STABILITY CRITERIA FOR THIN SUPERCONDUCTORS SUBJECTED TO EXTERNAL PERTURBATIONS

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Abstract

A criterion for the occurrence of flux jumps in superconducting thin sheets subjected to infinitesimally small perturbations was proposed by Hancox, leading to the elaboration of adiabatically stabilized filamentary conductors. In the present paper, we show how this criterion is modified for the case of a finite perturbation. We establish relations which allow to take into account the deviations of the different parameters of the superconductor (temperature, current density) from their equilibrium values. Each external perturbation being assumed as disturbing the initial values of the parameters, we determine the perturbations initiating either a limited or a full flux jump, even for a superconductor obeying to the classical criterion of adiabatic stability. This method allows to take into account the heat transfer to the surrounding medium, and thus to give a more general criterion for initiation of flux jumps in a superconducting sheet with external heat transfer.

I. Introduction

The flux jumps in hard superconductors were theoretically studied by several authors, either in thin sheets[1,2] or in bulk samples[3,4]. This paper is a tentative to extend the Hancox's stability criterion[1] of thin superconducting strips subjected to infinitesimally small external perturbations, to the triggering of flux jumps by finite perturbations. We will consider, for simplification, two time intervals : during the first, the external (magnetic or thermal) conditions are varied ; we shall call "external perturbation" the phenomena occurring during this time ; then, when the variations of the external conditions have been stopped, the internal induction and the temperature continue their evolution, during a second time interval. The main part of this paper is devoted to this second time interval. In order to take into account the heat transfer with the surrounding medium , it is necessary to study the dynamics of the jumps. It will be assumed that the strip is thin enough, allowing the temperature to be uniform at each time. The dynamics of the jumps are then governed by the phenomena of flux creep and flux flow, acting on the flux lines displacement. For simplification of the mathematical treatment, we shall consider only the flux flow.

II. Basic hypothesis and notations

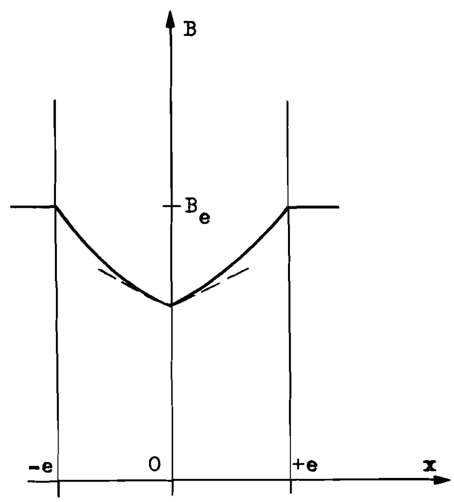

Fig. 1. Repartition of induction in the strip. The slope of the curve $B(x)_{x=0}$ is equal to $j_c(T)$.

A thin strip (fig. 1), limited by the planes of abscissae $x = e$ and $x = -e$, is subjected to a uniform external magnetic field, parallel to Oy, and assumed to be constant during the flux jump. At each time, the relative variations of the magnetic induction in the strip are small enough, leading to a value of critical current density depending only on temperature, that is $j_c(T)$. During each jump, the temperature increases from an initial temperature T_i until a final temperature T_f. The flux jump is said limited if $T_f < T_c$, and full if $T_f \geqslant T_c$. We shall only consider the range (T_i, T_f) if $T_f < T_c$, and (T_i, T_c) if $T_f > T_c$. The equilibrium between the driving force, the pinning force and the flux flow viscosity force acting on the fluxons , leads to the classical expression for the modulus of the critical current density :

$$j(x,t) = j_c[T(t)] + \frac{\eta[T(t)]}{\varphi_o} \cdot v(x,t) \quad (1)$$

In this expression, φ_o is the flux quantum (2.10^{-15} Wb), η the temperature dependent viscosity coefficient, and $v(x,t)$ the fluxoïd speed modulus, related to the electric field and induction moduli by :

$$v(x,t) = \frac{E(x,t)}{B(x,t)} \approx \frac{E(x,t)}{B_e} \quad (2)$$

where B_e is the value of B for $x = \pm e$. As an alternative, we may use in the formulae the flux flow resistivity, given by :

$$\rho_f = \frac{\eta}{\varphi_o B}$$

The two formulations being strictly equivalent, we will conserve in the following the viscosity.

