

"FORGY"

A COMPANION COMPUTER PROGRAM OF "TRIM"
TO CALCULATE FORCES AND ENERGY IN ELECTROMAGNETS*

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Abstract

FORGY is an auxiliary computer program of TRIM^{1, 2} which calculates the FORces and enerGY in a magnetic system. It reads a specified "dump" on a previously created TRIM magnetic tape and calculates the stored energy of the steel and air regions. Options exist to calculate the forces on mesh points which have an assigned current and calculate the forces on any curve formed by lines joining mesh points.

I. Forces on the Currents

The force exerted on a current element $id\ell$ by a magnetic field B is given by

$$d\vec{F} = id\vec{\ell} \times \vec{B}$$

The x and y components of the force per unit length are calculated for each mesh point which has an assigned current. In cylindrical geometry problems, the tension and total force at each current element are calculated by the equations

$$\begin{aligned} \text{Tension} &= RIB_z \\ \text{Z force} &= 2\pi RIB_r \end{aligned} \quad (1)$$

No net radial forces exist because of cylindrical symmetry.

II. Maxwell's Stresses in the Magnetic Field

Assume that the induction B is known as a function of H , say from the magnetization curve like that shown in Fig. 1.

The density of the magnetic energy is given by

$$U_m = \int_a^c \vec{H} \cdot d\vec{B} \quad (2)$$

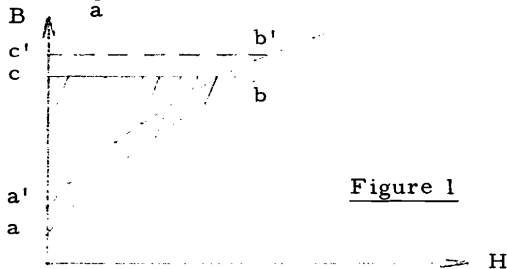


Figure 1

*Work performed under the auspices of the U. S. Atomic Energy Commission and the U. S. Naval Ship Research and Development Center.

The total energy of the magnetic field is given by the expression

$$W_m = \int_V U_m dV = \int_V \left(\int_a^c \vec{H} \cdot d\vec{B} \right) dV \quad (3)$$

which shows that the energy depends not only on the final value of H and B , but also on the magnetization curve. The volume force density may be derived from the energy. Consider a volume element dV fixed in space and assume a virtual displacement $\delta\vec{r}$ of the material. Because of this displacement, the value of H , B , and μ in dV will change. The shaded area in Fig. 1, representing the energy density, is replaced by the area inside the dashed line ($a'b'c'$). The infinitesimal change of U_m is composed of two parts.

$$\delta U_m = \vec{H} \cdot \delta\vec{B} - \int_0^H \delta_H \vec{B} \cdot d\vec{H} \quad (4)$$

The first term on the right gives the energy density change due to the real change in B . The second term is determined by the change of the magnetization curve. Assume that the residual magnetization is negligibly small so that one can write

$$\vec{B} = \mu\vec{H} \quad (B=0 \text{ when } H=0) \quad (5)$$

$$\delta_H \vec{B} = \delta\mu\vec{H} \quad (6)$$

Substituting this in Eq. (4), one obtains, using Eq. (3),

$$\delta W_m = \int_V \left[\vec{H} \cdot \delta\vec{B} - \int_0^H \delta\mu\vec{H} \cdot d\vec{H} \right] dV \quad (7)$$

Now assume that the location of the volume element dV is given by the vector \vec{r} . Due to the virtual displacement $\delta\vec{r}$, there is in dV material which before the displacement is located at $\vec{r} - \delta\vec{r}$. The μ in dV , after the displacement, is then

$$\mu(\vec{r} - \delta\vec{r}) = \mu(\vec{r}) - \delta\vec{r} \cdot \text{grad } \mu \quad (8)$$

or

$$\delta\mu = - \delta\vec{r} \cdot \text{grad } \mu \quad (9)$$

Substitution of this in Eq. (7) gives

$$\delta W_m = \int_V [\vec{H} \cdot \delta \vec{B} + \int_0^H \delta \vec{r} \cdot \text{grad} \mu \vec{H} \cdot d\vec{H}] dV \quad (10)$$

or

$$\frac{dW_m}{dt} = \int_V [\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \int_0^H (\text{grad} \mu \cdot \vec{u}) \vec{H} \cdot d\vec{H}] dV \quad (11)$$

where \vec{u} is the velocity of the virtual displacement.

