COIL CONFIGURATIONS IN SUPERCONDUCTING DIPOLES AND QUADRUPOLES*

G. Parzen and K. Jellett
Brookhaven National Laboratory
Upton, New York

Abstract

The limitations on the good-field aperture of superconducting dipoles due to approximating the cos θ current distribution using a finite number of ribbon-shaped conductors arranged in blocks are investigated. Results are presented for the good-field aperture limit due to the choice of the number of blocks per quadrant. Results are also presented for several ribbon configurations having 5, 6, and 7 blocks per quadrant, for which the lower undesired multipoles are eliminated by using either slight azimuthal displacements of the block or special multipole correction windings. A similar study is also made for quadrupoles having a cos 2θ current distribution. Results are given for quadrupoles having 2 and 3 blocks per octant.

I. Dipoles

A bending magnet with an exact cosine distribution and with a circular iron shield of infinite permeability will produce an exactly uniform magnetic field inside the current winding. However there are limitations in approximating the cosine current distribution using a finite number of current-carrying conductors, with the result that only a certain fraction of the aperture within the current winding has an acceptable uniform field. One then has the problem of arranging the finite number of current-carrying conductors so as to get the best approximation to the cosine current distribution, and thus the maximum good-field aperture.

This paper investigates the approach used by Sampson and co-workers[1] to approximate the current distribution which uses a rectangular ribbon, about 5/8 in. wide, that is wound in blocks. This geometry is roughly indicated in Fig. 1. It is likely that the same kind of considerations that enter into this approach also enter in other approaches that may use a different conductor shape.

In the geometry shown in Fig. 1, the ribbons are wound in blocks. Each block carries a total current approximately proportional to the cosine of the azimuthal angle, and the average current density in each block is also almost proportional to the cosine of the azimuthal angle.

One approach to improving the good-field aperture is to search for a choice of the number of blocks and ribbon distribution that causes most of the undesired lower multipoles in the field to vanish. The remaining important undesired multipoles which may be one or two in number can be

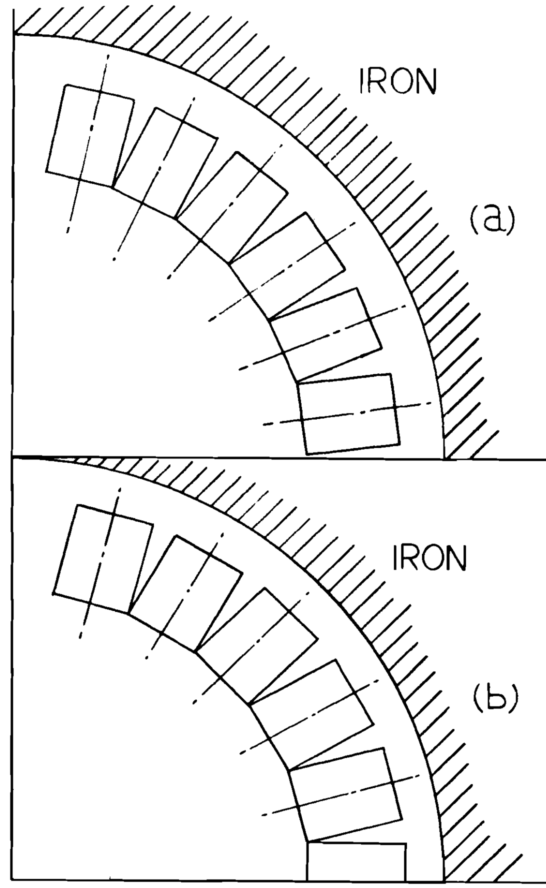

Fig. 1. Dipole geometry, (a) shows Beth arrangement, (b) shows Parzen-Jellett arrangement.

cancelled by special multipole windings designed for this purpose. For example, for a choice of 7 current blocks, there are ribbon distributions that make b_4 and b_8 vanish, where the b_n are the field multipoles defined by writing the median plane field as

$$B = B_o(1 + b_2x^2 + b_4x^4 + b_6x^6 + \dots) \qquad (1.1)$$

One can then get a fair amount of good-field aperture by using special cos 7θ and cos 3θ multipole windings to cancel out b_6 and b_2.