Due to the symmetry, only the phenomena for $x > 0$ will be studied.

The value of the temperature $T(t)$ is obtained by a "thermal equation" between the dissipation (loss density $E(x,t).j(x,t)$), the heat absorbed by thermal capacity, and the heat evacuated by heat transfer towards the external medium (of temperature T_b). The whole absorbed power per unit surface yoz is $2e\, C(T).\frac{dT}{dt}$, $C(T)$ being the specific heat ; the evacuated power per unit surface of the plane $x = e$ will be writen $f(T,T_b)$.

The evolution of the flux jump may be known by solving a set of equations including Maxwell equations, (1) and (2), and the thermal equation :

$$\int_o^e E(x,t).j(x,t).dx = e.C(T).\frac{dT}{dt} + f(T,T_b) \quad (3)$$

The exact solutions of this set of equations seems to be inextricable. So, we have looked for approximate solutions. Noticing that, during a jump, $j(x,t)$ is an increasing function of x having for $x = 0$ the value $j_c[T(t)]$, we expand the current density at the first order :

$$j[x,T(t)] \approx j_c[T(t)] + \alpha[T(t)]x \quad (4)$$

The equilibrium corresponds to $\alpha = 0$. For a given material, α is a function depending specifically on the flux jump studied, and particularly on the initial triggering conditions. In the following, we shall determine the $\alpha(T)$ functions, by equalling the expressions of $j(x,T)$ given by (1) and (4).

We shall note henceforth :

$$\alpha(T) = \alpha \quad j_c(T) = j_c \quad C(T)=C$$

$$\frac{df(T,T_b)}{dT} = \mathring{f}(T,T_b)$$

$$\frac{d\alpha}{dT} = \mathring{\alpha} \quad \frac{dj_c}{dT} = \mathring{j}_c \quad \frac{dT}{dt} = \dot{T}$$

III. Evolution equations of a flux jump

Using Maxwell's equations, and the superconductor being supposed to have the vacuum's permeability μ_o, one derives from (4) :

$$B(x,t) = B_e + \mu_o[j_c(x-e) + \frac{\alpha}{2}(x^2-e^2)]$$

and

$$\frac{E(x,t)}{x} = -\mu_o \dot{T}[\mathring{j}_c e + \frac{\mathring{\alpha} e^2}{2} - \frac{\mathring{j}_c x}{2} - \frac{\mathring{\alpha} x^2}{6}] \quad (5)$$

And by using (3) :

$$e\dot{T}[\mu_o\frac{e^2}{3}\frac{d}{dT}(\frac{j_c^2}{2} + \frac{5}{8} e j_c \alpha + \frac{1}{5}e^2\alpha^2) + C] + f(T,T_b) = 0 \quad (6)$$

As a result of our approximation (4), the expression of α obtained by using (1), (2) and (4), is x-dependent. A further approximation will be to choose for $\alpha(T)$ value the x-mean value :

$$\alpha[T(t)] = \frac{\eta(T)}{\varphi_o B_e} < \frac{E(x,t)}{x} > \quad (7)$$

From (5) and (7)

$$\alpha = -\frac{\eta(T)}{\varphi_o B_e}.\mu_o(\frac{3}{4} e \mathring{j} + \frac{4}{9} e^2 \mathring{\alpha}) \dot{T} \quad (8)$$

We shall consider only the processus where $\alpha > 0$ and $\dot{T} > 0$; this leads to the necessary relation :

$$\mathring{\alpha} < -\frac{27}{16}\frac{\mathring{j}}{e} \quad (9)$$

The variation $\alpha(T)$ and $T(t)$, that is the evolution of the jumps are completely described by solving the set of equations (6) and (8). As an application, we shall consider now the stability criterion of a strip.

IV. Stability criterion

This model allows to introduce easily the thermal or magnetic external perturbations, as triggering causes of flux jump. Suppose that a variation of external conditions (magnetic field, or temperature) induce a penetration of fluxons in the strip, perturbing thus the equilibrium induction repartition, corresponding to $\alpha = 0$.