A medium moving with velocity $u \ll c$, obeys the Maxwell's equations

$$\text{curl} \vec{E} = - \frac{\partial \vec{B}}{\partial t} + \text{curl} (\vec{u} \times \vec{B}) \quad (12)$$

$$\text{curl} \vec{H} = \vec{j} \quad (13)$$

Multiplying the first of these equations by $\vec{H} dV$ and the second by $\vec{E} dV$, subtracting and integrating over the whole system, gives

$$\begin{aligned} \int_V (\vec{H} \cdot \text{curl} \vec{E} - \vec{E} \cdot \text{curl} \vec{H}) dV = \\ - \int_V [\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{H} \cdot \text{curl} (\vec{u} \times \vec{B})] dV - \int_V \vec{E} \cdot \vec{j} dV. \end{aligned} \quad (14)$$

Since

$$\begin{aligned} \int_V (\vec{H} \cdot \text{curl} \vec{E} - \vec{E} \cdot \text{curl} \vec{H}) dV = \\ \int_V \text{div} (\vec{E} \times \vec{H}) dV = \int_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = 0, \end{aligned} \quad (15)$$

all that remains is

$$\int_V \vec{E} \cdot \vec{j} dV = - \int_V [\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{H} \cdot \text{curl} (\vec{u} \times \vec{B})] dV. \quad (16)$$

Using the vector identity

$$\begin{aligned} \text{div} \{ \vec{H} \times (\vec{u} \times \vec{B}) \} \\ = (\vec{u} \times \vec{B}) \cdot \text{curl} \vec{H} - \vec{H} \cdot \text{curl} (\vec{u} \times \vec{B}) \end{aligned} \quad (17)$$

and noting that

$$\int_V \text{div} \{ \vec{H} \times (\vec{u} \times \vec{B}) \} dV = \int_S \{ \vec{H} \times (\vec{u} \times \vec{B}) \} \cdot d\vec{S} = 0 \quad (18)$$

results in

$$\int_V \vec{E} \cdot \vec{j} dV = - \int_V [\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - (\vec{u} \times \vec{B}) \cdot \text{curl} \vec{H}] dV \quad (19)$$

or since

$$(\vec{u} \times \vec{B}) \cdot \text{curl} \vec{H} = (\vec{B} \times \text{curl} \vec{H}) \cdot \vec{u} \quad (20)$$

$$\int_V \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} dV = - \int_V \vec{E} \cdot \vec{j} dV + \int_V (\vec{B} \times \text{curl} \vec{H}) \cdot \vec{u} dV.$$

Substitution of this in Eq. (11) gives

$$\begin{aligned} \frac{dW_m}{dt} = - \int_V \vec{E} \cdot \vec{j} dV + \int_V [(\vec{B} \times \text{curl} \vec{H}) \\ + \int_0^H \text{grad} \mu \vec{H} \cdot d\vec{H}] \cdot \vec{u} dV. \end{aligned} \quad (21)$$

Let \vec{f} be the volume force density. The force acting on a volume element dV of the material is $\vec{f} dV$ and the work done by the volume forces is

$$\delta W = \int_V (\vec{f} \cdot \delta \vec{r}) dV \quad (22)$$

where $\delta \vec{r}$, as before, is the virtual displacement. This work is equal to the decrease in the magnetic energy.

$$- \delta W_m = \delta W = \int_V (\vec{f} \cdot \delta \vec{r}) dV \quad (23)$$

or

$$\frac{dW_m}{dt} = - \int_V (\vec{f} \cdot \vec{u}) dV \quad (24)$$

Ignoring the irreversible joule heat term

$$\int_V (\vec{E} \cdot \vec{j}) dV \quad \text{and comparing Eqs. (21) and (24)}$$

yields

$$\vec{f} = - \mu \vec{H} \times \text{curl} \vec{H} - \int_0^H \text{grad} \mu \vec{H} \cdot d\vec{H} \quad (25)$$

