ASSESSING AND CONTROLLING EDDY-CURRENTS IN CONDUCTORS OF FAST-PULSED COILS*

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Abstract

Measurement and analysis of the pulsed magnetic field of an early Berkeley ERA apparatus indicated significant undesirable perturbations due to induced eddy currents in the copper magnet conductors. Mathematical models were constructed which gave close estimates of the eddy currents and their associated magnetic field perturbations. Based on this analysis, the size and material of the conductors of a later ERA apparatus were selected to give suitably low field perturbations due to eddy currents. This has been confirmed by magnetic measurements.

Introduction

Compressors for electron-ring-accelerators[1] utilize rapidly-rising magnetic fields to contain and compress the high-current circulating ring of electrons. For the Berkeley Compressor 3 experiments of late 1969, the fields were produced by currents in the coil configuration shown in Fig. 1. The coils were wound of copper tubular conductor with dimensions and number of turns as given in Table I. The coils were sequentially excited by capacitor banks resulting in a field rise time of $\sim 800\ \mu$sec. Additional details of construction and circuitry previously have been presented elsewhere.[2-4]

It had originally been planned that similar coils and circuitry would be used for the forthcoming Compressor 5 experiments. However, experiments[5] on Compressor 4 indicated the presence of destructive collective radial oscillations which were attributable to field distortions arising from eddy currents in the copper conductors. More specifically, eddy currents in the conductors produce a shielding effect (analogous to trees in a stream) which reduces the magnetic flux density on the median plane opposite the conductors. This field reduction in turn produces in the near vicinity a significant change in the field index $n = -(r/B)(\partial B/\partial r)$ and in its radial derivative $\partial n/\partial r$. The collective radial oscillations can be suppressed by Landau-damping, for which the damping coefficient of the first radial mode is

$$E\frac{\partial \mathcal{S}_R}{\partial E} = -\frac{1}{\beta^2}\left[(1-\nu)\left(1-\frac{1-n}{\gamma^2}\right) + r\frac{\partial \nu}{\partial r}\right]\frac{\omega_0}{1-n} \qquad (1)$$

where $\nu = \sqrt{1-n}$ and $\partial\nu/\partial r = -(1/2\nu)(\partial n/\partial r)$. Local field variations near the conductors produced values of this coefficient which were of unacceptably small magnitude, as a result of strong cancellation between terms within the square bracket.

This prompted a review of the Compressor 5 design which indicated the need for reducing the magnitude of eddy-currents. Some aspects of this review were presented in an earlier paper.[6] This paper is primarily addressed to the practical problems of analytically estimating the eddy-current effects and using this knowledge to arrive at a coil configuration with suitably low eddy-current effects.

Eddy Currents and Field Perturbations

Upon coil energization, the magnetic field rise induces eddy-currents within conductive material which in turn produce a magnetic field component which generally tends to buck the applied magnetic field. Following is a simplified analysis suitable for initial estimates of the magnitude of this bucking magnetic field relative to the applied magnetic field.

Consider the case where (1) the curvature is neglected (i.e., the conductors are taken to be straight), (2) each conductor is subjected to a linearly-increasing magnetic field ramp starting with B=0 at t=0, (3) magnetic coupling of eddy currents between conductors is neglected (i.e., the bucking fields from one conductor do not influence eddy currents in other conductors), and (4) the conductors are taken as long cylindrical tubes as shown in Fig. 2.

At early times (soon after t=0), the applied magnetic flux is excluded from the conductor by surface eddy-currents having a distribution (in A/rad) of

$$\frac{di}{d\theta} = \frac{-Bb\ \sin\theta}{0.2\ \pi} \qquad (2)$$

for magnetic field B in gauss and conductor radius b in centimeters. The bucking magnetic field at an external point A due to this current

*Work supported by the U. S. Atomic Energy Commission.