When the external perturbation has stopped, the temperature of the strip is T_i. The variation between the induction repartition in the strip at the end of the external perturbation and the equilibrium repartition at the temperature T_i is assumed to be described by a particular value α_i of the parameter $\alpha(T)$. Each external perturbation is thus characterized by a couple of values (α_i, T_i).

In our model, we express the triggering condition of a jump by the criterion

$$\left(\frac{d\alpha}{dT}\right)_{\substack{\alpha = \alpha_i \\ T = T_i}} > 0 \quad \text{, that is } \mathring{\alpha}_i > 0 \qquad \text{(with } dT > 0) \qquad (10)$$

This condition does not mean that the penetration of the flux in the strip stops if $\mathring{\alpha}_i < 0$; an increment of penetrated flux may occur at the end of the external perturbation, for restoring the equilibrium conditions. But the criterion (5) expresses that the induction repartition, just at the end of the external perturbation, continues to diverge from the equilibrium value. This unstable behaviour characterizes the flux jumps.

The fig. 2 summarizes different possible evolutions after an external perturbation for different strips, at given α_i, T_i, T_c.

Eliminating $\dot{T}$ between (6) and (8), one obtains

$$\mathring{\alpha} = -\frac{\frac{3\,C}{\mu_o e^2} + j_c\,\mathring{j}_c + \frac{5}{8}\,e\alpha\,\mathring{j}_c - \frac{9}{4}\,\frac{\eta\, f(T,T_b)}{\alpha e^2\ \varphi_o\ B_e}\,\mathring{j}_c}{\frac{2}{5}\,e^2\alpha + \frac{5}{8}\,e j_c - \frac{4}{3}\,\frac{\eta\, f(T,T_b)}{\alpha e\ \varphi_o\ B_e}} \qquad (11)$$

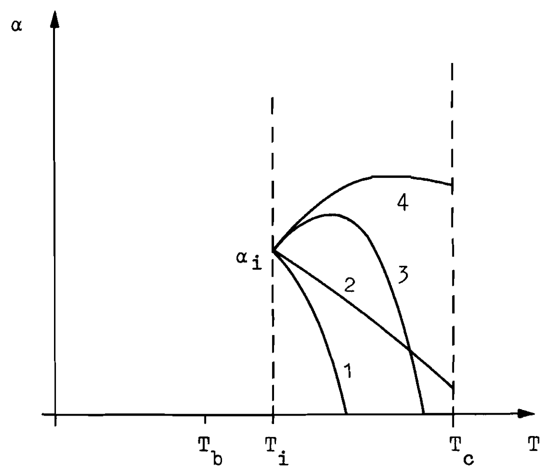

Fig. 2

Curve (1) : Restoration of the equilibrium without flux jump.

Curve (2) : Transition of the strip due to the external perturbation, but without flux jump.

Curve (3) : Flux jump, with restoration of the equilibrium.

Curve (4) : Flux jump, and transition to the normal state.

The application of the criterion (10) to the relation (11) allows to define the triggering conditions of the flux jumps. We shall use now the index i for indicating the values of the different parameters just at the end of the external perturbation, and we shall study successively the externally triggered and self-triggered flux jumps.

V. Externally triggered flux jumps

The flux jumps are externally triggered when they are produced by finite perturbations : $\alpha_i \neq 0$, $T_i \neq T_b$.

The representative curve of the function $\mathring{\alpha}_i$ versus f at given T_i and T_b is hyperbolic. The condition (9) is only satisfied for one branch of this hyperbola; after discussion, and as $\mathring{j} < 0$ for the materials studied here, only the branch corresponding to the positive values of the denominator of (11) is found to be convenient.

The application of the criterion (10) leads to :

$$\frac{3\,C_i}{\mu_o e^2} + j_{ci}\,\mathring{j}_{ci} < -\frac{5}{8} e\,\alpha_i\,\mathring{j}_{ci} + \frac{9}{4}\eta\frac{f(T_i,T_b)\,\mathring{j}_{ci}}{e^2\,\alpha_i\,\varphi_o\,B_e} \tag{12}$$

This formula allows to determine, for a given strip, the external perturbations leading to a flux jump, or inversely for a given perturbation, the value of e up to which the strip does not present flux jump. The two terms of the second member of (12) have opposite signs, showing if necessary that the heat transfer and the amplitude of α_i have opposite effects on the stability.

