TRANSFER FUNCTION FOR NAL MAIN RING
DIPOLE AND QUADRUPOLE MAGNETS

S. C. Snowdon
National Accelerator Laboratory*
Batavia, Illinois 60510

Abstract

The transfer function between the Fourier amplitudes of the central magnetic field and the excitation current has been calculated for NAL main-ring dipoles. A similar analysis also has been employed to obtain the transfer function between the Fourier amplitudes of the central gradient and the excitation current of the NAL main-ring quadrupoles. In each case the eddy currents in the laminations, vacuum chamber, and conductors are estimated and their effects in a magnetic circuit are found. The analysis also gives the impedances looking into the coil terminals.

Magnetic Circuit (Dipole)

Figure 1 indicates the magnet cross section under consideration. Let b designate the half width of the pole, h the half-gap height, p the effective pole height, w the yoke width assumed the same in both the top and side yoke, and ℓ the average length of the flux path through the top and side yokes. The effective pole half width is taken to be b+h. Hence the flux density B_o in the top and side yoke is given approximately by

$$\int_o^{b+h} H_y(x,h)\,dx = wB_o. \qquad (1)$$

Recognizing that the flux must be continuous across the boundary y = h, it is reasonable to let B in the pole vary with y according to

$$B(y) = \frac{1}{b}\int_o^b H_y(x,h)\,dx + \left[B_o - \frac{1}{b}\int_o^b H_y(x,h)\,dx\right]\cdot\frac{y-h}{p-h}. \qquad (2)$$

Appendix A indicates that the effect of eddy currents in magnet laminations is to introduce a sheet current in the z-direction lining the coil window of total amount $\int i_I d\ell$ integrated over the mean path length where

$$i_I = \frac{\lambda\delta}{8\pi\mu}\cdot\frac{\cosh\lambda\frac{\delta}{2}-1}{\sinh\lambda\frac{\delta}{2}}\,B, \quad \text{(emu)} \qquad (3)$$

δ is the lamination thickness, μ the permeability of the iron and

$$\lambda^2 = 4\pi\mu j\omega\sigma. \quad \text{(emu)} \qquad (4)$$

The conductivity of the iron is σ and ω is the angular frequency with which all magnetic and electrical field quantities are assumed to vary.

Appendix B indicates that eddy currents flowing in the vacuum chamber are represented by current sheets flowing in the z-direction of amount

$$i_c(x,y) = j\omega s U(x,y) \qquad (5)$$

where U(x,y) is the flux function. The coordinates x and y are given values only at the location of the chamber which will be assumed rectangular of half width a and half height h. Surface conductivity of the vacuum chamber is designated by s.

One may complete the magnetic circuit description using the scalar potential function V(x,y) corresponding to U(x,y) and the notion of a mean flux path in the iron. Thus for 0<x<a and (emu)

$$V(x,h) + \frac{1}{\mu}\int_h^p B(y)\,dy + \frac{\ell-p+h}{\mu}B_o = 2\pi NI - 4\pi\int_h^p i_I(y)\,dy - 4\pi(\ell-p+h)i_I - 4\pi\int_x^a i_c(x,h)\,dx - 4\pi\int_o^h i_c(a,y)\,dy. \quad (6)$$

For a<x<b+h the terms in i_c are dropped. Note that I is the excitation current and N the number of turns in the full gap. Note also that $i_I = i_I(p)$.

Appendix C gives the fields to be used in the region of the vacuum chamber. Outside the chamber the fields are obtained approximately as follows. First $4\pi i_c(a,h)$ is added to the vertical field

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at $x = a$, $y = h$. This field is matched to the solutions within the conductors as given in Appendix D. In summary, Appendices C and D and the continuation condition give

$$H_x(x,y) = \begin{cases} H_o \dfrac{\sum Q_n C_n \sinh Q_n x \sin Q_n y}{\sum Q_n C_n} & [0<x<a] \\ \text{Neglected} & [a<x<b+h] \end{cases} \quad (7)$$

$$H_y(x,y) = \begin{cases} H_o \dfrac{\sum Q_n C_n \cosh Q_n x \cos Q_n y}{\sum Q_n C_n} & [0<x<a] \\ A_1 \sinh P(x-a) + B_1 \cosh P(x-a) & [a<x<a+c] \\ A_2 \sinh P(x-a-c) + B_2 \cosh P(x-a-c) & \\ \text{etc.} & [a+c<x<a+2c] \end{cases} \quad (8)$$

where

$$P^2 = 4\pi\sigma_c j\omega, \qquad \text{(emu)} \quad (9)$$

$$Q_n \tan Q_n h = 4\pi j\omega s, \qquad \text{(emu)} \quad (10)$$

$$A_n \sinh Pc = -\frac{4\pi I}{h}\left[n-(n-1)\cosh Pc\right] - B_1(\cosh Pc-1), \quad (11)$$

$$B_n = -\frac{4\pi I}{h}(n-1) + B_1, \quad (12)$$

$$B_1 = H_y(a,h) + 4\pi j\omega s U(a,h) \quad (13)$$

$$C_n\left(Q_n \cosh Q_n a + 4\pi j\omega s \sinh Q_n a\right) \cdot \left(1 + \frac{\sin 2Q_n h}{2Q_n h}\right) = \frac{1}{h} \cdot \frac{\sin Q_n h}{Q_n h}. \quad (14)$$