Using the vector identity

$$\vec{H} \times \text{curl} \vec{H} = \frac{1}{2} \text{grad} (H^2) - (\vec{H} \cdot \nabla) \vec{H} \quad (26)$$

gives

$$\vec{f} = -\frac{1}{2} \mu \text{grad} (H^2) + \mu (\vec{H} \cdot \nabla) \vec{H} - \int_0^H \text{grad} \mu \vec{H} \cdot d\vec{H} \quad (27)$$

$$\vec{f} = (\vec{B} \cdot \nabla) \vec{H} - \frac{1}{2} \mu \text{grad} H^2 - \frac{1}{2} \int_0^H \text{grad} \mu dH^2 \quad (28)$$

It is not difficult to see from Fig. 1 that

$$\begin{aligned} \delta \int_0^H \vec{B} \cdot d\vec{H} &= \int_0^H \delta \mu \vec{H} \cdot d\vec{H} + \vec{B} \cdot \delta \vec{H} \\ &= \frac{1}{2} \int_0^H \delta \mu dH^2 + \frac{1}{2} \mu \delta H^2 \end{aligned} \quad (29)$$

Thus,

$$\begin{aligned} \text{grad} \int_0^H \vec{B} \cdot d\vec{H} &= \frac{1}{2} \int_0^H (\text{grad} \mu) dH^2 + \frac{1}{2} \mu \text{grad} (H^2) \end{aligned} \quad (30)$$

Using this relation and noting that $(\nabla \cdot \vec{B}) \vec{H} = 0$, one finally obtains

$$\vec{f} = (\vec{B} \cdot \nabla) \vec{H} + (\nabla \cdot \vec{B}) \vec{H} - \text{grad} \int_0^H \vec{B} \cdot d\vec{H} \quad (31)$$

Eq. (31) can be rewritten in the form

$$\begin{aligned} f_x &= \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \\ f_y &= \frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \\ f_z &= \frac{\partial T_{zx}}{\partial x} + \frac{\partial T_{zy}}{\partial y} + \frac{\partial T_{zz}}{\partial z} \end{aligned} \quad (32)$$

where³

$$\begin{aligned} T_{k,l} &= \vec{H}_k \cdot \vec{B}_l - \delta_{k,l} \int_0^H \vec{B} \cdot d\vec{H} \quad (k, l = x, y, z) \\ T_{k,l} &= T_{l,k} \\ \delta_{k,l} &= 1 \quad \text{when } k = l \\ \delta_{k,l} &= 0 \quad \text{when } k \neq l \end{aligned} \quad (33)$$

It shall now be shown that the nine components $T_{k,l}$ represent forces acting on unit elements of area (stresses). Since the magnet is in static equilibrium, the resultant force is zero and the resultant torque is zero.

$$\int_V \vec{f} dV + \int_S \vec{t} dS = 0 \quad (34)$$

$$\int_V (\vec{r} \times \vec{f}) dV + \int_S (\vec{r} \times \vec{t}) dS = 0 \quad (35)$$

where \vec{t} = surface force per unit area of S.

Introduce three vectors \vec{a} , \vec{b} , \vec{c} , which satisfy the equation

$$\begin{aligned} t_x &= \vec{a} \cdot \vec{n} = a_x n_x + a_y n_y + a_z n_z \\ t_y &= \vec{b} \cdot \vec{n} = b_x n_x + b_y n_y + b_z n_z \\ t_z &= \vec{c} \cdot \vec{n} = c_x n_x + c_y n_y + c_z n_z \end{aligned} \quad (36)$$

where \vec{n} is the unit outward normal to an element of S. Substitution of these relations in Eq. (34) gives

$$\int_V f_x dV + \int_S \vec{a} \cdot \vec{n} dS = \int_V (f_x + \nabla \cdot \vec{a}) dV = 0$$

$$\int_V f_y dV + \int_S \vec{b} \cdot \vec{n} dS = \int_V (f_y + \nabla \cdot \vec{b}) dV = 0 \quad (37)$$

$$\int_V f_z dV + \int_S \vec{c} \cdot \vec{n} dS = \int_V (f_z + \nabla \cdot \vec{c}) dV = 0$$

Hence,

$$\begin{aligned} f_x + \nabla \cdot \vec{a} &= 0 \\ f_y + \nabla \cdot \vec{b} &= 0 \\ f_z + \nabla \cdot \vec{c} &= 0 \end{aligned} \quad (38)$$