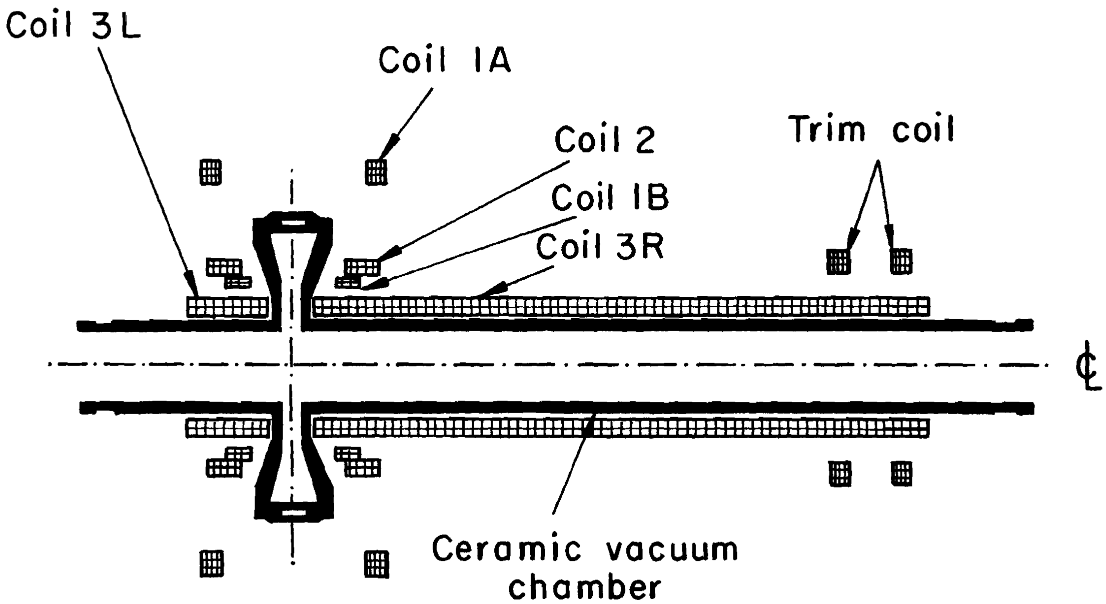

Fig. 1. ERA Compressor 3 longitudinal cross section showing coil arrangement.

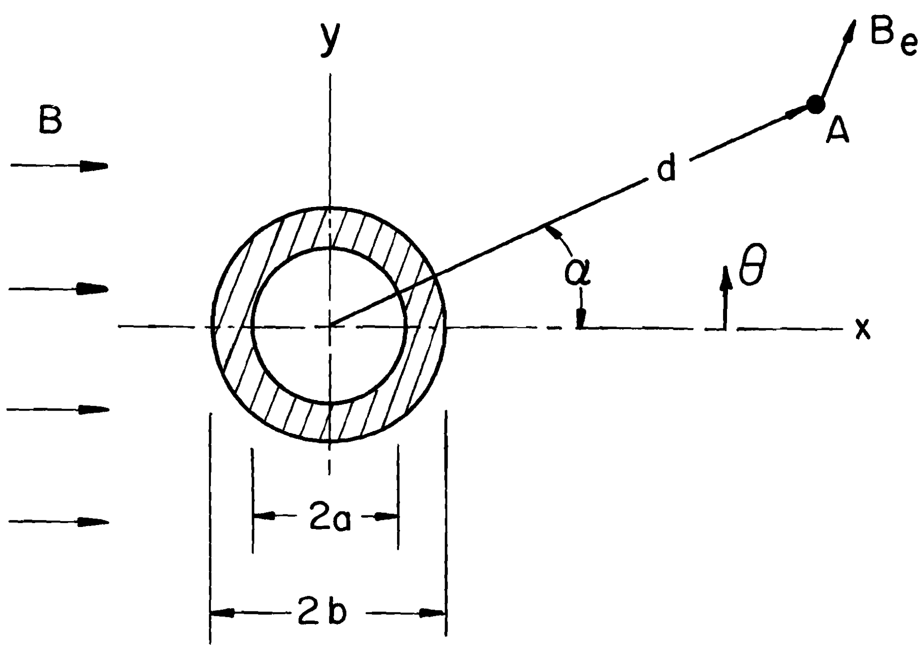

Fig. 2. Cross section of tubular conductor subjected to applied uniform magnetic field B.
Eddy currents in conductor produce bucking field B_e.

