RESIDUAL FIELDS IN SUPERCONDUCTING MAGNETS

M. A. Green
Institut für Experimentelle Kernphysik
Kernforschungszentrum, Karlsruhe
W. Germany

Abstract

The residual field phenomena in superconducting magnets becomes important when one must operate these magnets at low as well as high fields. Residual fields are caused by circulating currents in the superconducting filaments and circulating currents between the filaments through normal conducting metal. The former produces frequency independent fields; the latter type of residual field is frequency dependent.

The theory for residual fields in various kinds of superconducting magnets is presented. The reason for the undesirable multipole content of the residual field is explained and methods for reducing or eliminating the effects of residual fields in superconducting magnets are discussed.

I. Introduction

The development of superconducting magnets for physics application is at the same level of advancement as conventional magnets were 20 years ago. The performance of superconducting magnets has improved enough that we can consider their use in many pieces of physics hardware. It is well known that conventional magnets have residual fields which adversely affect their performance at low fields. The iron in conventional magnet shapes the field and is the cause of residual field. In superconducting magnets, the superconductor shapes the field and causes the residual field observed by a number of observers. [1,2,3]

The residual field found in copper iron magnets results from the hysteretic behaviour of the iron. In superconducting magnets, hysteretic behaviour is also responsible for the residual field effects. The hysteresis observed in superconductors is caused by circulating currents which flow in the superconductor and surrounding normal metals. Hence, the study of residual fields is directly related to the study of superconductor a.c. losses and instability which are caused by the same phenomena.

Residual fields are here defined as fields which are generated by circulating currents in the superconducting filaments. These fields have the following properties: 1) they have very long time constants, 2) they vary with previous flux changes. Penetration effects have been observed, 3) residual fields disappear when the superconductor temperature is raised above the critical temperature, 4) the residual field is very rich in higher multipole components.

Residual fields can be explained theoretically by using doublet theory. This theory, which is developed here, is adequate to show why residual fields behave as they do. Before proceeding the reader should be cautioned that all calculations are done in rationalized MKS units with $\mu_o = 4\pi \times 10^{-7}$.

II. Basic Theory

The field generated by a single current I traveling perpendicular to the x,y plane can be represented in complex form ($Z = x + iy$) as follows: [4,5]

$$H'^*(Z) = \frac{I}{2\pi i}\left(\frac{1}{Z-Z_c}\right) \qquad (1)$$

where $H'^*(Z)$ is the complex conjugate of the field $H'(Z)$, Z_c the location of the current, and Z the point where the field is calculated. Equation 1 can be extended to the calculation of field generated by circulating currents in superconducting filaments. The circulating currents in a superconducting filament can be represented by a current +I at Z_{c1} and a current -I at Z_{c2}. The distance between these currents is d and they are inclined at an angle α (see Fig. 1). We find that the field generated at Z is

$$H''^*(Z) = \frac{I}{2\pi i}\left[\frac{1}{(Z-Z_{c1})} - \frac{1}{(Z-Z_{c2})}\right] = \frac{I}{2\pi i}\left[\frac{d\ e^{i\alpha}}{(Z-Z_{c1})\ (Z-Z_{c2})}\right] \qquad (2)$$

If $Z_c = (Z_{c1}+ Z_{c2})/2$ and $|Z-Z_c| >> d$, then $Z-Z_{c1} \to Z-Z_c$ and $Z-Z_{c2} \to Z-Z_c$ hence