A second approach to improving the good-field aperture is to displace the current blocks in either the radial or azimuthal directions.

It was shown by G. Morgan that one can improve the good-field aperture considerably by introducing radial displacements, Δr_n, and azimuthal displacements, $\Delta\theta_n$, of the current blocks which are

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chosen to eliminate the undesired lower multipoles that may be present in the magnetic field. By a proper choice of the number of blocks and of the ribbon distribution, one can get an adequate good-field region using only azimuthal displacements, $\Delta\theta_n$, of the current blocks.

Calculations similar to those reported in this paper have also been done by Dahl, Morgan, and Sampson.[2,3]

1.1. Uniform Block Current Distributions

As an intermediate step, before considering the problem of current distributions wound with a rectangular ribbon, one may consider the more theoretical current distribution made out of rectangular blocks where within each block the current density is proportional to the cosine of the azimuthal angle at the center of the block.[4]

As one increases the number of blocks, one increases the region of good field. However, for a given number of blocks there appears to be a limit on the fraction of good-field aperture obtainable. Figures 1a and 1b show two block arrangements which appear to be optimum arrangements which were suggested by Beth[5] and by Parzen and Jellett.[6,7] These two block arrangements are optimum in the sense that it can be shown[5] that all undesired multipoles, b_n, in the magnetic field are absent up to $n = 4N_B - 2$. N_B is the number of blocks in one quadrant where one includes in the current blocks that carry zero current which are centered around $\theta = 90^o$.

In the two block arrangements, the width of each block is given by $\Delta\theta = \pi/2N_B$, and the current density in each block is proportional to the cosine of the azimuthal angle of the center of the block. The two arrangements can be obtained from each other by a rotation of $\Delta\theta/2$.

The fraction of the aperture within the current winding that has an acceptable good field is plotted against the number of blocks, N_B, in a quadrant in Fig. 2 for the Beth and Parzen-Jellett block arrangements. The results shown are for the case where the blocks have an inner radius of R_c = 1.5 in., and the radial block thickness, H_c, is 5/8 in. A circular iron shield is present with infinite permeability, whose inner surface is 1/8 in. past the current blocks. The results depend on the thickness of blocks, or on the ratio H_c to R_c; however this dependence is not very marked.

The criterion for the good-field region used was the restriction on the second derivative of the field that $B''/B_o < 1 \times 10^{-3}$ in.^{2} on the median plane. Changing the criterion would not change the results shown in Fig. 2 very much as when the median plane field starts to depart from uniformity, it does so suddenly and rapidly.

One sees that using N_B = 4 one can get about 54% of the aperture, and using N_B = 7 one can get about 70% of the aperture. These results for the theoretical current distribution may be regarded as ideal results which are close to the best one can do if one approximates these blocks by winding them out of a ribbon or other kind of conductor.

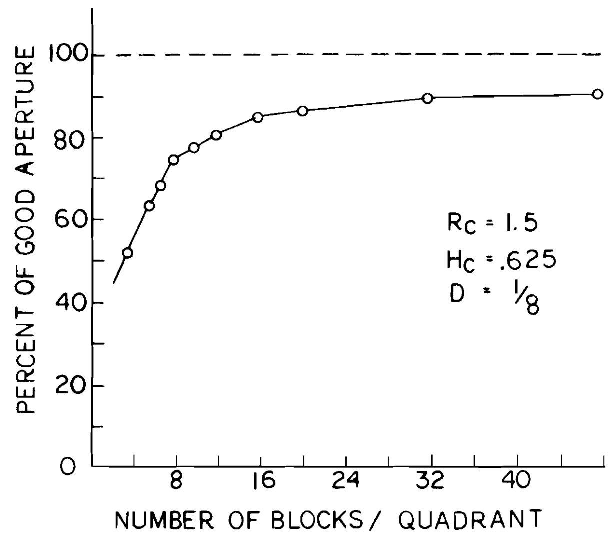

Fig. 2. The percent good aperture in dipoles with a uniform block current distribution vs the number of blocks per quadrant. R_c is the inner radius of the coil, H_c is the radial thickness of the blocks, d is the distance between the blocks and the iron shield, and all dimensions are in inches.