Appendix C gives B_1 in terms of H_o, Q_n, and C_n. In Eqs. (7-14) σ_c is the conductivity of the copper conductors, c is the horizontal width of the conductor, and H_o is the central magnetic field.

The magnetic circuit equations may now be evaluated. For $x = a$, Eq. (6) gives

$$V(a,h) + 4\pi j\omega s \int_0^h U(a,y)\,dy + \frac{k+1}{\mu}\left[\ell - \frac{1}{2}(p-h)\left(1-\frac{w}{b}\right)\right]B_o - \frac{k+1}{\mu}(p-h)\left[\left(B_{n_1+1}+\ldots B_{n_2}\right)\frac{\sinh Pc}{Pb} + \left(A_{n_1+1}+\ldots A_{n_2}\right)\cdot\frac{\cosh Pc-1}{Pb}\right] = 2\pi NI, \quad (15)$$

where the expression in V and U is evaluated in Appendix C and n_1 and n_2 are the integer parts of $(b-a)/c$ and $(b-a+h)/c$. For convenience

$$k = \frac{\lambda\delta}{2} \cdot \frac{\cosh\lambda\frac{\delta}{2} - 1}{\sinh\lambda\frac{\delta}{2}}. \quad (16)$$

The other circuit equation, Eq. (1), becomes

$$U(a,h) + \left(B_1+\ldots B_{n_2}\right)\frac{\sinh Pc}{P} + \left(A_1+\ldots A_{n_2}\right) \cdot \frac{\cosh Pc-1}{P} = wB_o. \quad (17)$$

After using Eqs. (11-13) and rearranging, Eqs. (15) and (17) become

$$V(a,h) + 4\pi j\omega s \int_0^h U(a,y)\,dy - \frac{(k+1)(p-h)\left(n_2-n_1\right)(\cosh Pc-1)}{\mu P b \sinh Pc} \cdot \left[H_y(a,h) + 4\pi j\omega s U(a,h)\right] + \frac{k+1}{\mu}\left[\ell - \frac{1}{2}(p-h)\left(1-\frac{w}{h}\right)\right]B_o = 2\pi I\left[N - \frac{(k+1)(p-h)\left(n_2^2-n_1^2\right)(\cosh Pc-1)}{\mu h P b \sinh Pc}\right], \quad (18)$$

and

$$U(a,h) + 2n_2\frac{(\cosh Pc-1)}{P\sinh Pc} \cdot \left[H_y(a,h) + 4\pi j\omega s U(a,h)\right] - wB_o = 4\pi I n_2^2 \frac{(\cosh Pc-1)}{Ph\sinh Pc}. \quad (19)$$

Appendix C relates the expressions in U, V and H_y to H_o. Thus Eqs. (18) and (19) provide two linear equations which may be solved simultaneously for H_o and B_o in terms of I. Thus

$$H_o = TI, \tag{20}$$

where T, the transfer function, is obtained by eliminating B_o from Eqs. (18) and (19). In addition, the sextupole term introduced by the vacuum chamber may be found by differentiating Eq. (8) twice. Thus, using Eq. (20)

$$H_o'' = T\frac{\sum Q_n^3 C_n}{\sum Q_n C_n} I. \tag{21}$$

ac Impedance of Coil

The inductance of the windings may be found by solving Eqs. (18) and (19) for B_o and using it to construct the flux linkage per unit current. Thus

$$L = \frac{2NwB_o}{I} \text{ (Magnet length)}. \quad \text{(emu)} \tag{22}$$

The ac resistance of the conductors is found by finding the power loss of series connected conductors in a deep slot. For the main-ring magnets it seems reasonable to calculate this loss assuming that the conductors are paired by twos in the layers of a deep slot. This calculation follows the reasoning of Appendix D. The conductor height is taken to be the average height of the inner and outer conductor heights. The result of this calculation yields the resistance R. Then the ac impedance of the magnet is

$$Z = R + j\omega L. \tag{23}$$

The inductance, of course, will have an imaginary part and contribute to the actual resistance seen between the coil terminals.