$$H''^*(Z) = \frac{I\ d\ e^{i\alpha}}{2\pi i(Z-Z_c)^2} \qquad (3)$$

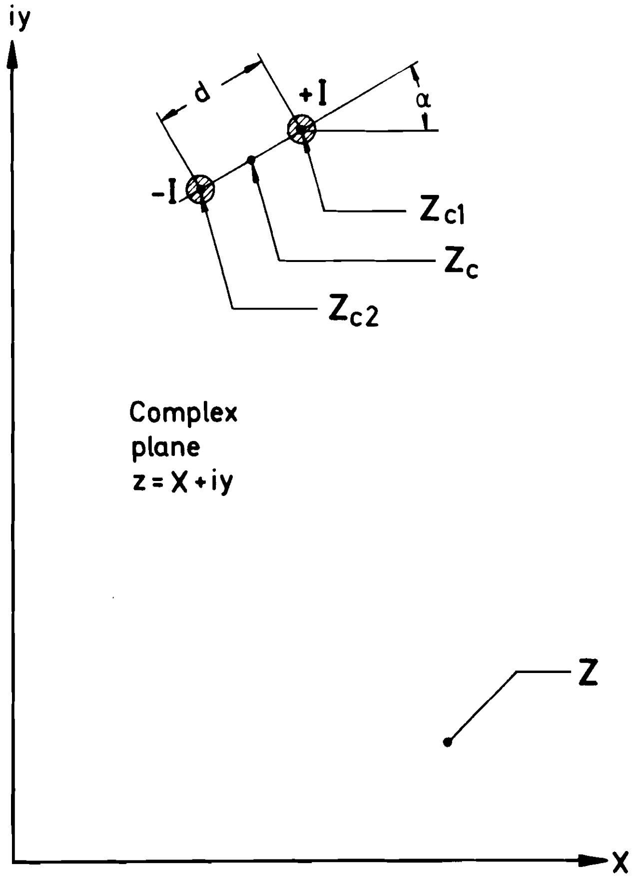

Fig. 1: A simple finite current doublet in the complex plane.

Equation 3 is the classical doublet equation from hydrodynamics.[6] The strength of the doublet which is defined as Γ = Id and the doublet angle α are functions of the previous flux history of the superconducting filament. For a round filament with a radius a = D/2 we find that the doublet strength factor is;

$$\Gamma = 4 \int_0^a J_c(x)\ (a^2-x^2)^{1/2} x\ dx \qquad (4)$$

The functional relationship $J_c(x)$ is often difficult to find because J_c is a function of T and H in the superconductor. Parts of the superconductor which have had no magnetic field change have no circulating currents in them. Thus we find that $J_c(x)$ really is a J_c (H, $\dot{H}$, T) where H, $\dot{H}$ and T are functions of x (see Fig. 2).

If the superconducting filament diameter D is small, one can assume that the magnitude of J_c is uniform wherever it exists in the filament. This J_c is a function of the local field H which occurs at the filament boundary x = a.

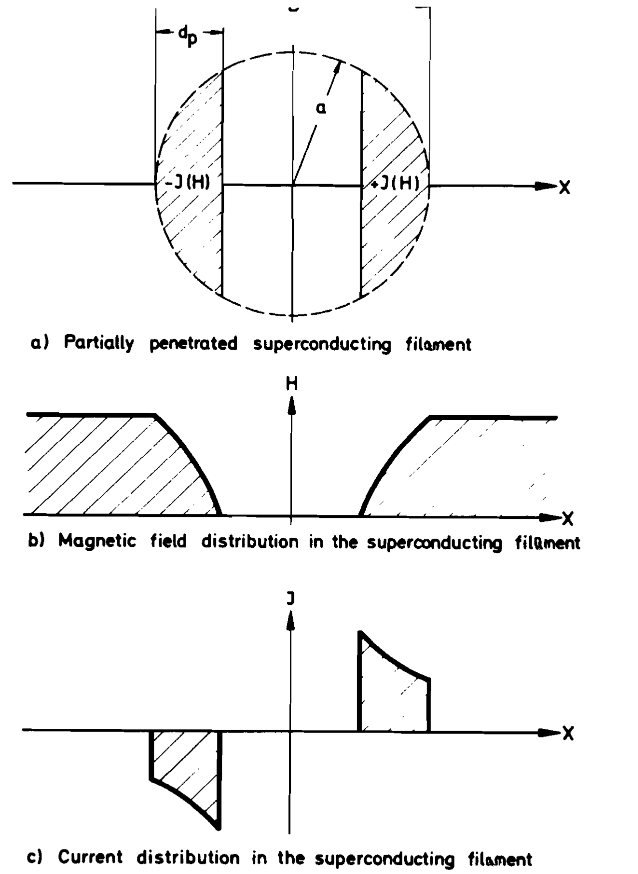

Fig. 2: Current and field distribution in a superconducting filament.