1.2. Integerized Block Current Distribution

Another theoretical current distribution that is worth considering before attacking the problem of the ribbon-wound current distribution, is to take the block current distribution discussec in Sec. 1.1 and to make the current densities in the blocks in the ratio of whole integers, rather than being exactly proportional to the cosine of the azimuthal angle. This theoretical example shows one effect of using a ribbon-wound current distribution, where the current density is proportional to the number of ribbon turns in each block, and thus with a finite number of ribbon turns there is a limit on how close one can get to an exact cosine distribution.

The current densities in the blocks can be made in the ratio of whole integers by taking the current density S_n in block n as

$$S_n = (A \cos \theta_n + 0.5)_{\text{integer part}} , \qquad (1.2)$$

where θ_n is the center of the block and A is some integer like 15. S_n can be thought of as the number of ribbons in each block, if the block is wound out of ribbons.

Because of the departure from the pure cosine distribution, it is no longer true that the first $4N_B - 2$ multipoles except the dipole are zero. The extent to which the lower multipoles are

TABLE I. Results for two dipoles having 7 blocks and 5 blocks per quadrant for which some multipoles vanish. The b_m are the multipoles in units of 10^{-3} in.$^{-m}$.

$b_m/10^{-3}$	7-Block Dipole		5-Block Dipole	
	Integerized Uniform Blocks	Ribbon Blocks	Integerized Uniform Blocks	Ribbon Blocks
b_2	0.000	-0.707	-1.065	-2.735
b_4	0.000	-0.014	0.000	0.037
b_6	0.179	0.168	-0.073	-0.060
b_8	0.000	-0.011	0.012	0.011
b_{10}	0.008	0.004	0.004	0.003
b_{12}	0.008	0.003	-0.002	0.002
b_{14}	0.000	0.003	0.000	0.005
b_{16}	-0.001	0.000	0.000	-0.029
b_{18}	0.000	0.000	0.009	0.189
Percent good aperture b_2, b_6, corrected		68%	Percent good aperture b_2, corrected	64%
Ribbon Distribution	15,14,13,11,9,6,3			22,21,18,13,7
Total Ribbons/Quadrant		71		70

excited by integerizing the current distribution can be estimated by doing a simple Fourier analysis of the current distribution along the inside circle which can be done analytically.

There are interesting cases where some of the lower multipoles vanish. The most interesting case seems to be when there are 7 current blocks in a quadrant, and there are then choices of the ribbon distribution corresponding to choices of A in Eq. (1.2) which make b_2, b_4, and b_8 vanish. This indicates that for the case of 7 blocks, one may be able to obtain fairly large good-field regions by introducing a separate cos 7θ winding to eliminate b_6.

In Table I results are given for the field shape for the case of a 7-block Beth distribution, and one sees that by cancelling out the b_6 and b_2 terms one can get a 63% good-field region, where, according to the results of Sec. 1.1, 73% is about the maximum achievable with $N_B = 7\frac{1}{2}$.

In Table I, results are given for the integerized uniform current block case, and for the case when the blocks are assumed to be wound of ribbons. One notes that the vanishing of b_4 and b_8 is not disturbed very much when ribbons are used instead of uniform current blocks.

Another moderately interesting case is the 5-current block in a quadrant case. There is here a choice of A that makes b_4 vanish, and one may be able to obtain a reasonable good-field region by introducing a separate cos 3θ winding to eliminate b_2. Table I shows the field shape for the case of a 5-block Parzen-Jellett distribution, and one sees that by cancelling out the b_2 term one can get a 62% good-field region, although this 62% is in part due to the helpful cancellation caused by the b_6 term.