Numerical Results for Dipole Magnets

The relations necessary to solve Eqs. (18) and (19) have been coded for the CDC 6600 in a program HITRANS, which yields basically H_o/I and B_o/I for various frequencies and assumed permeabilities. The results are almost independent of the permeability which attests to the fact that the effects of eddy currents in the conductors and the vacuum chamber dominate over the eddy current effects in the magnet laminations. A reasonable choice of permeability might be the value giving the observed AMPFAC. If T(0) designates the transfer function for dc, then

$$\text{AMPFAC} = \frac{2\pi N}{hT(0)}. \tag{24}$$

A second transfer function is produced from the T of Eq. (22) by noting that every magnet coil is paralleled by a 10-ohm resistor. Thus the effective transfer function TE is

$$TE = \frac{10}{10+Z} \cdot T. \tag{25}$$

Numerical results[1] for NAL B1 and B2 main-ring dipoles are shown graphically in Figs. (5-8) using reasonable estimates for the pole width, yoke thickness, etc. A permeability of 3000 was used since this gives approximately the observed AMPFAC.

One should recognize that the approximations inherent in the use of an equivalent magnetic circuit and of a deep slot ac resistance calculation lead to uncertainties in all the above estimates. Comparison of the calculated results with measurements[2] made on the B1 magnet indicate agreement to better than 2 percent for frequencies below 10 Hz and above 1000 Hz. The measured results between these two limits are low by as much as 17 percent at 100 Hz.

Magnetic Circuit (Quadrupole)

Figure 9 indicates the cross section of the quadrupole under consideration. Let a designate the aperture radius, b the horizontal half aperture of the vacuum chamber, d the distance from the quadrupole center to the inner edge of the coil in the horizontal slot, h the half height of the coil in the horizontal slot, ℓ the length of the mean flux path in the horizontal octant of the yoke, w the average width of the horizontal octant, and N the number of coil turns in the full slot. For simplicity the quadrupole is considered symmetrical with d representing the mean of the horizontal and vertical coil distances.

If U(x) designates the flux per unit length of the quadrupole passing through the median plane from x = 0, the quadrupole center, to x = x, then the average flux density B_o in the horizontal octant of the yoke is to be determined by

$$U(d) + U_{coil} = wB_o, \tag{26}$$

where Appendix D may be used to give the flux passing through the coil. Thus

$$U_{coil} = \frac{\cosh Pc - 1}{P \sinh Pc} \cdot \left[NH(d) + \frac{\pi N^2}{h} I \right] \quad \text{(emu)} \tag{27}$$

where

$$p^2 = 4\pi j\omega\sigma_{coil} \quad \text{(emu)} \tag{28}$$

c is the horizontal width of the conductor in the horizontal slot, and H(d) is the magnetic field at the edge of the horizontal coil. Although easily modified for other cases, it is assumed that only one layer of conductors exists between the median plane and the pole.

To complete the magnetic circuit description one evaluates the ampere integral along the flux line beginning at $x = x$, $y = 0$ through the vacuum chamber. In the iron the mean flux path will be used. Thus, remembering that in the ideal quadrupole the potential on the iron is given by $1/2B'a^2$ where B' is the gradient, one has approximately

$$\frac{1}{2}a^2H'(x) + \frac{\ell}{\mu}B_0 = 2\pi NI - 4\pi\int_{(1)}^{(2)} i_c dt - 4\pi\int_{(2)}^{(3)} i_c dt - 4\pi i_I \ell, \tag{29}$$

where from Appendix B the surface current density of eddy currents in the vacuum chamber is

$$i_c = j\omega s U(x), \tag{30}$$

s being the surface conductivity, and from Appendix A the effective surface current density due to eddy currents in the iron is

$$4\pi i_I = \frac{k}{\mu} B_0. \tag{31}$$

From the geometry of the ideal quadrupole dt along the iron surface (assumed to be coincident with the vacuum chamber surface) is

$$dt = \frac{x}{\sqrt[4]{x^4+a^4}} dx. \tag{32}$$

Along the flux line at x = b one has approximately

$$\int_{(2)}^{(3)} dt = \frac{1}{2}\frac{a^2}{b}. \tag{33}$$

Thus, remembering that

$$U(x) = \int_0^x H(x)dx \tag{34}$$

one has from Eq. (29)

$$\frac{1}{2}a^2U''(x) + 4\pi j\omega s\int_x^b \frac{x}{\sqrt[4]{x^4+a^4}} U(x)dx + 4\pi j\omega s\frac{a^2}{b}U(b) + \frac{(k+1)}{\mu}\ell B_0 = 2\pi NI. \quad \text{(emu)} \tag{35}$$