A particular case corresponds to the thermally insulated strip that is $f(T,T_b) \equiv 0$. We have plotted on Fig. 3, in reduced coordinates, the values of e up to which the insulated strip does not present flux jump, versus the characteristic parameters of the perturbations. One sees that an external perturbation can have simultaneously stabilizing effects, by increasing the temperature up to T_i, and degradating effects, by deviating the induction repartition from its equilibrium value.

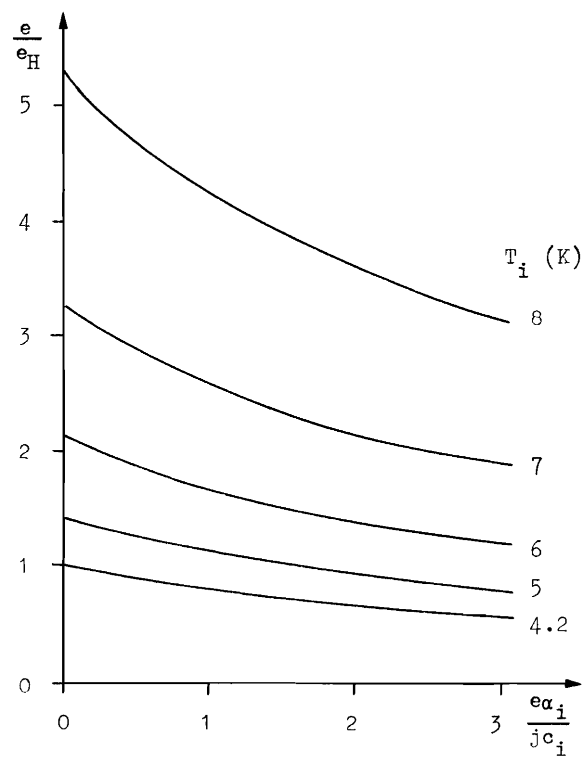

Fig. 3. In reduced coordinates : Critical value of e up to which the strip does not present flux jump, versus the parameters (α_i, T_i), of the external perturbation. The value e_H corresponds to the self-triggered flux jumps, at T_i = 4.2 K. This diagram is valid for a material with critical temperature T_c = 9.2 K.

Always for the thermally insulated strip, further information about the evolution of the jumps may be easily derived. The relation (6) can be written:

$$\frac{\mu_o e^2}{3}\frac{d}{dT}\left(\frac{j_c^2}{2} + \frac{5}{8} e\alpha j_c + \frac{1}{5} e^2\alpha^2\right) + C = 0 \tag{13}$$

The integration of (13) gives the evolution of α during the jump :

$$\alpha = -\frac{25}{16}\frac{j_c}{e} + \frac{1}{2\,e}\cdot\sqrt{-\frac{15}{64} j_c^2 + 10\, j_c^2(T_b) - \frac{60}{\mu_o e^2}\int_{T_b}^{T} C\,dT + A_i} \tag{14}$$

where A_i is a constant fixed by the initial perturbation :

$$A_i = \frac{60}{\mu_o e^2}\int_{T_b}^{T_i} C\,dT + 4e^2\alpha_i^2 + \frac{25}{2} e\alpha_i\, j_{ci} + 10\left[j_{ci}^2 - j_c^2(T_b)\right] \tag{15}$$

The temperature T_b has been arbitrarly introduced in the definition of A_i, such as $A_i = 0$ when $\alpha_i = 0$ and $T_i = T_b$. The expression (14) allows to know the limit of the perturbations leading to a "full" flux jump. The end of a limited flux jump is obtained when $\alpha(T_f) = 0$ (with $T_f > T_c$). The limit of the full flux jumps corresponds to $\alpha(T_f) = 0$ with $T_f = T_c$. One finds easily that this limit is obtained when A_i has the critical value :

$$(A_i)_c = \frac{60}{\mu_o e^2}\int_{T_b}^{T_c} C\,dT - 10\, j_c^2(T_b) \tag{16}$$

that is :

$$-\frac{3}{\mu_o e^2}\int_{T_i}^{T_b} C\,dT + \frac{e^2\alpha_i^2}{5} + \frac{5}{8} e\alpha_i\, j_{ci} + \frac{j_{ci}^2}{2} = 0$$

The condition for full flux jumps when $f(T,T_b) \neq 0$ is more difficult to obtain, since it would be at once necessary to solve (11), which needs a numerical integration.