1.3. Stacked Ribbon Current Distribution

One way[1] of approximating a cosine current distribution is to wind the coil in blocks out of a superconducting ribbon whose width is the thickness of the blocks, and the number of ribbons in a block is proportional to the cosine of the azimuthal angle n. For a choice of a given number of blocks, the results of Sec. 1.1 give an upper limit to the present good-field aperture one can achieve. For example, using 6 blocks in the Beth arrangement, one has an upper limit of about 68% good aperture. One does not usually obtain this upper limit, primarily because the integer number of ribbons per block cannot approximate the cos θ distribution exactly.

The proposed[8] ISABELLE dipoles require a 5 cm good aperture for an 8 cm inner diameter coil or 62.5%. Thus 6 blocks per quadrant may be sufficient for the ISABELLE dipoles.

A Six-Block Dipole. Figure 3 shows a possible 6-block dipole with the ribbon distribution of 14, 13, 12, 9, 7, 3, or a total of 58 ribbons in a quadrant. The ribbons are all parallel to each other and parallel to the radius to the center of the block, and each rectangular ribbon has one corner touching the inside circle of the coil. In Fig. 3, ten inert non-current-carrying ribbons have

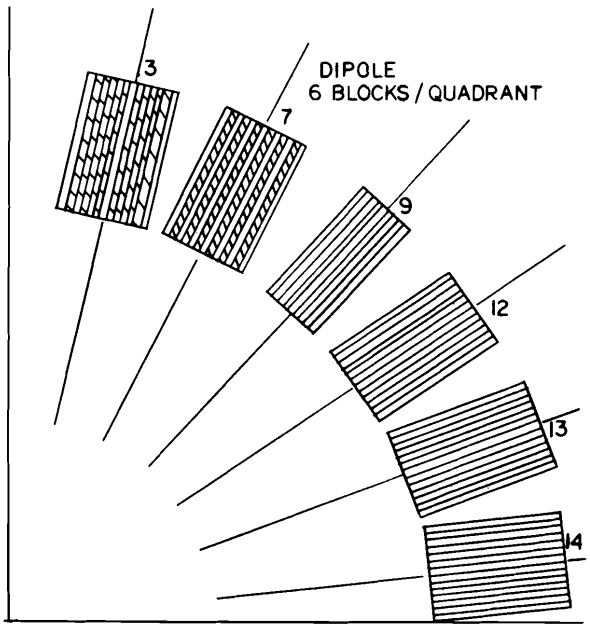

Fig. 3. A six-block dipole. The crosshatched ribbons are inert non-current-carrying ribbons used as spacers.

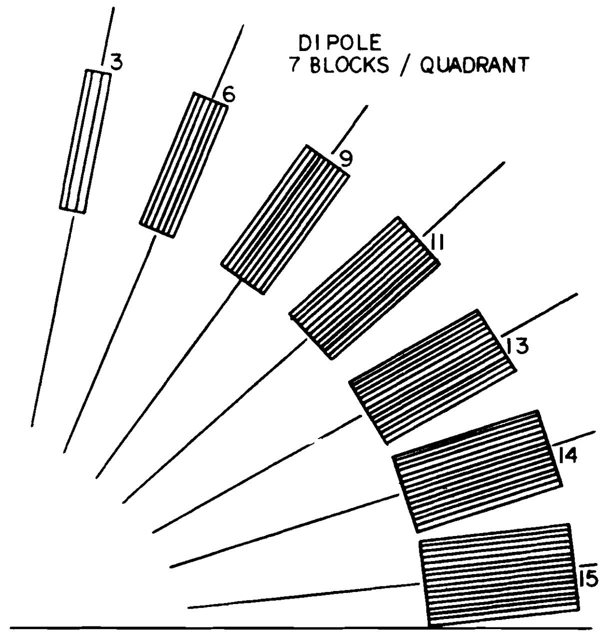

Fig. 4. A seven-block dipole showing the ribbon distribution.