In the simplified model, $J_c(x)$ is either $+J_c(H)$, $-J_c(H)$, or zero. Using this model one finds that:

$$\Gamma = \varepsilon_o \left[\frac{\pi D^3}{8} \right] J_c(H) \qquad (5)$$

when the filament has been fully penetrated by a particular $\dot{H}$, $\varepsilon_o = 4/(3\pi) \approx 0.423$. Hence, for this case d = 0.423 D and $I = J_c \pi D^2/8$. Equation 5 can be used even when a superconductor has not been fully penetrated by a flux change. Figure 3 shows ε_o as a function of the penetration distance d_p. Using our simplified model we find that

$$d_p \approx \frac{\Delta H}{J_c(H)} = \frac{\Delta B}{\mu_o J_c(B)} \qquad (6)$$

the effect of multiple penetrations can also be calculated using figure 3.[7]

Doublet strength factors may also be calculated for superconducting filaments which carry a transport current. The addition of a transport current, in general, only affects the magnitude of

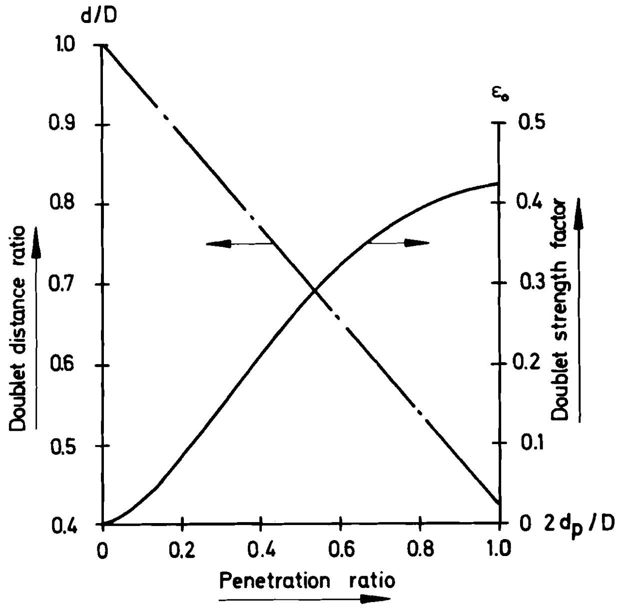

Fig. 3: Doublet strength factor Vs superconductor penetration.

the circulating current I not the doublet distance d. When the filament carries a transport current one finds that;

$$\Gamma = (1 - \delta)\varepsilon_0 \left[\frac{\pi D^3}{8}\right] J_c(H) \qquad (7)$$

where

$$\delta = \frac{J_T}{J_c(H)} \qquad (8)$$

J_T, the transport current density, is defined as the transport current I_T of the filament divided by its area $\pi D^2/4$. J_T can be no larger than $J_c(H,T)$, hence $0 \leq \delta \leq 1$.

III. Residual fields in solenoid magnets

The theory shown in the previous section can be extended to solenoid magnets. I shall not dwell on this point because my main field of interest is superconducting dipole and quadrupole magnets. Instead I will present the residual field generated by a simple circular loop of radius b which lies in the x, y plane at w = 0. Along the w axis only the H_w component of field exists; this is

$$H''_w = \frac{\Gamma}{r^2} \cos(\theta - \alpha) \qquad (9)$$

where Γ is defined as in the previous section, $r = \sqrt{b^2 + w^2}$, α is the doublet angle with respect to the x, y plane, and $\theta = \tan^{-1}(w/b)$. The above equation should be compared with the field generated on the w axis by a simple circular loop carrying a transport current which is

$$H'_w = \frac{I\, b^2}{2r^3} \qquad (10)$$

Equations 9 and 10 can be integrated over a real solenoid. The angle α in equation 9 is a function of the previous magnetic field direction in the coil. One may define $\alpha = \phi + \pi/2$, where ϕ is the flux line angle with respect to the x, y plane. When $\alpha \neq 0$, one can see that behaviour of equation 9 with respect to w is quite different from the behaviour of equation 10. Equation 9 also behaves differently from equation 10 with respect b regardless of the value of α. The conclusion that one may draw from this is as follows: the residual field which is generated by circulating currents in a solenoid does not have the same structure as the field generated by the transport current. As a result, superconducting solenoids which are designed to produce uniform fields in a particular region will not produce uniform residual fields in that region. This can result in undesirable field inhomogeneities even at high fields if the superconductor diameter is large enough.