distribution is

$$B_\rho = \frac{-Bb^2 \cos\alpha}{d^2}$$

$$B_\alpha = \frac{-Bb^2 \sin\alpha}{d^2} \tag{3}$$

with minus sign indicating the bucking nature of the field. If we look only at $\alpha = 0$, the bucking field due to eddy currents is parallel to the applied field and of magnitude

$$B_e = \frac{Bb^2}{d^2} . \tag{4}$$

In contrast, at late times when the eddy currents are fully established, the current density in the conductor is

$$j = \frac{10^{-8} B'y}{\rho} , \tag{5}$$

where $B' = B/t$ and ρ is electrical resistivity (Ω-cm) of the conductor. For this current distribution, the bucking field due to eddy currents at an external point with $\alpha = 0$ is of magnitude

$$B_e = \frac{2\text{x}10^{-9} B'}{\rho d^2} \iint_S y^2 ds ,$$

where the integration extends over the cross-section of the conductor. For the tubular cylindrical conductor, this reduces to

$$B_e = \frac{10^{-9}\pi B' (b^4 - a^4)}{2\rho d^2} \tag{6}$$

It is interesting to determine the characteristic flux penetration time, $t = T_c$, at which the solutions (4) and (6) are equal (i.e., the time at which the two asymptotic solutions intersect), namely

$$T_c = \frac{10^{-9}\pi}{2\rho} \cdot \frac{b^4 - a^4}{b^2} . \tag{7}$$

Equation (6) can then be rewritten as

$$B_e = B'T_c \cdot \frac{b^2}{d^2} = \frac{Bb^2}{d^2} \cdot \frac{T_c}{t} \tag{8}$$

Noting that this equation differs from eq. (4) only by the last factor of T_c/t, it is evident that the relative magnitude of the eddy current bucking field can be expressed as

$$\frac{B_e}{B} = \frac{b^2}{d^2} \cdot f_d , \tag{9}$$

where f_d is an eddy-current decay factor that for early times ($t \ll T_c$) is determined from eq. (4) as $f_d = 1$ and for late times ($t \gg T_c$) is determined from eq. (8) as $f_d = T_c/t$. This relationship is plotted in Fig. 3.

At intermediate times near T_c, the foregoing equations are not valid and the relationship is shown approximately by the dashed curve in Fig. 3. A full solution at intermediate times involves[7] finding multiple roots of equations in Bessel functions and is considered beyond the scope of this paper. Figure 3 can be used to compare the approximate relative magnitude of eddy-current field perturbations for alternate materials and configurations.

Computer Model

How much reduction in eddy currents is required? For the ERA compressor case, the answer was found by adapting an existing interactive computer program as follows.[8]

The existing program computed magnetic field and other parameters as a function of time for several multi-turn axisymmetric coils excited by capacitive discharge and/or inductive coupling. To simulate eddy currents, two closely-spaced turns are series-connected in opposition so as to be magnetically coupled to the prevailing field passing between them. For example, as shown in Fig. 4, the turns at r_1 and r_2 couple axial field passing between them while turns at z_1 and z_2 form a pair that couples radial field. Twenty-eight (28) of these eddy-current simulation circuits were included into the computer model (8 in Coil 2, 4 in Coil 1B, and 16 in Coil 3) with one-half of them oriented for radial field and the remainder for axial field.

This computer model was then applied[9] to the copper-conductored compressor configuration described earlier. Resistances and inductive coupling of the eddy-current simulation circuits were adjusted so that the magnetic field pattern closely matched the fields for the real coils as measured by W. W. Chupp, J. M. Peterson and J. B. Rechen. The first radial Landau-damping coefficient per eq. (1) was then computed and is shown vs r for a typical compression cycle, as the "100%" curve in Fig. 5. To avoid beam instability in our case, the magnitude of this coefficient should not drop below $\sim 500\ \mu\text{sec}^{-1}$. However, as can be seen, this coefficient passes