been inserted in the sixth block, and six inert ribbons have been inserted in the fifth block. Very often the addition of inert ribbon spacers does not change the results very much, but in this case it does. With 6 blocks in a quadrant one can eliminate the multipole b_2 up to b_{12} by slight azimuthal displacements of the blocks as was demonstrated by G. Morgan. The first possibly troublesome multipole is b_{14}. The azimuthal displacement of the blocks introduces an appreciable b_{14}, of the order of -0.012×10^{-3}. Introducing the inert ribbons reduces the amount of azimuthal correction required and thus the b_{14} introduced by the correction. In addition the amount of b_{14} present before correction is reduced. The ribbon distributions in the sixth and fifth blocks are given by 1000001000001 and 1010101010101, where 0 indicates an inert ribbon and 1 indicates a current-carrying ribbon. The improvement of the field produced by introducing inert ribbons in the 6-block case was pointed out by Dahl.[2]

One can eliminate the b_2 up to b_{12} multipoles by slight azimuthal displacements of the 6 blocks and the results are given in Table II. A good-field aperture of 68% is obtained. The ribbon thickness used was 0.0258 in. The inner radius of the current blocks is $R_c = 1.5$ in. and the width of the ribbon is $H_c = 0.625$ in.

A Seven-Block Dipole. Figure 4 shows a possible 7-block dipole with the ribbon distribution 15, 14, 13, 11, 9, 6, 3 and with 71 ribbons in a quadrant.

One can eliminate all the multipoles b_2 to b_{14} by a slight azimuthal displacement of the seven blocks. In this case no inert ribbons were added since it did not appear that inert ribbons would improve the good-field results. Inert ribbons could be added for mechanical support reasons without changing the results very much.

The results are shown in Table II. A good-field aperture of 76% is obtained. Seven-block coils having a similar good-field aperture with less ribbon turns per quadrant and a thicker ribbon are possible. The inner radius of the current blocks is $R_c = 1.5$ in. and the width of the ribbons is $H_c = 0.625$ in.

II. Quadrupoles

A quadrupole with an exact cos 2θ current distribution and with a circular iron shield of infinite permeability will produce an exact quadrupole magnetic field inside the current winding. However, there are limitations in approximating the cos 2θ current distribution using a finite number of ribbon-shaped conductors arranged in blocks, each block having an average current density proportional to cos 2θ. This geometry is indicated in Fig. 5.

It is found that using 2 blocks in an octant can give up to about 67% good-field aperture which may be sufficient for the proposed[8] ISABELLE quadrupoles. Three blocks per octant can give up to about 77% good aperture. The results given below

TABLE II. Results for two dipoles having 6 and 7 blocks per quadrant. The b_m are the multipoles in units of 10^{-3} in.$^{-m}$. The $\Delta\theta_n$ are the block displacements in milliradians required to eliminate the lowest 6 or 7 undesired multipoles. W is the thickness of the ribbon in inches. The ribbon current is the current required in amperes to obtain a field of 40 kG if no saturation of the iron is present. The ribbon is assumed to be 5/8 in. wide, the inner radius of the coil is 1.5 in. and the coil is 1/8 in. from the iron shield.

$b_m/10^{-3}$	6-Block Dipole		7-Block Dipole	
	Uncorrected	Corrected	Uncorrected	Corrected
b_2	-0.757	0.026	-0.707	0.008
b_4	-0.881	-0.006	-0.014	0.009
b_6	0.209	0.007	0.168	0.001
b_8	-0.035	-0.004	-0.011	-0.001
b_{10}	0.027	0.001	0.004	0.001
b_{12}	-0.001	0.000	0.003	0.000
b_{14}	0.003	-0.002	0.003	0.000
b_{16}	0.000	0.000	0.000	-0.001
b_{18}	-0.003	-0.003	0.000	-0.001
b_{20}	0.002	0.003	0.000	0.000
Ribbon Distribution	14,13,12,9,7,3		15,14,13,11,9,6,3	
$\Delta\theta_1$		0.627		0.99
$\Delta\theta_2$		-0.937		-0.46
$\Delta\theta_3$		2.861		-0.92
$\Delta\theta_4$		9.431		-2.18
$\Delta\theta_5$		18.93		0.78
$\Delta\theta_6$		43.63		4.72
$\Delta\theta_7$				6.20
Percent Good Aperture	21	68	42	76
W		0.0258		0.0209
Total Ribbon/Quadrant		58		71
Ribbon Current for 40 kG		3042		2480