The rotational equilibrium expression (35) gives

$$\int_V (yf_z - zf_y) dV + \int_S (yt_z - zt_y) dS = 0 \quad (39)$$

or

$$\int_V (yf_z - zf_y) dV + \int_S (y\vec{c} - z\vec{b}) \cdot \vec{n} dS = 0 \quad (40)$$

which gives

$$y f_z - z f_y + \nabla \cdot (y \vec{c} - z \vec{b}) = 0 \quad (41)$$

or

$$y f_z - z f_y + y \nabla \cdot \vec{c} - z \nabla \cdot \vec{b} + c_y - b_z = 0 \quad (42)$$

Making use of the fact that $f_z + \nabla \cdot \vec{c} = 0$ and $f_y + \nabla \cdot \vec{b} = 0$ [Eq. (38)], one finds $c_y = b_z$.

In the same way, it can be shown that $a_y = b_x$ and $a_z = c_x$. Comparison of Eqs. (32) and (38) gives

$$\begin{aligned} -\vec{a} &= T_{xx} \vec{i} + T_{xy} \vec{j} + T_{xz} \vec{k} \\ -\vec{b} &= T_{yx} \vec{i} + T_{yy} \vec{j} + T_{yz} \vec{k} \\ -\vec{c} &= T_{zx} \vec{i} + T_{zy} \vec{j} + T_{zz} \vec{k} \end{aligned} \quad (43)$$

where \vec{i} , \vec{j} , and \vec{k} are unit vectors in the x, y, and z directions. Eq. (36) then becomes

$$\begin{aligned} -t_x &= T_{xx} n_x + T_{xy} n_y + T_{xz} n_z \\ -t_y &= T_{yx} n_x + T_{yy} n_y + T_{yz} n_z \\ -t_z &= T_{zx} n_x + T_{zy} n_y + T_{zz} n_z \end{aligned} \quad (44)$$

One sees from these relations that the diagonal terms act in the direction of the normal to the surface element and are, therefore, pressures or tensions. The remaining six components are shearing stresses.

The "Maxwell" tensor given by Eqs. (32) and (33) is a symmetric tensor and can, therefore, be reduced to three components by transformation to the principal axes. To find these principal axes, one solves the determinant

$$\left| T_{k,l} - \delta_{k,l} p \right| = 0 \quad (45)$$

where p is the force per unit area of the surface parallel to the normal. Using Eq. (33), one obtains

$$-\left(p + \int_0^H \vec{B} \cdot d\vec{H} \right)^3 + \left(p + \int_0^H \vec{B} \cdot d\vec{H} \right)^2 \vec{H} \cdot \vec{B} = 0 \quad (46)$$

or

$$p_1 = \vec{H} \cdot \vec{B} - \int_0^H \vec{B} \cdot d\vec{H} = \int_{B_0}^B \vec{H} \cdot d\vec{B} \quad (47)$$

$(B_0 = B \text{ when } H = 0)$

$$p_{2,3} = - \int_0^H \vec{B} \cdot d\vec{H} \quad (48)$$

On the principal axes, the Maxwell stress tensor takes the form

$$T = \begin{vmatrix} \int H dB & 0 & 0 \\ 0 & - \int B dH & 0 \\ 0 & 0 & - \int B dH \end{vmatrix} \quad (49)$$

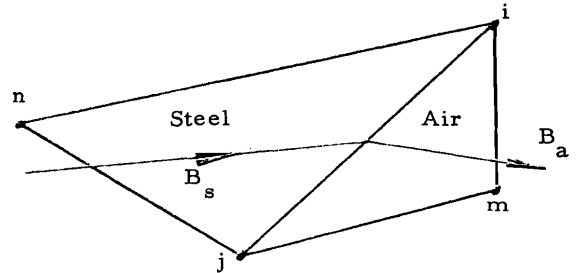
Substitution of this in Eq. (44) gives

$$\begin{aligned} \vec{t}_x &= -\vec{n}_x \int H dB = -\vec{n}_x p_x \\ \vec{t}_y &= +\vec{n}_y \int B dH = +\vec{n}_y p_y \\ \vec{t}_z &= +\vec{n}_z \int B dH = +\vec{n}_z p_z \end{aligned} \quad (50)$$

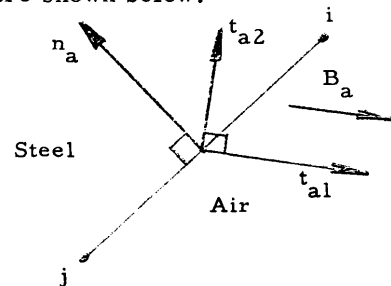
The principal axes are so oriented that the coordinate axis corresponding to the root p_1 is parallel to B, while the other two axes corresponding to p_2 and p_3 are perpendicular to B.