IV. Residual fields in two dimensional dipole and quadrupole magnets with no iron

Equations 1 and 3 may be expanded by a Taylor series. The expansion of equation 1 is as follows:

$$H'^*(Z) = \sum_{N=1}^{\infty} a'_N Z^{N-1} \qquad (11a)$$

where

$$a'_N = \frac{-I}{2\pi i} Z_c^{-N} \qquad (11b)$$

The expansion of equation 3 is as follows:

$$H''^*(Z) = \sum_{N=1}^{\infty} a''_N Z^{N-1} \qquad (12a)$$

where

$$a''_N = \frac{-\Gamma e^{i\alpha}}{2\pi i} N Z_c^{-(N+1)} \qquad (12b)$$

The radius of convergence for both power series is $|Z_c|$. Most dipoles and quadrupoles of interest are symmetrical (see Fig. 4). As a result 11b and 12b can be further simplified. If one defines $Z_c = r_c e^{i\theta_c}$ we find that

$$a'_N = -\frac{T\, I}{\pi i} \cos(N\theta_c) r_c^{-N} \qquad (11c)$$

$$a'_N = \ldots \quad \text{when } N = T(2P+1), \; P = 0, 1, 2, \ldots$$

$$a'_N = 0 \quad \text{when } N \neq T(2P+1), \; P = 0, 1, 2, \ldots$$

and (12c)

$$a''_N = -\frac{T\,\Gamma}{\pi i} N \cos((N+1)\theta_c - \alpha) r_c^{-(N+1)}$$

when $N = T(2P+1)$
$P = 0, 1, 2, \ldots$

$$a''_N = 0$$

when $N \neq T(2P+1)$
$P = 0, 1, 2, \ldots$

where T is defined as the fundamental multipole number (T = 1 is a dipole; T = 2 is a quadrupole).

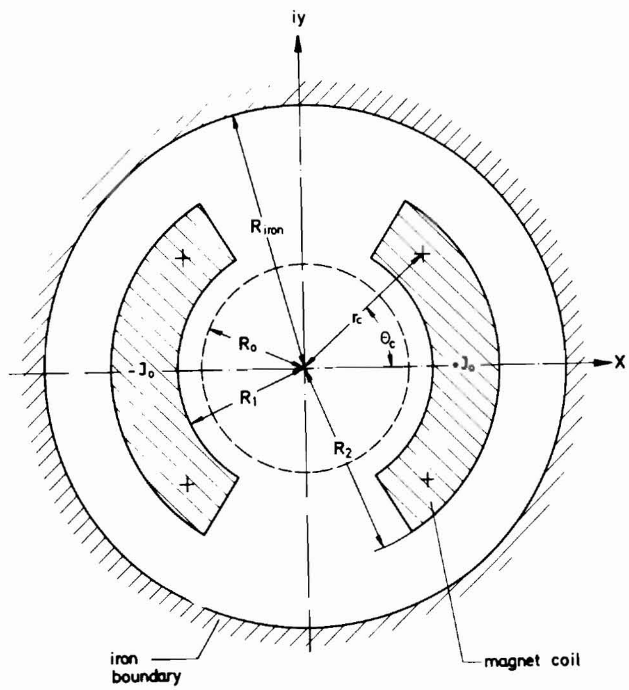

Fig. 4: A single layer symmetrical 1 dipole magnet with an iron shell (the crosses mark points of symmetry).

Equations 11c and 12c can be integrated over the two dimensional coil section. The integral of 11c gives the field generated in the magnet by the transport current; the integral of 12c gives the residual field.