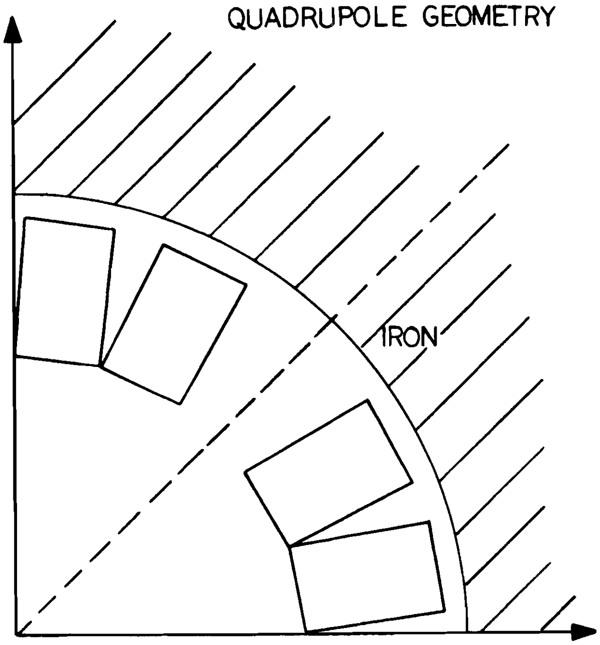

Fig. 5. Quadrupole geometry.

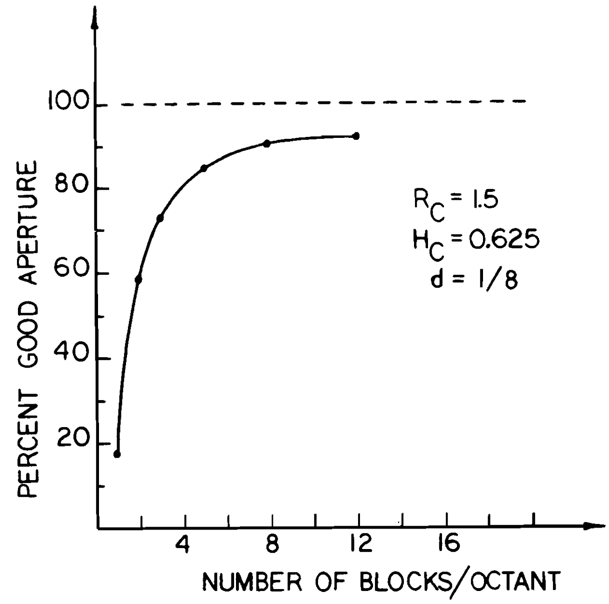

Fig. 6. The percent good aperture in quadrupoles with a uniform block current distribution vs the number of blocks per octant. R_c is the inner radius of the coil in inches, H_c is the radial thickness of blocks in inches, and d is the distance between the blocks and the iron shield in inches.

for actual ribbon distributions give a 66% good-field aperture for a 2-block case with a 21, 13 ribbon distribution, and an 82% good-field aperture for a 3-block case with a 14, 11, 6 ribbon distribution.

2.1. cos 2θ Block Current Distribution

For a given number of blocks per octant, one can find what appears to be a good approximation to the largest good-field aperture obtainable by considering the theoretical current distribution made out of rectangular blocks,[9] where within each block the current density is proportional to cos 2θ and each block has an equal azimuthal extent, including the block that may carry zero current.

In Fig. 6, the fraction of the aperture within the current winding that has an acceptable good field is plotted against the number of blocks, N_B, in an octant. In order to make all possible block arrangements lie on the same curve, N_B includes the blocks carrying zero current. Thus the 2-block arrangement shown in Fig. 5 has $N_B = 2\frac{1}{2}$ since it has in an octant a half block carrying zero current.

The criterion for the good field region used was the restriction that the gradient change across the aperture $\Delta B'/B'$, obeys $\Delta B'/B' < 10^{-3}$. Changing the criterion would not change the results shown in Fig. 6 very much, as when the median plane field starts to depart from being linear, it does so suddenly and rapidly.