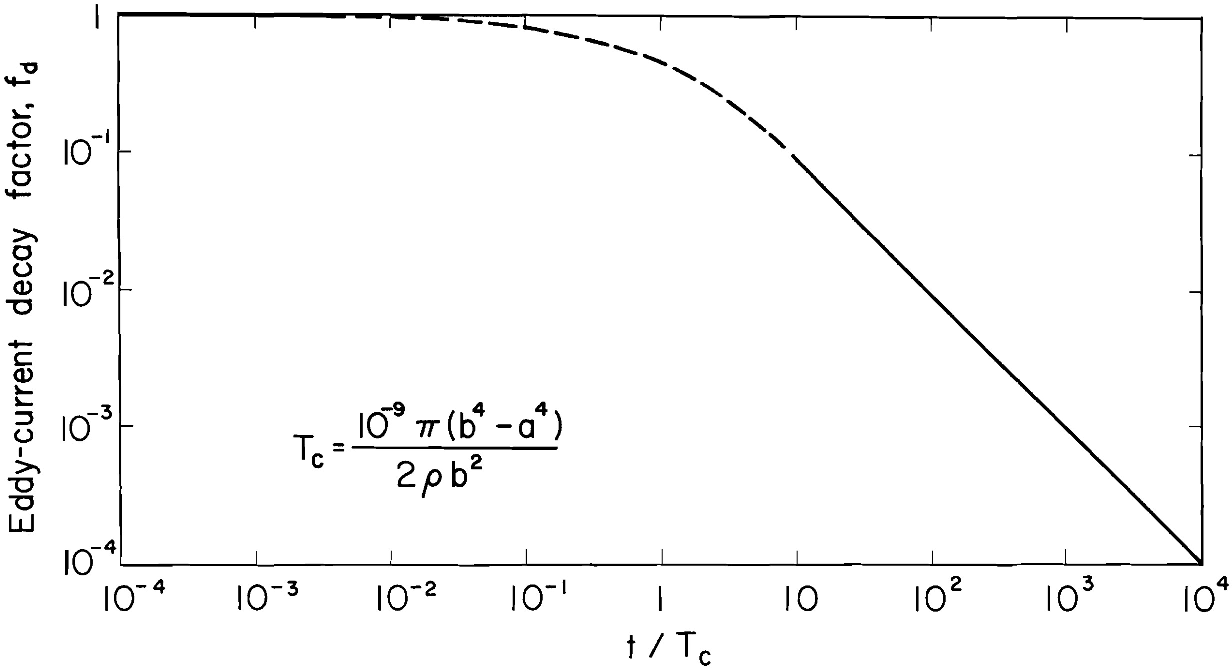

Fig. 3. Eddy-current damping factor f_d vs time (normalized to characteristic flux penetration time T_c) for magnetic field ramp $B = B' t$.

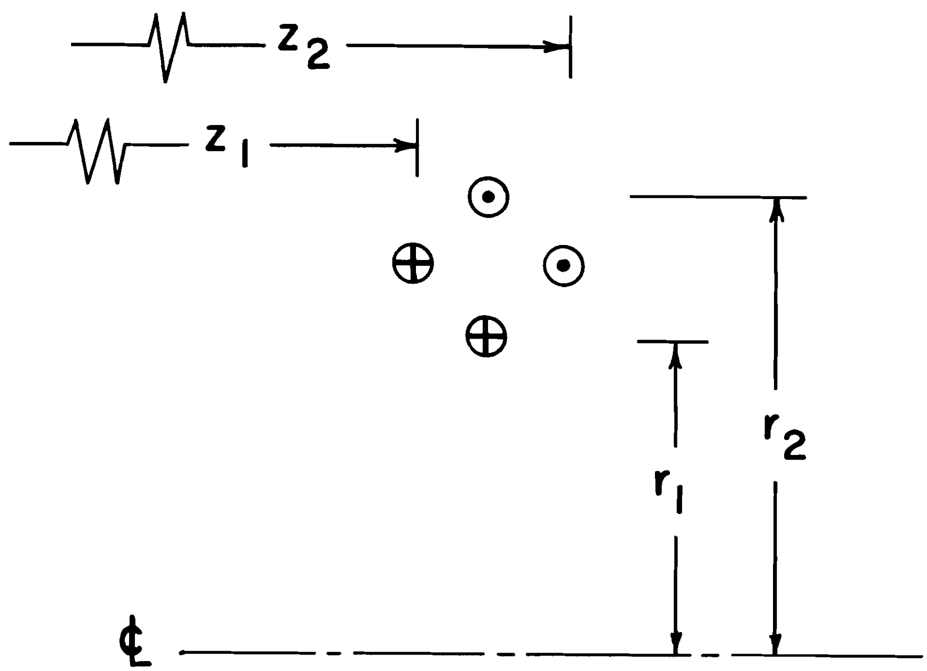

Fig. 4. Eddy-current simulation circuits for computer model. Pair of turns at r_1, r_2 couple the axial component of the prevailing magnetic field while pair of turns at z_1, z_2 couple the radial component.