V. Residual field in two dimensional dipole and quadrupole magnet with a circular iron shell

Equations 11 and 12 can be extended to include a circular iron shell of radius R and with $\mu = \infty$.[4] When the center of the shell coincides with the center of the coil, the field can be calculated by simply reflecting the current off the shell. Thus we find the expansion for a symmetrical dipole carrying transport current only is as follows:

$$H'^{*}(Z) = \sum_{N=1}^{\infty} c'_N Z^{N-1} \tag{13a}$$

where $c'_n = a'_N + b'_N$; a'_N is defined by equation 11c and b'_N is defined as follows:

$$b'_N = \frac{T\,I}{\pi i} \cos(N\theta_c) \frac{r_c}{R^{2N}} \tag{13b}$$

when $N = T(2P+1)$
$P = 0, 1, 2, \ldots$

$$b'_N = 0$$

when $N \neq T(2P+1)$
$P = 0, 1, 2, \ldots$

One should note the similarity of the θ terms in equation 11c and 13b. This is the key to magnets which will generate a good field whether or not the iron shell is present.[8]

Reflection of a current doublet is not as simple as the reflection of a single current.[9] The doublet strength factor Γ_r of the reflected doublet is different from the Γ of the non-reflected doublet. The doublet angle α_r of a reflected doublet is different from the α of the non-reflected doublet (see Fig. 5). α_r and Γ_r can be related to α and Γ as follows:

$$\alpha_r = \alpha + \pi + 2\theta_c \tag{14a}$$

and

$$\Gamma_r = \frac{R^2}{r_c} \Gamma \tag{14b}$$

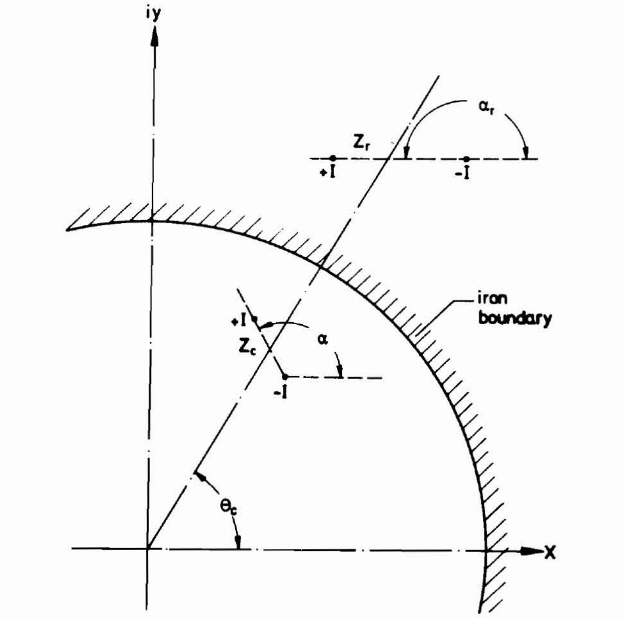

Fig. 5: Reflected doublet off of a circular iron shell.

Using equation 14a and 14b one finds that:

$$H''^{*}(Z) = \sum_{N=1}^{\infty} c''_N \, Z^{N-1} \qquad (15a)$$

where $c''_N = a''_N + b''_N$; a''_N is defined by equation 12c and b''_N is defined as follows:

$$b''_N = \frac{T\,\Gamma}{\pi i} N \cos((N-1)\theta_c + \alpha)\frac{r_c^{\,N}}{R^{2N}} \qquad (15b)$$

when $N = T(2P+1)$, $P = 0, 1, 2, \ldots$

$$b''_N = 0$$

when $N \neq T(2P+1)$, $P = 0, 1, 2, \ldots$

A superconducting dipole or quadrupole will generate fields with circulating and transport currents at the same time. The resulting field can be found by adding equations 13a and 15a. Hence, we find that:

$$H^{*}(Z) = \sum_{N=1}^{\infty} c_N \, Z^{N-1} \qquad (16)$$

where $c_N = c'_N + c''_N$. If one desires induction instead of field then

$$B^{*}(Z) = \mu_0 H^{*}(Z) \qquad (17)$$

VI. Computer calculations of residual fields

Equations 11c, 12c, 13b, 15b and 16 are used directly in a computer program to calculate the magnetic fields generated by a complex magnet coil. The program calculated the residual field in the aperture of the Karlsruhe Dipole D-2a coil shown in figure 6. A detailed description of this magnet is given elsewhere in these proceeding, [10] hence only the data which is important for calculating residual fields is given here. The coil has the following properties:

1) The useful aperture radius is 30 mm; the inner coil radius (the radius of convergence for the power series) is 40 mm.

