RMS EMITTANCE AND THE LAMPF BEAM*

by

Donald W. Mueller

University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

ABSTRACT

The LAMPF preinjector, with very careful adjustment can produce a beam with almost constant current density, uniform emittance spread across the beam, and good linearity in divergence. Intensity as a function of velocity difference from the local centroid velocity at any point is roughly Maxwellian. The rms emittance has obvious physical significance if we idealize this special case.

The velocity term of the rms emittance is the square of the rms velocity deviations from the local velocity centroid and is therefore proportional to the local temperature, i.e., that for a macroscopically small element of volumn of the beam.

The phase space can thus be populated in a physically significant way and the difficulty of assigning envelope values of r and $\dot{r}$ required by the KV concept is eliminated.

The LAMPF preinjector, with very careful adjustment, can produce a beam with unusually constant current density, uniform emittance spread across the beam, and good linearity of divergence. The beam has cylindrical symmetry up to the quadrupoles beyond the exit of the column. The emittance velocity distribution is at least roughly Maxwellian. It is interesting to discuss the idealized version of this special case in terms of the rms emittance, a constant of the motion.

The rms transverse emittance is:

$$\left\{R^2(V^2 - \dot{R}^2)\right\}^{\frac{1}{2}} \tag{1}$$

where

$$R = \left\{\frac{\sum_{i=1}^{N} r_i^2}{N}\right\}^{\frac{1}{2}} \tag{2}$$

and

$$V = \left\{\frac{\sum_{i=1}^{N} \dot{r}_i^2}{N}\right\}^{\frac{1}{2}} \tag{3}$$

r_i is the distance from the axis to the ith particle. Independence of θ is assumed.

We consider the motions of the particles in a macroscopically small element of volume of the beam. The motions can be described in terms of the velocity of the center of mass of the small element and the velocities of the individual particles relative to the center of mass of the small element.

The straight line, $v_r = (\dot{R}/R)r$ gives the radial component of the center of mass of the small element of the beam at radius r. Note that we are considering the case of uniform spatial density, uniform emittance spread and linearly increasing divergence of the local center of mass motion.

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The motion of the ith particle at radius, r_i, with radial velocity, $\dot{r}_i$, has the velocity, Δv_i, relative to the center of mass of the small element of volume around the ith particle. This relative velocity is

$$\Delta v_i = \dot{r}_i - (\dot{R}/R) r_i$$

so

$$(\Delta v_i)^2 = \dot{r}^2 - 2 r_i \dot{r}_i (\dot{R}/R) + (\dot{R}/R)^2 r_i^2 \quad .$$

Thus, the mean square velocity relative to the center of mass of the small element of volume around r_i is

$$\sum_{i=1}^{m} \frac{\Delta v_i^2}{m} = \frac{\sum_{i=1}^{m} \dot{r}_i^2}{m} - \frac{2(\dot{R}/R) \sum_{i=1}^{m} r_i \dot{r}_i}{m} + \frac{(\dot{R}/R)^2 \sum_{i=1}^{m} r_i^2}{m} \quad . \qquad (4)$$

When we multiply the term on the left by $\frac{1}{2}m_0$ we have the mean energy per ion for the motion relative to the local center of mass. This is therefore, the local temperature, $k_o T$. In our special case this temperature is uniform throughout the beam, so

$$\frac{\sum^{\text{over } m_1} \Delta v_{i_1}^2}{m_1} = \frac{\sum^{\text{over } m_2} \Delta v_{i_2}^2}{m_2} = \frac{\sum_{i=1}^{N} \Delta v_i}{N} \quad . \qquad (5)$$

Also using (2), we get by differentiating R^2

$$2R\dot{R} = \frac{\sum_{i=1}^{N} 2 r_i \dot{r}_i}{N} \quad . \qquad (6)$$

When we extend the summation of (4) over the whole beam we have

$$\frac{\sum_{i=1}^{N} \Delta v_i^2}{N} = \frac{\sum_{i=1}^{N} \dot{r}_i^2}{N} - \frac{2(\dot{R}/R) \sum_{i=1}^{N} r_i \dot{r}_i}{N} + \frac{(\dot{R}/R)^2 \sum_{i=1}^{N} r_i^2}{N} \quad . \qquad (7)$$

Thus, in accordance with (5)

$$\frac{\sum^{\text{over } m_i} \Delta v_i^2}{m_i} = \frac{\sum_{i=1}^{N} \Delta v_i}{N} = V^2 - \dot{R}^2 \quad .$$

Thus, the term $V^2 - \dot{R}^2$ of (1) is proportional to the local temperature of the beam or simply to the temperature of the beam since the postulate is that the temperature is uniform.

One way in which the LAMPF column is run is with the rms radius of the beam constant within the column. The beam current is reduced to provide focusing to compensate for the emittance spreading at the edges of the beam.