Differentiating Eq. (35) gives

$$\frac{1}{2}a^2U'''(x) - 4\pi j\omega s\frac{x}{\sqrt[4]{x^4+a^4}}U(x) = 0. \quad \text{(emu)} \tag{36}$$

Solution

By definition the flux line passing through the origin may be designated by $U(0) = 0$. A quadrupole field is further designated by $U'(0) = 0$, and $U''(0) = H_0'$. Thus, if, for convenience,

$$U(x) = H_0'u(x), \tag{37}$$

then the conditions on U(x) are

$$u(0) = 0, \quad u'(0) = 0, \quad u''(0) = 1 \tag{38}$$

and the differential equation is

$$u'''(x) = \begin{cases} Q(x)u(x), & 0<x<b \\ 0 & b<x<d \end{cases} \tag{39}$$

where

$$Q(x) = \frac{8\pi j\omega s}{a^2}\frac{x}{\sqrt[4]{x^4+a^4}}. \tag{40}$$

Numerical integration using a Runge-Kutta procedure yields u(x) in the interval $0<x<b$. Outside b Eq. (39) yields directly

$$u(x) = u(b) + \frac{1}{2}\alpha(x^2-b^2). \tag{41}$$

The constant α may be found using Eq. (35) once for x = b and once for x = 0. Subtraction yields

$$\alpha = 1 + \int_0^b Q(x)u(x)dx. \tag{42}$$

Transfer Function and Coil Impedance

Having found u(b) and α one may write Eq. (26) and Eq. (35) for x = 0 in the form

$$\left[u(b) + \frac{1}{2}\alpha(d^2-b^2) + \alpha c d N\frac{(\cosh Pc-1)}{Pc\sinh Pc}\right]H_o'$$

$$- wB_o = \frac{\pi c N^2}{h} \cdot \frac{(\cosh Pc-1)}{Pc\sinh Pc} I \quad (43)$$

and

$$\left[\frac{1}{2}a^2\alpha + \frac{4\pi j\omega s a^2}{b} u(b)\right]H_o' + \frac{k+1}{\mu} \ell B_o$$

$$= 2\pi NI. \quad (44)$$

Eqs. (43) and (44) may be solved simultaneously for H_o' and B_o. The transfer function is then given by

$$T = \frac{H_o'}{I} \quad \text{(emu)} \quad (45)$$

and the complex inductance by

$$L = 4Nw\frac{B_o}{I} \times \text{(quadrupole length)}. \quad \text{(emu)} \quad (46)$$

From this inductance and the ac resistance R of the conductors paired by twos in the coil slot one has for the coil impedance

$$Z = R + j\omega L. \quad (47)$$

The inductance, of course, will have an imaginary part and contribute to the actual resistance seen between the coil terminals.

Numerical Results for Quadrupole Magnet

The relations necessary to solve Eqs. (43) and (44) have been coded for the CDC 6600 in a program GITRANS. The complex values of T in gauss/cm/A and Z in ohms is given for each assumed frequency in the range 0 to 1000 Hz. A second transfer function is produced from that of Eq. (45) if each quadrupole is paralled by a resistor R_o. The effective transfer function is

$$TE = \frac{R_o}{Z+R_o} \cdot T \quad (48)$$

and the effective impedance is

$$ZE = \frac{R_o Z}{Z+R_o}. \quad (49)$$

For each frequency T, Z, TE and ZE are given. Computer results for the 7-ft NAL main-ring quadrupole are shown in Figs. (10-11).

References

1. S.C. Snowdon, "Transfer Function between Magnetic Field and Excitation Current in Main Ring Bending Magnets," National Accelerator Laboratory Report TM-325 (1971).
2. J.E. Griffin, private communication (1971).
3. S.C. Snowdon, "Transfer Function and Coil Impedance for Main Ring Quadrupole Magnets," National Accelerator Laboratory Report TM-352 (1972).

Appendix A
Eddy Currents in Magnet Lamination

Figures 2a, 2b, and 3 give more detail of the infinite lamination on which the eddy current calculation will be based. If x designates the coordinate orthogonal to y and z as shown in Fig. 3, then only B_x is needed and is governed by

$$\frac{\partial^2 B_x}{\partial z^2} - 4\pi\mu\sigma j\omega B_x = 0. \quad \text{(emu)} \quad \text{(A-1)}$$

Letting

$$\lambda^2 \equiv 4\pi\mu\sigma j\omega, \quad \text{(A-2)}$$

one has

$$B_x = \mu H\frac{\cosh \lambda z}{\cosh \lambda\frac{\delta}{2}}, \quad \text{(A-3)}$$

where μ is the permeability of the iron, σ the conductivity of the iron, and δ the lamination thickness.