2) The coil consists of 420 turns each of which forms a calculation point for calculating α, ϕ, Γ, a''_N, and b''_N. These calculation points lie at the geometric center of each turn.

3) The coil outer radius is 76 mm. Iron shells of radii varying from 76 mm to 200 mm were used in the calculation.

4) The coil shown in figure 6 will generate a transport current field which is better than 1 part in 1000 inside the 30 mm useful aperture radius.

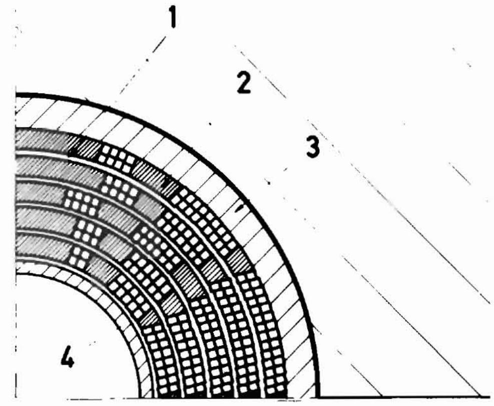

Fig. 6: Karlsruhe dipole D-2a cross section 1) cooling channel, 2) epoxy spacer, 3) coil support ring, 4) magnet cold bore tube.

The superconductor used in the calculations has a J_c versus B characteristic which is shown in figure 7. The IMI soldered cable which will be used in dipole D-2a is believed to have similar J_c versus B characteristics.[11] The diameter of the superconducting filaments is assumed to be 10 μm (the actual dipole D-2a superconductor will have filament diameters ranging fom 10 - 12 μm) the assumed normal metal to superconductor ratio is 1 to 1, and the cable packing factor (including insulation) is about 0.25. The computer program assumes that the superconducting filaments are totally decoupled from one another. (This assumption is valid in real magnets if one measures residual fields some time after a flux change.)

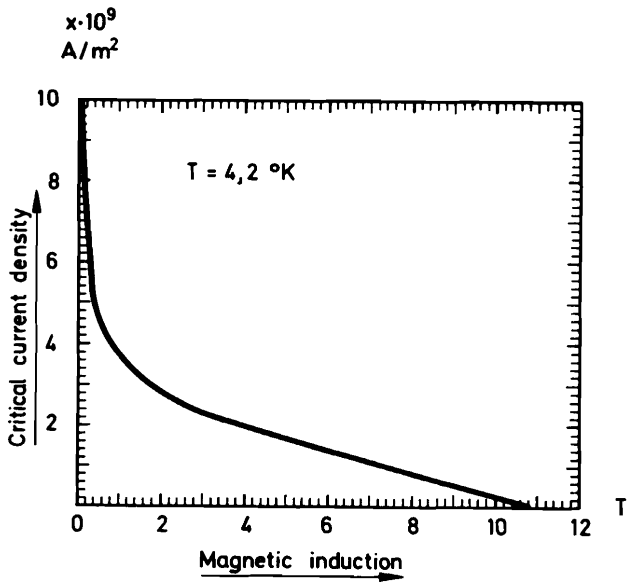

Fig. 7: J_c Vs B for a typical Nb-Ti superconductor at 4.2 K.

The program will start at any initial field and calculate after up to 2 flux changes (At the initial field the program assumes that the flux is excluded from the filament.) The program includes the effects of transport current and notifies the programer if at sometime during the magnet cycle, the magnet current exceeds the critical current. The program defines the flux line angle ϕ in the counterclockwise direction from the positive x axis. By definition $\alpha = \phi + \pi/2$.