VI. Self-triggered flux jumps

The self-triggered flux jumps correspond to the limit case of infinitesimally small perturbations, i.e. $\alpha_i = 0$, $T_i = T_b$. Equation (11), for $T \approx T_b$, must be solved by an expansion method. Noting δT a small increment of temperature above the temperature T_b, an expansion of α and f leads to

$$\begin{aligned} \alpha(T_b + \delta T) &= \delta T . \mathring{\alpha}(T_b) \\ f(T_b + \delta T, T_b) &= \delta T . \mathring{f}(T_b, T_b) \end{aligned} \tag{17}$$

When $\delta T \rightarrow 0$, the equation (11) leads to :

$$\frac{5}{8} e^3 j_{ci} [\mathring{\alpha}(T_b)]^2 + \left[\frac{3C_i}{\mu_o} + e^2 j_{ci} \mathring{j}_{ci} - \frac{4}{3} \frac{e \eta_i \mathring{f}(T_b,T_b)}{\varphi_o B_e}\right] \mathring{\alpha}(T_b) - \frac{9}{4} \frac{\eta_i \mathring{j}_{ci} \mathring{f}(T_b,T_b)}{\varphi_o B_e} = 0 \tag{18}$$

where the index i refers to the values of the parameters when $T_i = T_b$.

If

$$\frac{3 C_i}{\mu_o} + e^2 j_{ci} \mathring{j}_{ci} < 0 \tag{19}$$

the equation (18) has positive solutions when $\mathring{f}$ is external to the interval between the roots $\mathring{f}_1$ and $\mathring{f}_2$ of the determinant $(0 < \mathring{f}_1 < \mathring{f}_2)$; one has to consider only the solution of (18) which, by continuity, gives the solution of the problem with $f \equiv 0$. This solution holds with the condition (9), only when $0 < \mathring{f} < \mathring{f}_1$. This leads to :

$$\frac{4}{3} \frac{e \eta_i}{\varphi_o B_e} \mathring{f}(T_b,T_b) < \frac{3 C_i}{\mu_o} - \frac{71}{64} e^2 j_{ci} \mathring{j}_{ci} - \sqrt{\left(\frac{3C_i}{\mu_o} - \frac{71}{64} e^2 j_{ci} \mathring{j}_{ci}\right)^2 - \left(\frac{3C_i}{\mu_o} + j_{ci} \mathring{j}_{ci} e^2\right)^2} \tag{20}$$

As it can be seen directly by applying (10) to (11) when $f(T,T_b) \equiv 0$, the condition (19) refers to the self-triggered flux jumps of a thermally insulated strip. If the condition (20) does not hold, the strip is adiabatically stable without heat transfer, and is consequently stable when $f(T,T_b) \neq 0$. This explains that (19) and (20) have to be considered both together for the occurrence of flux jumps.

The condition (20) corresponds directly to the stability criterion of Hancox[1], calculated for a thermally insulated current -carrying strip. Using the Hancox's method for a strip carrying no current (case of this paper), it is easy to find (20).

On the fig. 4, we have plotted the variations of the values of e up to which the strip is stable, versus $\frac{\eta_i}{\varphi_o B_e} \mathring{f}(T_b,T_b)$ for typical values of the other parameters.

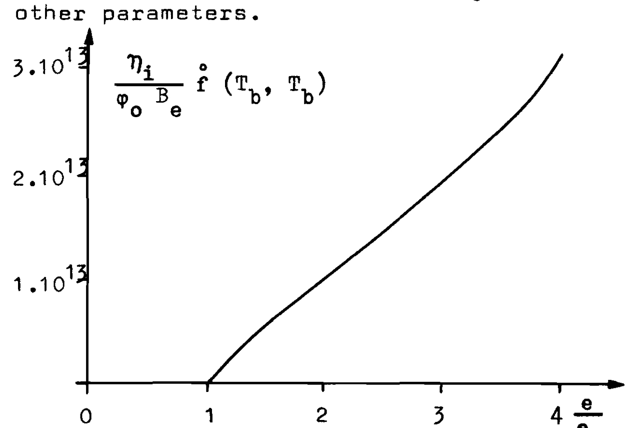

Fig. 4. Typical influence of the heat transfer on the critical value of e up to which there is not self-triggered flux jumps, calculated for a Nb-Ti alloy, at $B_e = 2.4$ T and $T_b = 4.2$ K. The critical current density is given by [7] and the specific heat by [8]. The value of e_H is approximately 10^{-4}m.