From Fig. 6, one sees that using 2 blocks per octant in the Beth arrangement, $N_B = 2\frac{1}{2}$, gives 67% good-field aperture, while 3 blocks would give 77%. These results are not very different from those for the dipole if one compares geometries having equal numbers of blocks in a quadrant or octant.

The ISABELLE storage ring quadrupoles which require a 62% good-field aperture could then possibly use only 2 blocks per octant, and 3 blocks per octant would certainly be sufficient.

2.2. Stacked Ribbon Current Distribution

One way[1] of approximating a cos 2θ current distribution is to wind the coils in blocks out of a superconducting ribbon whose width is the thickness of the blocks, and the number of ribbons in a block is proportional to cos 2θ as is shown in Figs. 7 and 8. For a choice of a given number of blocks, the results of Sec. 2.1 give an approximate upper limit to the percent good-field aperture one can achieve. For example, using 2 blocks in an octant in the Beth arrangement, one has an upper limit of about 67% good aperture, while using 3 blocks per octant yields 77% good aperture.

Two-Block Quadrupoles. Figure 7 shows a possible 2-block quadrupole with 21 ribbons in the first block and 13 ribbons in the second block, and 34 ribbons in an octant.

In Fig. 7, eight inert non-current-carrying ribbons have been inserted in the second block so that both blocks are the same size. Generally

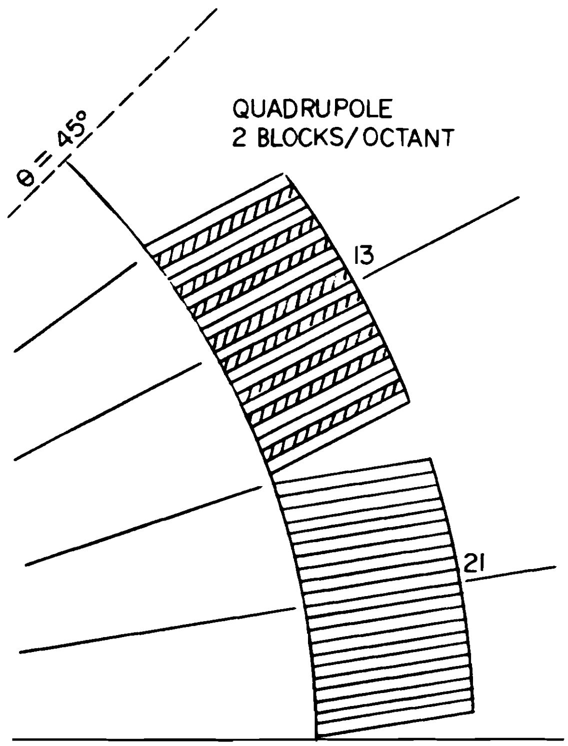

Fig. 7. A two-block quadrupole showing the ribbon distribution. The crosshatched ribbons are inert non-current-carrying ribbons used as spacers.

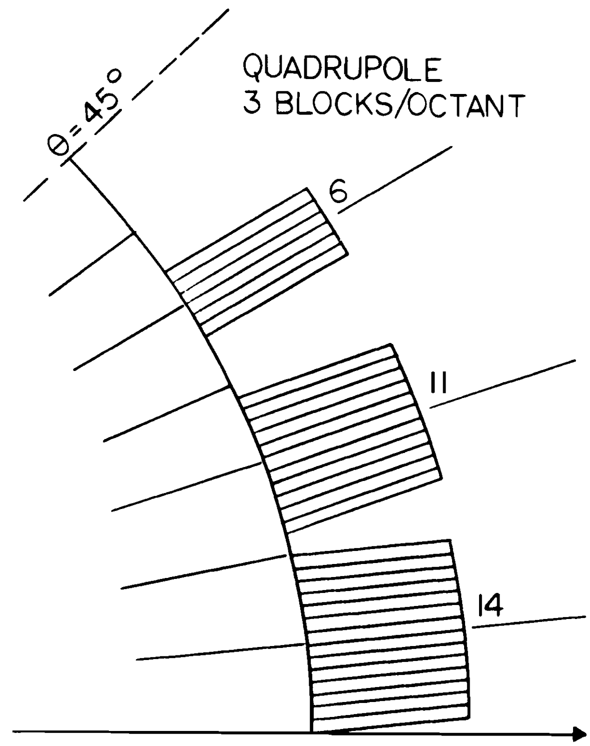

Fig. 8. A three-block quadrupole showing the ribbon distribution.