Table 1 compares the transport current and circulating current field (residual field) after the computer charged the magnet from 0 to 0.4 T by increasing the transport current in the coil from 0 to 175 A. The superconductor in the coil has a penetration induction of about 0.06 T. Nearly all of the superconductor in is fully penetrated by the flux when the central induction reaches 0.2 T. The multipole ratios given in Table 1 illustrate the fact that a magnet which is designed to produce a good dipole field will have a residual field which is rich in higher multipoles. The predicted 98 % sextupole at the 30 mm radius is typical of the sextupole measured at Berkeley and Rutherford.[1,2]

Figure 8 shows the dipole component of residual induction as a function of the central induction generated by the transport current. At the start of the first cycle there are no circulating currents hence no residual induction. As the superconductor in the coil gets penetrated by the rising field, the residual field rises.

Table 1: Computer calculations of transport current and residual inductions at a current of 175 A in dipole D-2a with no iron

	Transport current induction	Residual induction
Dipole component	0.4 T	1.52×10^{-4}T
Multipole number	Multipole ratio*	
1	1.0000	1.0000
2	-	-
3	0.0003	0.9863
4	-	-
5	-0.0000	0.0001
6	-	-
7	0.0002	0.0548
8	-	-
9	-0.0001	-0.0264
10	-	-
11	-0.0002	0.0547

*at a radius of 30 mm

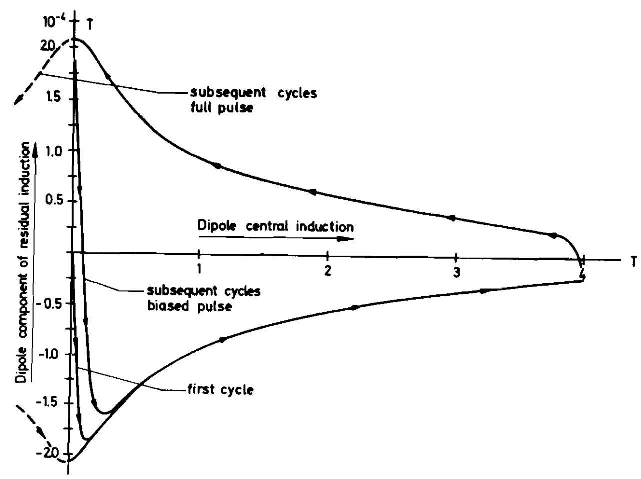

Fig. 8: A residual field hysteresis loop.

Soon most of the filaments are penetrated; the residual field is now controlled by the J_c which is a function of the local H. As the central field rises, so does the transport current which generates it. The combined effect of reduced critical currents and increased transport current reduces the residual field generated at high fields. The dipole component of residual field generates a hysteresis loop. The width of this loop at zero field is analogous to the coersive force in iron. The residual field hysteresis loop area is proportional to the a.c. loss in the magnet.

The relationship between the residual field and the a.c. losses is an important one because it means that magnets with low a.c. losses will also have low residual fields.

The addition of an iron shell changes the residual field. The a.c. losses are also slightly changed but in a different way. Residual field changes due to the iron are illustrated in figure 9 (the magnet was excited from 0 to 4 T and back to zero in all cases). The primary reason for the change of sign of the residual field was due to the change in α when the iron shell is added. The kind of behaviour shown in the computer results has not been observed experimentally to my knowledge. The hysteresis loop generated by one of the iron cases would have a smaller area than the one shown in figure 8. This does not mean that the a.c. losses are smaller. The a.c. losses are a measure of the magnitude of circulating currents. The residual field is the sum of positive and negative fields generated by these circulating currents.