VII. Relation between perturbations and the triggering conditions of a flux jump.

The preceding method allows to define the triggering conditions of a flux jump. The problem which has to be solved now is to calculate the temperature variations and the disturbance of induction repartition generally simultaneously produced by a given thermal or magnetic external perturbation. It seems that the general method would be to describe the induction repartition by the expression (4) at each time of the external perturbation, and to use the approximation (7). Introducing the variation of surface induction $B_e(t)$ and its time derivative $\dot{B}_e$, and assuming the external thermal perturbations equivalent to a volumic heat input $\dot{Q}(t)$, it is easy to establish a set of two differential equations, extension of (6) and (8) :

$$\alpha = -\frac{\eta}{\varphi_o B_e(t)} \left[\mu_o \left(\frac{3}{4} e \frac{dj_c}{dt} + \frac{4}{9} e^2 \frac{d\alpha}{dt}\right) - \frac{dB_e}{dt}\right]$$

$$\dot{T}\left[\mu_o \frac{e^3}{3} \frac{d}{dT}\left(\frac{j_c^2}{2} + \frac{5}{8} e j_c \alpha + \frac{1}{5} e^2\alpha^2\right) + eC\right] - \frac{dB_e}{dt}\left(j_c \frac{e^2}{2} + \alpha\frac{e^3}{3}\right) - eQ(t) + f(T,T_b) = 0 \quad (21)$$

This set of equations can be solved in particular cases ; for example, assuming simultaneously isothermal process, no heat input, a uniform time-variation of B_e and that the critical current density does not depend on B, one obtains as an asymptotic solution, when the perturbation has been applied for a long time :

$$\alpha(t) \approx \frac{\eta}{\varphi_o B_e} \frac{dB_e}{dt} \quad (22)$$

More merely, by using the "flux flow resistivity" $\rho_f = \frac{B_e \varphi_o}{\eta}$,

$$\alpha(t) \approx \frac{1}{\rho_f} \frac{dB_e}{dt} \quad (23)$$

For a material where the flux flow resistivity is known, it is thus possible to evaluate the value of $\alpha(t)$ during this particular perturbation.

VIII. Conclusions

In this paper, we have developed a method for taking into account the influence of the amplitude of external magnetic or thermal perturbations, as triggering causes of flux jumps in thin superconducting strips.

For a thermally insulated strip, we find that only very large and particular perturbations, acting mainly on the induction repartition, may lessen significantly the boundary value of the thickness corresponding to stability without perturbation (Hancox criterion).

For the strip with heat transfer, the stability depends on the flux flow resistivity ρ_f [5] classically related, in real materials for high values of the electric field, to the normal state resistivity ρ_n, by :

$$\rho_f = \rho_n \cdot \frac{B}{\mu_o H_{C2}} \quad (24)$$

Using this expression and considering for example a heat transfer directly to liquid helium, lead to a boundary value of the thickness only a slightly higher than the Hancox value. But it must be noticed that for the real high pinning superconductors such as Nb-Ti, the expression (1) is a very crude approximation for the low values of the electric field corresponding to the self-triggered flux jumps. Indeed, the flux creep phenomenon then plays a leading part; the apparent flux flow resistivity is strongly reduced, and depends on the electric field[6], ratios of more than 10^{-3} between the measured apparent flux flow resistivity and the value ρ_f given by (24) have already been observed. This means that probably the condition of occurrence of self-triggered flux jumps, given by (19), with $\frac{\eta i}{\varphi_o B_e} = \rho_f$, does not hold, the strip being more stable than predicted.

At last, it must be noticed that we restricted ourselves in this paper to the setting up of the stability conditions. In addition, it would be possible, by solving numerically (11), to know $\alpha(T)$, and with (6), to deduce $T(t)$, that is the kinetics of the flux jump in a strip.

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