The only value of B_x that can be used in the equivalent magnetic circuit analysis is the average value. From Eq. (A-3) one has for the average flux density

$$B = \frac{2\mu H}{\lambda\delta} \tanh \lambda\frac{\delta}{2}. \quad \text{(A-4)}$$

Figure 4 indicates the manner in which the infinite slab calculations are to be utilized for laminations of finite extent. In particular, the net current flowing along the edge of the lamination is assumed to be that current flowing vertically in one half of the infinite lamination. Thus the current per unit length is

$$4\pi i_I = H-H(0) = H\left(1-\frac{1}{\cosh \lambda\frac{\delta}{2}}\right). \quad \text{(emu)} \quad \text{(A-5)}$$

Eliminating H between Eqs. (A-4) and (A-5) one arrives at

$$i_I = \frac{\lambda\delta}{8\pi\mu} \cdot \frac{\cosh\lambda\frac{\delta}{2} - 1}{\sinh\lambda\frac{\delta}{2}} B. \quad \text{(emu)} \qquad \text{(A-6)}$$

Appendix B
Current Flow in Thin Sheets

Figures 2c and 2d indicate the spatial relationships for the present calculation. The only assumption made is that the conducting sheet is sufficiently thin that the induced electric field does not vary throughout its thickness. For the vacuum chamber wall thickness of interest this means that the frequencies must be less than 100 kHz.

A gauge for which $\nabla \cdot \bar{A} = 0$ is used for the vector potential. Then $A_z(x,y)$ is adequate for the description of the fields. Thus

$$E_z = -j\omega A_z. \quad \text{(emu)} \qquad \text{(B-1)}$$

Hence, the current density

$$J_z = \sigma E_z = -j\omega\sigma A_z, \quad \text{(emu)} \qquad \text{(B-2)}$$

where σ is the conductivity of the sheet. Hence, if i_c designates the current flowing in the sheet per cm and d is the thickness, one has

$$i_c = J_z d = -j\omega\sigma d A_z. \quad \text{(emu)} \qquad \text{(B-3)}$$

Letting $s = \sigma d$ the surface conductivity and

$$U(x,y) = -A_z(x,y) \quad \text{(emu)} \qquad \text{(B-4)}$$

the flux function, one has

$$i_c(x,y) = j\omega s U(x,y). \quad \text{(emu)} \qquad \text{(B-5)}$$

Appendix C
Eddy Current Fields from Rectangular Chamber in Ideal Dipole Magnet

An idealized calculation is made of the gap field in Fig. 1. The conductors are replaced by a sheet current I placed against the side wall of a window frame magnet of aperture 2a and gap 2h. Permeability of iron is assumed infinite. In order to utilize this calculation in the main text, the current I is expressed in terms of the central field H_o.

By superposition of elementary solutions of Laplace's equation one may choose

$$V(x,y) = 4\pi I \sum C_n \cosh Q_n x \sin Q_n y \qquad \text{(C-1)}$$

for the scalar potential function and

$$U(x,y) = 4\pi I \sum C_n \sinh Q_n x \cos Q_n y \qquad \text{(C-2)}$$

for the flux function. From Eq. (C-1) or Eq. (C-2) one has

$$H_x(x,y) = 4\pi I \sum Q_n C_n \sinh Q_n x \sin Q_n y \qquad \text{(C-3)}$$

and

$$H_y(x,y) = 4\pi I \sum Q_n C_n \cosh Q_n x \cos Q_n y. \qquad \text{(C-4)}$$

At y = h the boundary condition is

$$H_x(x,h) = 4\pi i_c(x,h), \qquad \text{(C-5)}$$

which from Appendix B gives

$$\sum C_n \left(Q_n \sin Q_n h - 4\pi j\omega s \cos Q_n h\right) \cdot \sinh Q_n x = 0. \qquad \text{(C-6)}$$

Equation (C-6) may be satisfied if Q_n is the solution

$$Q_n \tan Q_n h = 4\pi j\omega s. \qquad \text{(C-7)}$$

At x = a the boundary condition is

$$H_y(a,y) = \frac{2\pi I}{h} - 4\pi i_c(a,y), \qquad \text{(C-8)}$$

which from Appendix A gives

$$\sum C_n \left(Q_n \cosh Q_n a + 4\pi j\omega s \sinh Q_n a\right) \cdot \cos Q_n y = \frac{1}{2h}. \qquad \text{(C-9)}$$

The functions $\cos Q_n y$ form an orthogonal set since the Q_n satisfy Eq. (C-7). Therefore,

$$C_n \left(Q_n \cosh Q_n a + 4\pi j\omega s \sinh Q_n a\right) \cdot \left(1 + \frac{\sin 2Q_n h}{2Q_n h}\right) = \frac{1}{h} \cdot \frac{\sin Q_n h}{Q_n h}. \qquad \text{(C-10)}$$