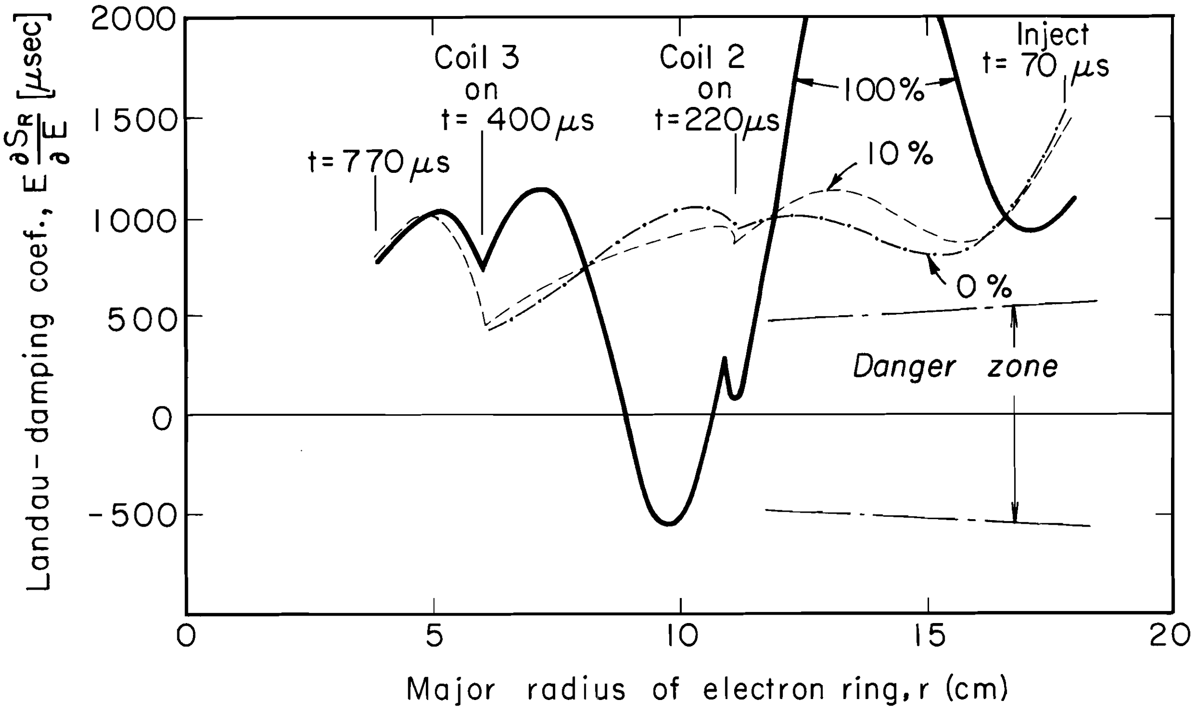

Fig. 5. Computed values of the Landau-damping coefficient of the first radial mode vs electron ring major radius for a typical ERA compression cycle. The "100%" curve corresponds to the Compressor 3 configuration of Fig. 1 with copper conductors. Two remaining curves correspond to computed case where eddy currents are arbitrarily reduced to 10% and 0% of those for the "100%" case.

through zero, which is unsatisfactory. The fields were then recomputed with eddy-current magnitudes successively reduced, yielding the corresponding Landau-damping coefficients also shown in Fig. 5. This led to the conclusion that eddy currents should be reduced to no more than 10% of their former magnitude.

Determining New Coil Configuration

The problem now shifted to finding a coil configuration for which the eddy currents would not exceed 10% of those of the copper configuration.

As the first step, the relative magnitude of the eddy-current bucking fields on the median plane were estimated for Coils 2 and for Coils 3L and 3R of the copper configuration. Specifically, the value of B_e/B was determined from Fig. 3 and eq. (9) for each conductor (with $\alpha = 0$) and then summed for all conductors in the coil. This calculation is summarized in Table II. Considering the approximations in the analysis, it was pleasing to note that the calculated values agreed well with the measured values--for instance, for Coils 3L and 3R at $t = 200$ μsec, the calculated value of $\Sigma B_e/B$ was 0.09 while the corresponding measured value was 0.08 with a measurement uncertainty of $\sim \pm 0.02$.