III. Applying the Maxwell Stresses to TRIM

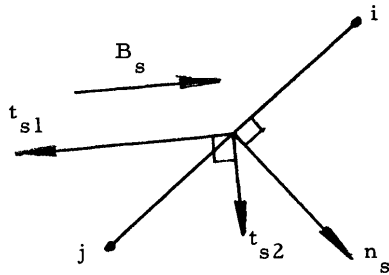
The computer program TRIM generates a triangular mesh of various size and shape triangles. A general interface between air and steel is shown below as the line ji common to both triangles and extending to infinity perpendicular to the plane of the paper.



A typical field line is also shown. Consider the Maxwell stresses of the AIR triangle on the surface ji. The outward normal \vec{n}_a and stresses t_{a1} and t_{a2} are shown below.



Now consider the Maxwell stresses of the STEEL triangle on the surface ji . The outward normal n_s and stresses t_{s1} and t_{s2} of the STEEL triangle on ji are shown below.



By Eq. (50), the magnitude of the stresses are:

$$p_{a1} = \int_0^B HdB = \int_0^B \frac{B}{\mu_0} dB = \frac{B^2}{2\mu_0} \quad (51)$$

$$p_{a2} = \int_0^H BdH = \int_0^H \mu_0 HdH = \frac{\mu_0 H^2}{2} = \frac{B^2}{2\mu_0} \quad (52)$$

$$p_{s1} = \int_{B_0}^B HdB = \text{area above the curve of Fig. 1.} \quad (53)$$

$$p_{s2} = \int_0^H BdH = \text{area below the curve of Fig. 1.} \quad (54)$$

Note that p_{s1} is the energy density of the steel and p_{s2} may be called the coenergy density of the steel. It is apparent that the stresses depend not only on the present values of B and H in the steel, but also on the B - H characteristics of the steel.

The forces of each stress are obtained by multiplying by the area perpendicular to the stress. These forces may then be resolved into normal and tangential forces which are added algebraically and then resolved into x and y components.

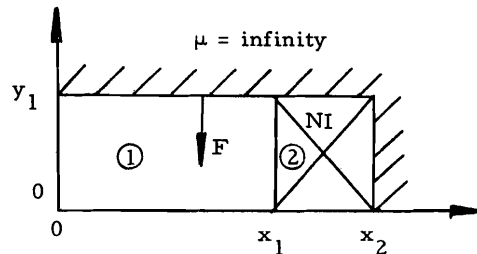
IV. Specifying the Desired Calculations

The computer program TRIM produces a magnetic tape which contains variables that specify the mesh geometry in "dump" number 1. "Dumps" 2, 3, . . . specify a particular solution and contain quantities such as the vector potential, the inverse of the permeability, etc. for each mesh point. The "dump" number desired must be specified as input to FORGY. A single digit punched into the input card determines if the

forces are to be calculated on mesh points having assigned currents. There is no need to specify which mesh points have assigned currents since the program searches for them. Another digit determines if the forces are to be calculated on an AIR-STEEL interface. If so, the mesh row (column) coordinates must be specified as well as the B - H table that was used when the problem was solved by TRIM. A third digit determines if the stored energy in the AIR and STEEL is to be calculated. Again, the B - H table is needed for this calculation.

V. Perfect Dipole Magnet Infinitely Long (TTEST) Test Case

The cross section of one quarter of this magnet is shown below.



The general equation to be solved is:

$$\nabla \times \nabla \times \vec{A} = \mu \vec{j} \quad (55)$$

which reduces to

$$\frac{\partial^2 \vec{A}}{\partial x^2} = -\mu \vec{j} \quad (56)$$

Hence,

$$\frac{\partial A}{\partial x} = -\mu j x + C_1 \quad (57)$$

$$A_n = -\frac{\mu_n}{2} j_n x^2 + \mu_n C_{n1} x + C_{n2} \quad (58)$$

$$\frac{B_n}{\mu_n} = -j_n x + C_{n1} \quad (59)$$

where n is the region number.