This expression for C_n formally completes the solution. One desires, however, to eliminate the current I by expressing it in terms of the central field. Thus, from Eq. (C-4)

$$H_o = 4\pi I \sum Q_n C_n. \qquad \text{(C-11)}$$

Expressions needed in the main text follow.

$$V(a,h) + 4\pi j\omega s \int_0^h U(a,y)\,dy$$

$$= H_o \frac{\sum \frac{\left(\frac{\sin Q_n h}{Q_n h}\right)^2}{\left(1+\frac{\sin 2Q_n h}{2Q_n h}\right)}}{\sum Q_n C_n}, \tag{C-12}$$

and

$$H_y(a,h) + 4\pi j\omega s U(a,h)$$

$$= H_o \frac{\sum \frac{1}{\left(1+\frac{2Q_n h}{\sin 2Q_n h}\right)}}{\sum h Q_n C_n}, \tag{C-13}$$

and

$$U(a,h) = H_o \frac{\sum C_n \sinh Q_n a \cos Q_n h}{\sum Q_n C_n}. \tag{C-14}$$

Thus, given Q_n as the solution of Eq. (C-7) and C_n from Eq. (C-10) the desired expressions are seen to be expressed in terms of H_0 the central field.

Appendix D
Field in Dipole Magnet Within Conductors in Gap

An ideal situation is envisaged in which the magnet poles have infinite permeability, zero conductivity, and extend to infinity. Although easily modified for other cases, it is further assumed that only one layer of conductors exists between the median plane and the pole.

If the conductors are counted with the index n, n = 1 being the conductor beginning at the edge of the vacuum chamber (removed), then

$$H_{yn} = A_n \sinh Px + B_n \cosh Px, \tag{D-1}$$

where x = 0 at the left-hand edge of each conductor of the right-coil. Reasoning similar to that of Appendix A gives

$$P^2 = 4\pi j\omega\sigma, \tag{D-2}$$

where σ is the conductivity of the copper. Further, let c be the horizontal width of each conductor.

Continuity of H_y between conductors gives

$$H_{y,n+1}(0) = H_{y,n}(c) \tag{D-3}$$

or

$$B_{n+1} = A_n \sinh Pc + B_n \cosh Pc. \tag{D-4}$$

The ampere integral around the nth conductor gives

$$H_{yn}(0) - H_{yn}(c) = \frac{4\pi I}{h}, \quad \text{(emu)} \tag{D-5}$$

or

$$B_n - A_n \sinh Pc - B_n \cosh Pc = \frac{4\pi I}{h}. \quad \text{(emu)} \tag{D-6}$$

Equations (D-4) and (D-6) may be solved in terms of B_1 which in turn is the magnetic field in the aperture at the beginning of the conductors. Thus Eqs. (D-4) and (D-6) may be solved in terms of B_1, which in turn is the magnetic field in the aperture at the beginning of the conductors. Thus

$$A_n \sinh Pc = -\frac{4\pi I}{h}\left[n-(n-1)\cosh Pc\right] - (\cosh Pc - 1)B_1 \tag{D-7}$$

and

$$B_n = -\frac{4\pi I}{h}(n-1) + B_1. \tag{D-8}$$

Of course, B_1 would, in turn be related to I by a simple ampere integral around all the conductors. The intention, however, is to utilize the magnetic fields within the conductor as one component in a magnetic circuit. Hence B_1 and I are intentionally separated.