The next step was to arrive at a configuration for which the value of B_e/B did not exceed 10% that of the copper configuration. Examination of Fig. 3 and eq. (9) showed that this could be accomplished by either increased resistivity, decreased diameter, decreased wall thickness, or a combination thereof. Mechanical strength was a significant consideration, so most of the reduction in our case was accomplished by increased resistivity. For Coils 3L and 3R, the copper conductor was directly replaced by 0.375 in. o.d. x 0.065 in. wall #304 stainless-steel tubing which was rolled square to 0.340 in. across flats. For Coil 2, the copper conductor was directly replaced by 0.250 in. o.d. x 0.065 in. wall #304 stainless tubing. The calculated relative bucking fields for the new coils are also given in Table II. Their values are ~3% of the earlier

Table I. Compressor 3 coil parameters.

	COILS				
	Pair #1A	Pair #1B	Pair #2	#3L	#3R
No. of turns	24	18	24	24	150
Conductor size					
Outside, square (in.)	.255	.188	.255	.340	.340
Hole, round (in.)	.124	.115	.124	.184	.184
Mean radius (cm)	32.3	14.2	16.7	10.0	10.0

Table II. Estimate of relative eddy-current bucking field for two ERA compressor coil designs.

	COILS			
	"Earlier" copper		"New" stls. stl.	
	2	3L + 3R	2	3L + 3R
Resistivity, ρ [10^{-6} Ω-cm]	1.73	1.73	72	72
Outside radius, b [cm]	0.368*	0.493*	0.317	0.47*
Inside radius, a [cm]	0.158	0.234	0.152	0.31*
Characteristic flux penetration time, T_c, eq. (6), [μsec]	119	212	2.1	3.9
Time t at which electron ring passes coil [μsec]	70	200	70	200
t/T_c	0.59	0.94	33	51
Decay factor, f_d, from Fig. 3	0.6	0.5	0.03	0.02
No. of conductors	24	24 (3L) 150 (3R)	24	24 (3L) 150 (3R)
Nearest conductor, d_{min} [cm]	9.35	4.41	9.35	4.41
uniformly spaced to				
Furthest conductor, d_{max} [cm]	14.2	16.8 (3L) 103.0 (3R)	14.2	16.8 (3L) 103.0 (3R)
Σ (b^2/d^2), all conductors	0.049	0.180	0.037	0.163
Relative bucking field: $\Sigma \frac{B_e}{B} = f_d \cdot \Sigma \frac{b^2}{d^2}$	3×10^{-2}	9×10^{-2}	1×10^{-3}	3×10^{-3}

*Radius with approximately same second moment as actual shape.

values, which meets the desired criterion. Coils of the new geometry and material were fabricated and installed. Their magnetic field was measured by J. B. Rechen and J. M. Peterson, and the eddy-current effects were found to be very small, as anticipated.

If a further reduction in eddy currents had been necessary, a conductor consisting of a thinner-walled stainless-steel tubing overwrapped with multiple strands of insulated small-diameter wire would have been considered.

Since the perturbations were very small with the new configuration, the question arose whether their effect still needed to be simulated in the computer model described in the previous section. In the interest of accuracy, it was decided to retain the eddy-current simulation circuits but with their resistance and inductance based on an analytical solution[10] of the case in which the eddy currents are fully established ($t \gg T_c$).

Conclusion

Eddy currents in the conductors of fast-pulsed coils can produce significant perturbations in the magnetic field. For the ERA compressor under consideration, the magnitudes of the perturbations were estimated and suitable new configurations with reduced eddy effects were devised by use of the techniques presented herein. Magnet measurements confirmed that the eddy currents were significantly reduced. These same techniques should be applicable to other coil geometries as well.

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7. For instance, see W. R. Smythe, Static and Dynamic Electricity, (McGraw-Hill, N.Y., 1968) 3rd Ed., Sect. 10.23 for fields inside the tube. This solution could be extended to find fields outside the tube.

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