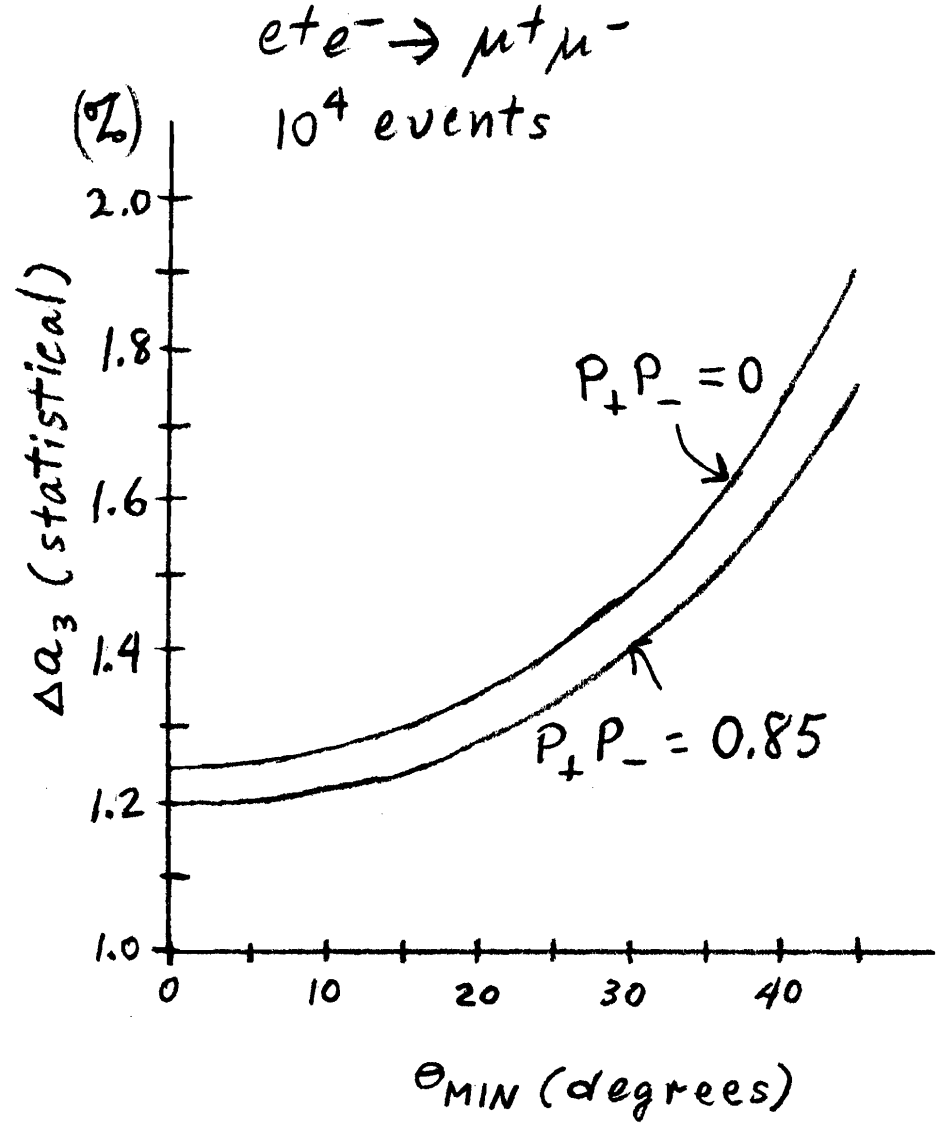


FIGURE 3
e+e− → μ+μ− (8000 events)
——— N+ + N− (P+P− = 0.8)
– – – N+ + N− (P+P− = 0)
(N− − N+)×10
(Weinberg Model)
Number of Events per (Δθ = 10-deg., Δφ = 15-deg.)
θ (degrees)
φ̄ = 82.5°
67.5°
52.5°
P+P− = 0
37.5°
22.5°
7.5°
10ΔN
0
40
80
120
160
200
240
280
320
0
10
20
30
40
50
60
70
80

FIGURE 4



$e^+e^- \to \mu^+\mu^-$
10^4 events
(%)
Δa_3 (statistical)
2.0
1.8
1.6
1.4
1.2
1.0
0
10
20
30
40
$P_+P_- = 0$
$P_+P_- = 0.85$
θ_{MIN} (degrees)