At the boundary of region ① and ②, the tangential components of H are continuous and the vector potentials are equal. Solving the resulting four equations for C_{11} , C_{12} , C_{21} , C_{22} , and substituting into Eqs. (58) and (59) yields

$$A_1 = \mu_0 j (x_2 - x_1) x \quad (60)$$

$$B_1 = \mu_0 j (x_2 - x_1) \quad (61)$$

$$A_2 = \frac{-\mu_0}{2} j (x_2^2 - xx_2 + x_1^2) \quad (62)$$

$$B_2 = -\mu_0 j x + \mu_0 j x_2 \quad (63)$$

The stored energy in Region ① is then

$$\frac{W_1}{L} = \int_0^{y_1} \int_0^{x_1} \frac{B_1^2}{2\mu_0} dx dy = \frac{\mu_0}{2} (NI)^2 \left(\frac{x_1}{y_1} \right) \quad (64)$$

Similarly in Region ②, the stored energy is

$$\frac{W_2}{L} = \frac{\mu_0}{6} (NI)^2 \left(\frac{x_2 - x_1}{y_1} \right) \quad (65)$$

Since the field is normal to the pole at $y = y_1$, the normal force per unit length for Region ① is given by:

$$\frac{F_y}{L} = - \int_0^{x_1} p_1 dx \quad (66)$$

where by Eq. (51),

$$p_1 = \int_0^B H dB = \frac{B_1^2}{2\mu_0} \quad (67)$$

Hence,

$$\frac{F_y}{L} = - \frac{B_1^2 x_1}{2\mu_0} = - \frac{\mu_0 (NI)^2 x_1}{2y_1^2} \quad (68)$$

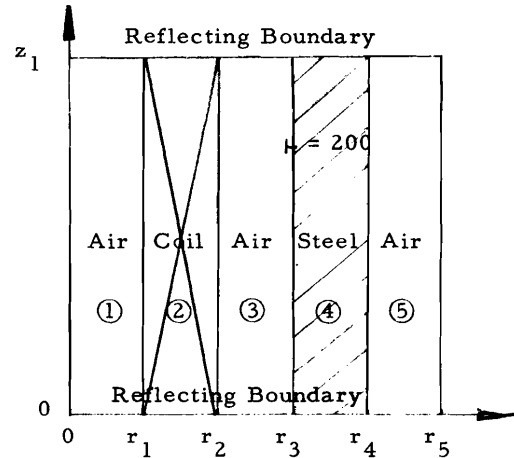
Similarly, for Region ②, the normal force is

$$\frac{F_y}{L} = - \int_{x_1}^{x_2} p_1 dx = - \frac{\mu_0 (NI)^2 (x_2 - x_1)}{6y_1^2} \quad (69)$$

The numerical results of the energy and force calculations are given in Table I. The energy is given in joules/meter and the forces are in newtons for the rectangular geometry test case. Agreement between the total value calculated by FORGY and the exact value is seen to be better than 0.2 percent.

VI. Cylindrical Geometry Test Case

The cross section of a small section of this case is shown below.



The z axis is an axis of rotation. Hence this case consists of an infinitely long cylindrical coil having inner radius, r_1 , and outer radius, r_2 , inside of an infinitely long steel cylinder of inner radius, r_3 , and outer radius, r_4 .

The general equation (55) reduces to

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rA)}{\partial r} \right) = -\mu j \quad (70)$$

This results in

$$\left. \begin{aligned} rA_\theta &= -\mu j \frac{r^3}{3} + C_{n1} \frac{r^2}{3} + C_{n2} \\ B_z &= -\mu jr + C_{n1} \end{aligned} \right\} \quad (71)$$

The constants C_{n1} and C_{n2} for region (n) are obtained by solving the ten equations in ten unknowns obtained by applying the conditions that the B_z/μ and A_θ are single valued at r_1 , r_2 , r_3 , and r_4 , and that rA_θ is zero at $r = 0$ and r_5 .