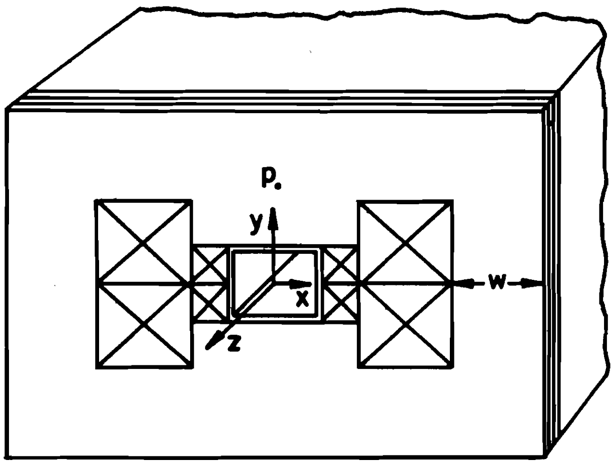

Fig.1 Dipole Cross Section

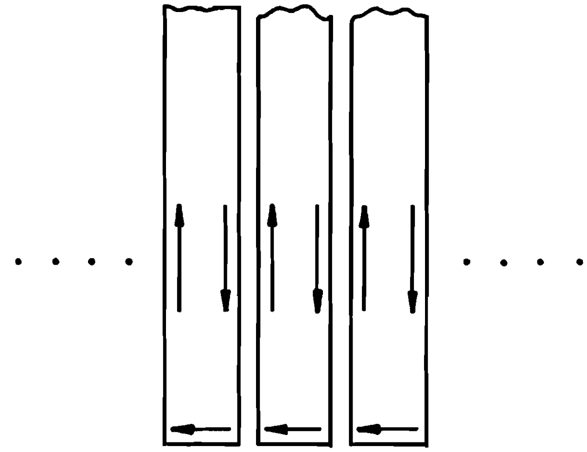

Fig.4 Semi-Infinite Slab Lamination Edge View

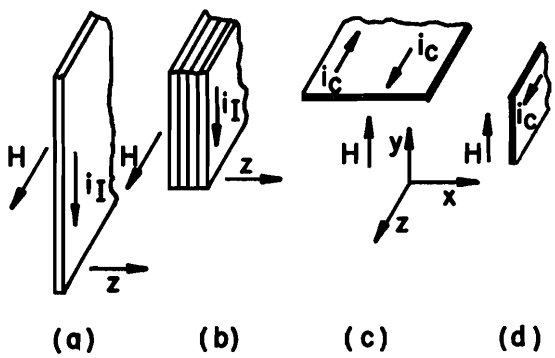

Fig.2 Lamination (I) and Vacuum Chamber (C) Details

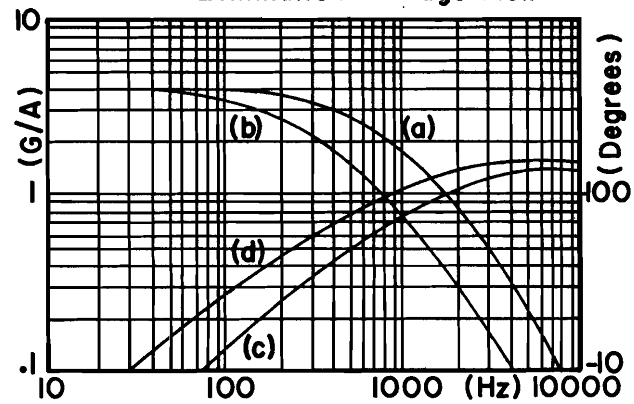

Fig.5 Transfer Function (B1)
(a) Magnitude (no shunt)
(b) Magnitude (10Ω shunt)
(c) Phase (no shunt)
(d) Phase (10Ω shunt)

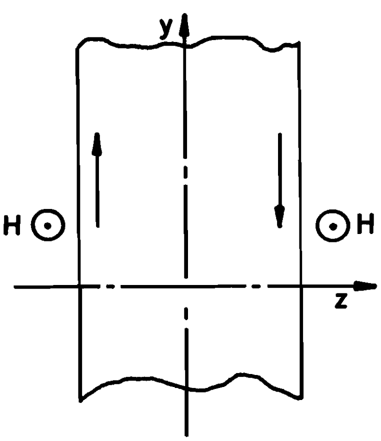

Fig.3 Infinite Slab Lamination Edge View
Magnetic Field is Directed up out of Paper

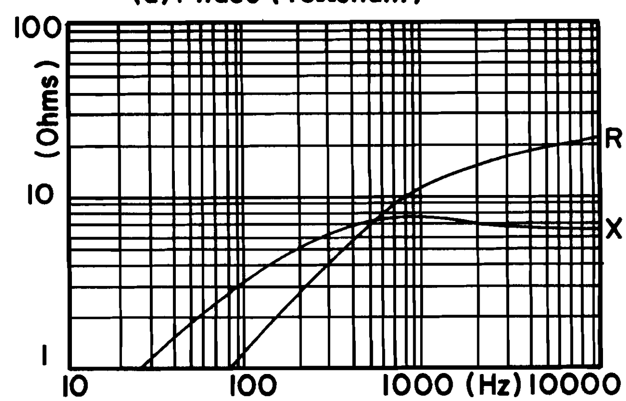

Fig.6 Impedance Function (B1) (no shunt)
$R_0 = .0059\ \Omega$
$L_0 = .0061$ Hy

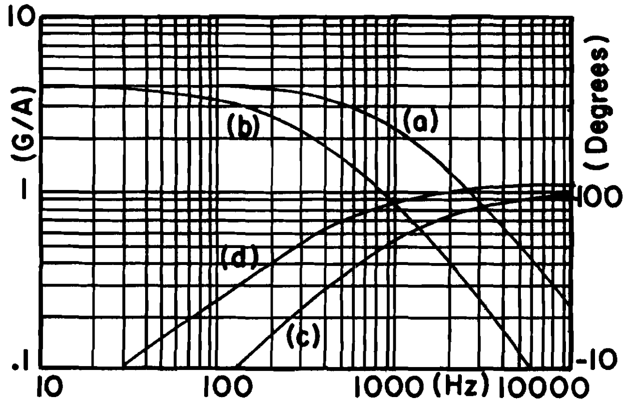

Fig. 7 Transfer Function (B2)
(a) Magnitude (no shunt)
(b) Magnitude (10Ω shunt)
(c) Phase (no shunt)
(d) Phase (10Ω shunt)