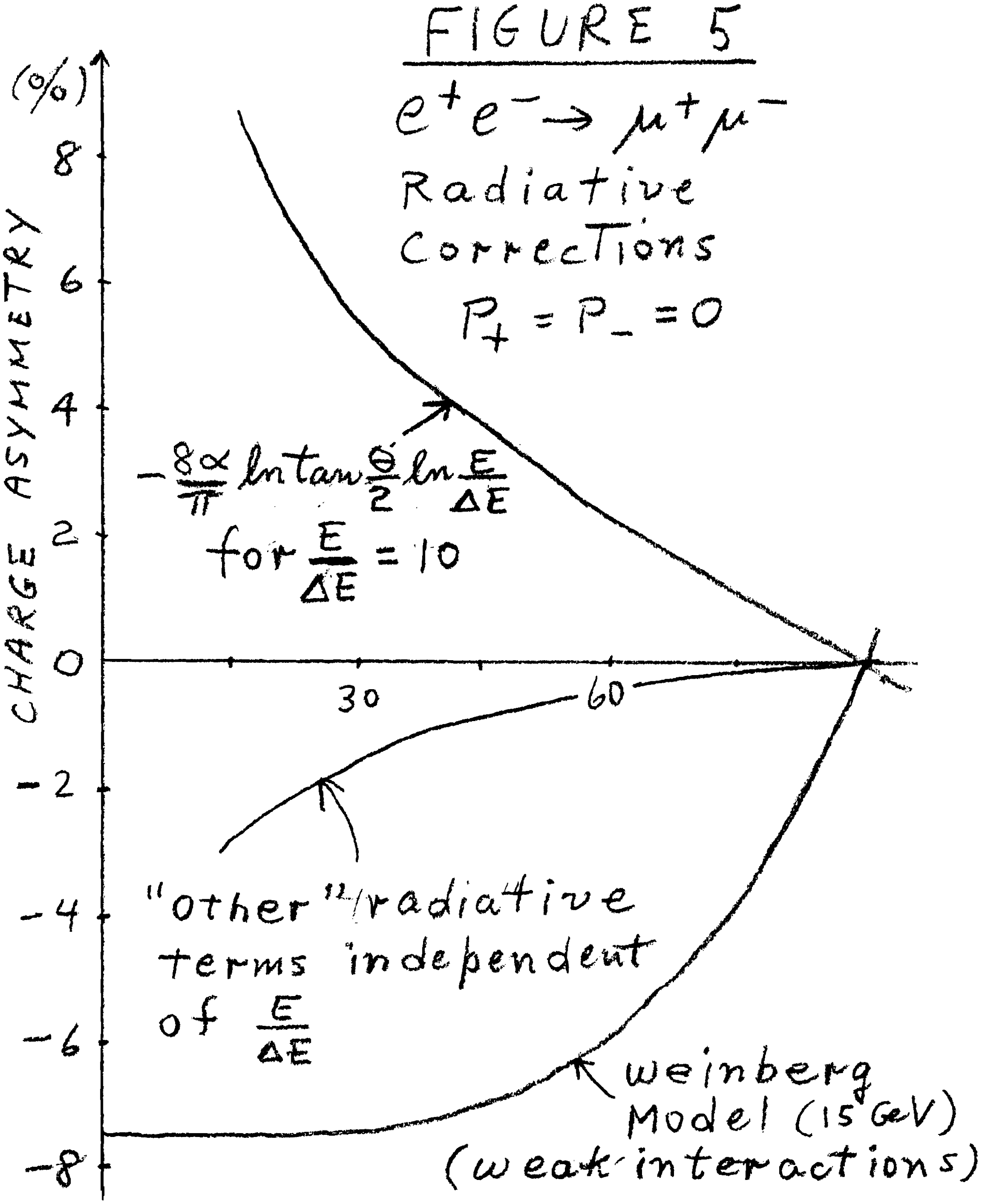


FIGURE 5
$e^+e^- \to \mu^+\mu^-$
Radiative Corrections
$P_+ = P_- = 0$
(%)
CHARGE ASYMMETRY
8
6
4
2
0
-2
-4
-6
-8
30
60
$-\frac{8\alpha}{\pi} \ln \tan \frac{\theta}{2} \ln \frac{E}{\Delta E}$ for $\frac{E}{\Delta E} = 10$
"other" radiative terms independent of $\frac{E}{\Delta E}$
Weinberg Model (15 GeV) (weak interactions)