The z components of the field for each region turn out to be

$$B_1 = B_3 + \mu_0 j (r_2 - r_1) \quad (72)$$

$$B_2 = B_3 + \mu_0 j (r_2 - r) \quad (73)$$

$$B_3 = -\mu_0 j \left(r_2^3 - r_1^3 \right) \quad (74)$$

$$\left/ 3 \left[r_5^2 + r_4^2 \left(\frac{\mu_4}{\mu_0} - 1 \right) - r_3^2 \left(\frac{\mu_4}{\mu_0} - 1 \right) \right] \right.$$

$$B_4 = \mu_4 B_3 / \mu_0 \quad (75)$$

$$B_5 = B_3 \quad (76)$$

Integrating the energy density over each region results in the total energy for each region.

$$W_1 = \pi Z_1 B_1^2 r_1^2 / 2\mu_0 \quad (77)$$

$$W_2 = \frac{\pi Z_1}{12} \left\{ 3\mu_0 j^2 (r_2^4 - r_1^4) - 8 \left(j B_3 + \mu_0 j^2 r_2 \right) (r_2^3 - r_1^3) + 6 \left(\frac{B_3^2}{\mu_0} + 2j B_3 r_2 + \mu_0 j^2 r_2^2 \right) (r_2^2 - r_1^2) \right\} \quad (78)$$

$$W_3 = \pi Z_1 B_3^2 (r_3^2 - r_2^2) / 2\mu_0 \quad (79)$$

$$W_4 = \pi Z_1 B_4^2 (r_4^2 - r_3^2) / 2\mu_4 \quad (80)$$

$$W_5 = \pi Z_1 B_5^2 (r_5^2 - r_4^2) / 2\mu_0 \quad (81)$$

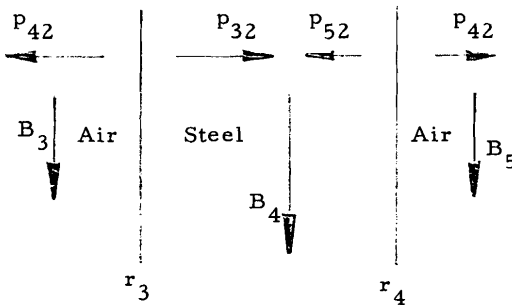
From Eq. (50) and the figure below, it follows that the radial force per unit radian on the steel at r_3 is

$$\frac{F_r}{\theta} = r_3 \int_0^{z_1} (p_{32} - p_{42}) dz \quad (82)$$

$$\text{where } p_{n2} = \int_0^{H_n} B_n dH_n \quad (83)$$

Hence,

$$p_{32} = \int_0^{H_3} B_3 dH_3 = \frac{B_3^2}{2\mu_0} \quad (84)$$



Also,

$$p_{42} = \frac{B_4^2}{2\mu_4} = \frac{\mu_4^2 B_3^2}{2\mu_4 \mu_0} = \left(\frac{\mu_4}{\mu_0} \right) \frac{B_3^2}{2\mu_0} \quad (85)$$

Therefore, at r_3 the force is

$$\frac{F_r}{\theta} = -r_3 \int_0^{z_1} \left(\frac{\mu_4}{\mu_0} - 1 \right) \frac{B_3^2}{2\mu_0} dz \quad (86)$$

$$= - \left(\frac{\mu_4}{\mu_0} - 1 \right) \frac{B_3^2 z_1 r_3}{2\mu_0} \quad (87)$$

Similarly, at r_4 the force is

$$\frac{F_r}{\theta} = r_4 \int_0^{z_1} (p_{42} - p_{52}) dz \quad (88)$$

$$= \left(\frac{\mu_4}{\mu_0} - 1 \right) \frac{B_3^2 z_1 r_4}{2\mu_0} \quad (89)$$

The resultant z component of the force at r_3 and r_4 is zero since the areas perpendicular to p_{31} , p_{41} , and p_{51} are zero. The result of this test case is shown in Table 1. Agreement between the force calculated by FORGY and the exact value is about 0.6 percent. The unit of energy is joules and the unit of force is newton/radian.

VII. General Cylindrical Geometry Case

The real merit of this program is to calculate the steel forces on complicated geometries such as the cylindrical model of the streamer chamber magnet at the Zero Gradient Synchrotron. The magnet geometry and flux lines are shown in Fig. 2. The normal components of the forces on the iron surfaces are shown in Fig. 3. Because of the difficulty of showing the tangential components of the forces in Fig. 3, the largest values are given in Table 2. The forces have units of newtons per radian.