speaking, the addition of inert ribbon spacers does not change the results very much, but in this case it does. With only 2 blocks in an octant one can eliminate the b_5 and b_9 multipoles by slight azimuthal displacements of the blocks as was demonstrated by G. Morgan. (Note that because of the symmetry only the multipoles b_1, b_5, b_9, b_{13}, etc. can be present.) The first possibly troublesome multipole is b_{13}, and in this case because the block width is comparable to $2\pi/14$, the addition of the inert ribbons considerably reduces the size of b_{13}. The eight inert ribbons were inserted in the second block in the configuration 101101011010110101101, where 0 indicates an inert ribbon and 1 indicates a current-carrying ribbon.

One can eliminate the b_5 and b_9 multipoles by slight azimuthal displacements of the 2 blocks and the results are given in Table III. A good-field aperture of 66% is obtained. The ribbon thickness used was 0.021 in.

One can reduce the number of ribbons/octant and obtain a larger ribbon thickness. Another 2-block configuration considered was one having the ribbon distribution of 16, 10 and with 26 ribbons/octant. The ribbon thickness is then 0.027 in. The results for this case are given in Table III. A good-field aperture of 66% is obtained. The second block contains six inert ribbons which are distributed as 1011010110101101.

A Three-Block Quadrupole. Figure 8 shows a possible 3-block quadrupole with 31 ribbons in an octant and a ribbon distribution of 14, 11, 6.

In this case, no inert ribbon spacers were added to block 2 or block 3 since it did not appear that inert ribbons would improve the good field result. Inert ribbon spacers could be added for mechanical support reasons, without changing the results very much.

One can eliminate the b_5, b_9, and b_{13} multipoles by a slight azimuthal displacement of the 3 blocks, and the results are given in Table III. A good-field aperture of 82% is obtained. The ribbon thickness used was 0.023 in.

Acknowledgements

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TABLE III. Results for 3 quadrupoles having 26, 31, and 4 ribbon turns per octant. The b_m are the multipoles in 10^{-3} in.$^{-m}$. The $\Delta\theta_n$ are the block displacements in milliradians required to eliminate the lowest 2 or 3 undesired multipoles. W is the thickness of the ribbon in inches. The ribbon current is the current required in amperes to obtain a gradient of 10 kG/cm. The ribbon is assumed to be 5/8 in. wide, the inner radius of the coil is 1.5 in., and the coil is 1/8 in. from the iron shield.

$b_m/10^{-3}$	2-Block Coil		2-Block Coil		3-Block Coil	
	Uncorrected	Corrected	Uncorrected	Corrected	Uncorrected	Corrected
b_5	0.134	-0.0006	-0.1633	-0.0021	-0.2563	0.0157
b_9	0.000	-0.0039	0.0000	0.0008	0.0037	-0.0006
b_{13}	-0.0133	-0.0162	-0.0123	-0.0089	0.0001	0.0000
b_{17}	-0.0504	-0.0508	-0.0549	-0.0546	-0.0005	-0.0006
b_{21}	-0.0006	-0.0005	-0.0013	-0.0013	0.0056	0.0054
b_{25}	-0.0001	-0.0001	-0.0001	0.0000	-0.0016	-0.0018
Ribbon Distribution	21, 13		16, 10		14, 11, 6	
$\Delta\theta_1$		0.362		-0.413		-0.289
$\Delta\theta_2$		0.465		-0.665		-0.700
$\Delta\theta_3$		--		--		-0.681
Percent Good Aperture	66		66		82	
W		0.021		0.027		0.023
Total Ribbons/Octant	34		26		31	
Ribbon Current for 10 kG/cm	3330		4350		3680	