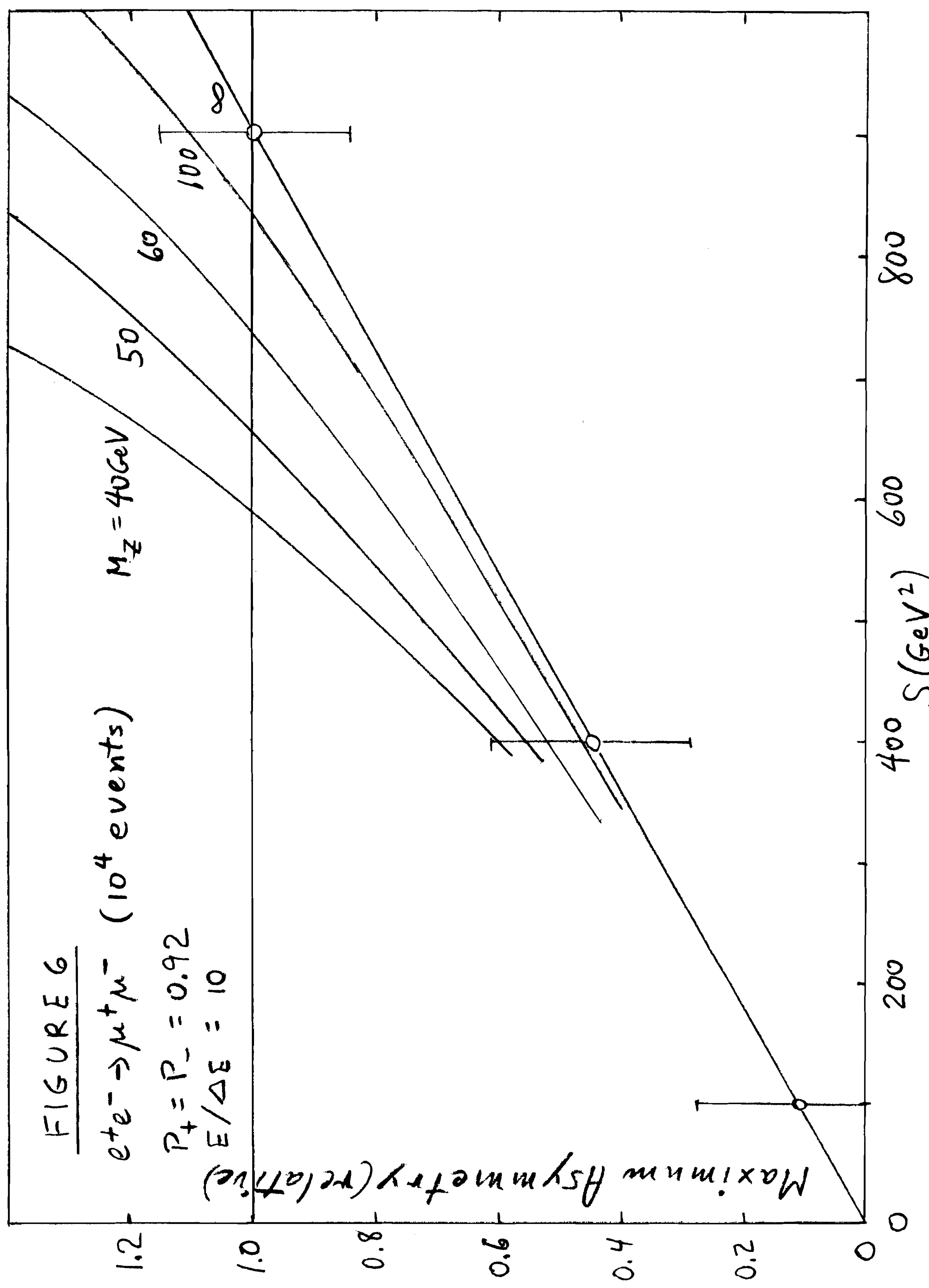


FIGURE 6
e+e- → μ+μ- (10^4 events)
P+ = P- = 0.92
E/ΔE = 10
M_Z = 40GeV
50
60
100
∞
Maximum Asymmetry (relative)
S (GeV²)
0
200
400
600
800
0.2
0.4
0.6
0.8
1.0
1.2