Summary and Conclusions

It has been shown how the computer program FORGY uses the results of TRIM to calculate the forces on each mesh point which has an assigned current. The Maxwell stress tensor has been derived for a general case and the stress components, and hence, the resultant forces on the steel surface depend not only on the present value of B and H, but also on the magnetization curve of the steel. Two cases, calculated by FORGY were compared with analytical calculations and the results agreed to better than one percent. A third case, which could not be calculated analytically,

ically, shows the real usefulness of this program. It should be noted that TRIM assumes no volume deformation due to the magnetic forces and this assumption also applies to FORGY.

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- ² John S. Colonias, "Calculations of Two Dimensional Magnetic Fields by Digital Display Techniques," UCRL-17340, April 28, 1967.
- ³ It has been pointed out to the authors by James W. Butler of the Applied Mathematics Division of Argonne that Eqs. (33) were derived by Emil Cohn. (DAS Electromagnetische Feld, Leipzig 1900, p. 517, Eqs. 3)

Table 1
Rectangular Geometry Test Case

Region	FORGY Energy	Exact Energy	% Diff.	FORGY Force on Pole	Exact Force on Pole	% Diff.
1	513.184	513.402	- .04	-20206.82	-20212.68	0.0
2	84.193	85.567	-1.61	- 3316.16	- 3368.78	-1.6
Total	597.38	598.969	-0.27	-23522.98	-23581.46	-0.2

Cylindrical Geometry Test Case

Region	FORGY Energy	Exact Energy	% Diff.
1	3.968	3.935	+0.8
2	3.262	3.272	-0.3
3	$.5279 \times 10^{-4}$	$.5345 \times 10^{-4}$	-1.2
4	$.1506 \times 10^{-1}$	$.1497 \times 10^{-1}$	+0.6
5	--	$.9621 \times 10^{-4}$	--
Total in air	7.230	7.207	+0.3
Total in air and steel	7.245	7.222	+0.3

r	FORGY Force	Exact Force	% Diff.
r ₃	-8.174×10^{-4}	-8.125×10^{-4}	-0.6
r ₄	$+10.90 \times 10^{-4}$	$+10.833 \times 10^{-4}$	-0.6

Table 2

R	Z	F_r	F_z	R	Z	F_r	F_z
(inch)		(newton/radian)		(inch)		(newton/radian)	
32.227	37.0	-2864.	155524.	1.250	94.0	-101.	- 2060.
34.682	37.0	-8770.	153275.	3.750	94.0	-128.	- 6754.
37.136	37.0	-2471.	142588.	6.250	94.0	14.	-11300.
39.591	37.0	367.	127547.	8.750	94.0	161.	-15956.
42.045	37.0	1091.	108648.	11.250	94.0	376.	-20757.
44.500	37.0	127.	85362.	13.750	94.0	674.	-25787.
46.955	37.0	- 768.	60673.	16.250	94.0	844.	-31315.
49.409	37.0	- 622.	39246.	18.750	94.0	803.	-37677.
51.864	37.0	312.	23749.	21.250	94.0	957.	-44773.
54.318	37.0	546.	12511.	23.750	94.0	1536.	-52410.
56.773	37.0	47.	6102.	26.250	94.0	2556.	-61257.
35.0	95.5	92011.	-19184.	28.750	94.0	4870.	-71860.
35.0	98.5	77166.	- 4530.	31.250	94.0	11852.	-83903.
35.0	101.5	62133.	- 2790.	33.750	94.0	36825.	-90326.
35.0	104.5	55701.	- 2132.	36.286	109.0	-6464.	-17165.
35.0	107.5	39102.	1489.	38.857	109.0	1009.	-14174.
31.0	17.5	- 399.	472.	41.429	109.0	2306.	-10910.
31.0	20.5	-2216.	1526.	44.000	109.0	2152.	- 7691.
31.0	23.5	-8154.	3885.	46.571	109.0	1626.	- 4638.
31.0	26.5	-27645.	5683.	49.143	109.0	993.	- 2091.
31.0	29.5	-58955.	5501.	51.714	109.0	317.	- 516.
31.0	32.5	-84630.	8952.				
31.0	35.5	-95254.	27057.				