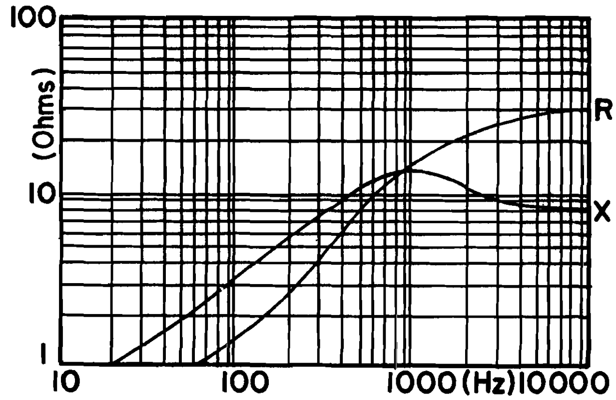

Fig. 8 Impedance Function (B2)
(no shunt)
R_0=.0073
L_0=.0082 Hy

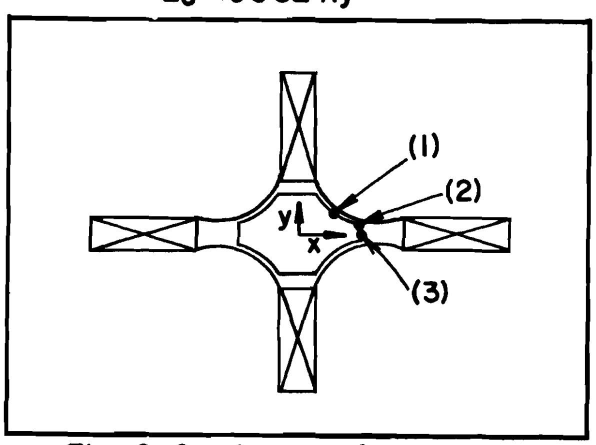

Fig. 9 Quadrupole Cross Section

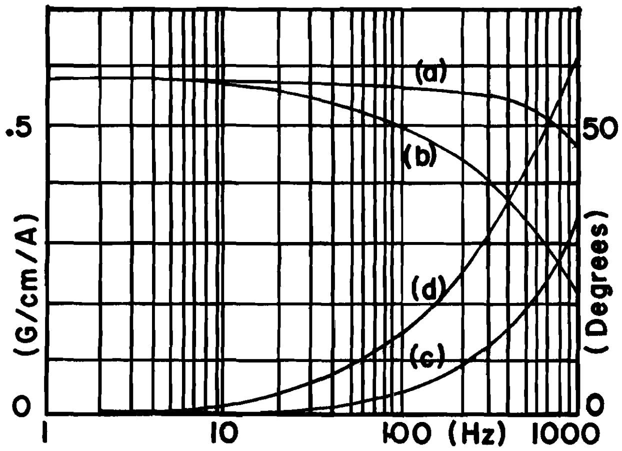

Fig. 10 Transfer Function (7ft)
(a) Magnitude (no shunt)
(b) Magnitude (5Ω shunt)
(c) Phase (no shunt)
(d) Phase (5Ω shunt)

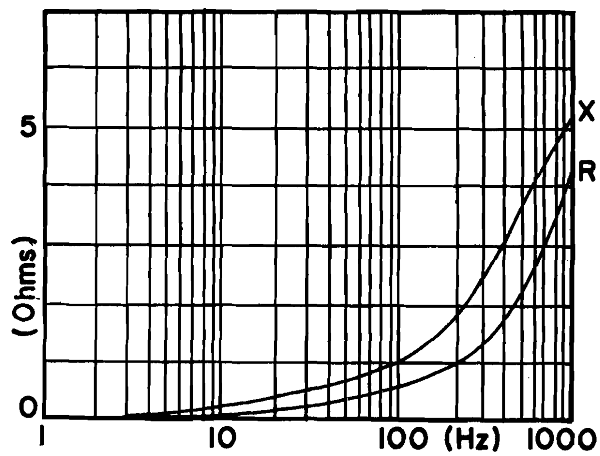

Fig. 11 Impedance Function (7ft)
(no shunt)
R_0=.0049Ω
L_0=.0032 Hy