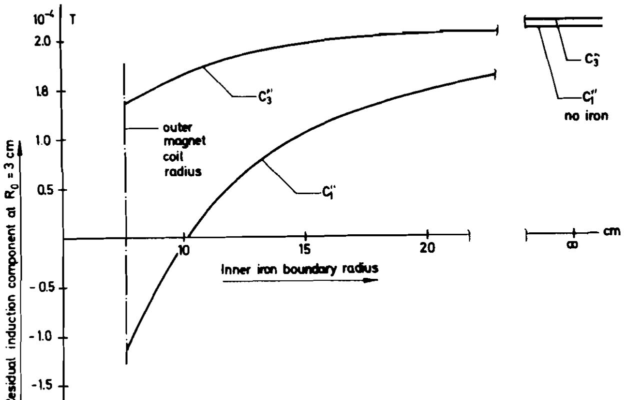

Fig. 9: Residual field components Vs inner iron radius.

VII. The effect of time dependent circulating currents in superconducting magnets

Circulating currents in superconducting magnets are not limited to the superconducting filaments. Circulating current may also take to form of coupled currents between superconducting filaments and ordinary normal metal eddy currents. The theory presented here can be used to calculate the order of magnitude of fields generated by superconductor coupled currents and ordinary eddy currents as well.

Ordinary eddy currents have been the cause of magnet aberrations in conventional fast cycling synchrotrons for years. The magnitude of an ordinary eddy current is proportional to the impressed voltage and inversely proportional to the resistance of the circuit. The field generated by an eddy current is proportional to the current. This field can be calculated for a tube or rod as follows:

$$H_{eddy} \approx \frac{\tau_e}{t_{rise}} H_{max} \qquad (18)$$

when $\tau_e << t_{rise}$, where t_{rise} is the rise time for a field from 0 to H_{max} and τ_e is the eddy current time constant. The eddy current time constant for a tube of diameter D and wall thickness s is;

$$\tau_e \approx \frac{\mu_o \, D \, s}{2N \, \rho_v} \qquad (19a)$$

the eddy current time constant for the wire or rod of diameter D is;

$$\tau_e \approx \frac{\mu_o \, D^2}{8N \, \rho_v} \qquad (19b)$$

where ρ_v is the volume resistivity (ohm m), N is the exciting field multipole number and $\mu_o = 4\pi \times 10^{-7}$.

The field generated by eddy currents inside a rod or tube has the same basic multipole structure as the exciting field [12] (no new multipoles are added, but the higher multipoles are decayed with respect to the lower ones). Outside a tube or rod the eddy current field structure resembles that of the superconductor residual field. Thus we find that one can use doublet theory if Γ is replaced by:

$$\Gamma \approx 0.4 \, D^2 \, H_{eddy} \qquad (20)$$

Multicore superconductors will exhibit coupled current effects, which are caused by large superconductor normal metal eddy currents. These currents do not behave precisely like eddy currents because the current can be limited by the J_c of the superconductor. When the time constant of the coupled currents is much longer than the superconducting magnet rise time, the coupled current behaves like a superconducting circulating current. When the reverse is true the coupled current behave like ordinary eddy currents. Coupled currents in superconductors will also give rise to fields which are similar in structure to the residual field. They will be frequency dependent if the superconductor is twisted or transposed. These fields should be of little importance in d.c. magnets. In a.c. magnets, such as superconducting synchrotron magnets, the field generated by the superconductor coupled currents can cause some problems. To first order, the magnitude of the coupled current field is proportional to the a.c. loss due to coupled currents. [13]

VIII. Summary

Residual fields can be controlled at injection into a superconducting synchrotron. The problem is compounded by the fact that superconductors vary in properties from batch to batch. Twenty percent variations in critical current are not uncommon amoung superconductors from a single manufacturer. Superconductor manufacturers are in general unconcerned about the low field properties of their material; furthermore, the diameters of the filaments in a matrix are often not uniform.

Residual field can be reduced by 1) decreasing the superconductor filament diameter, 2) reducing the circulating coupled current by increasing matrix resistivity, and 3) tailoring the metallurgical processes to reduce the superconductor low field J_c. A much greater understanding of the superconductor low field J_c is clearly needed. Measurements at Berkeley indicate low field J_c can be quite high. [14]

Residual field and a.c. loss calculations show that the type of superconductor used in a synchrotron magnet will often be determined by residual fields, not a.c. losses, when low injection fields are required. One also finds that magnet to magnet variations of residual field are very important (one should avoid building a synchrotron with superconductor supplied by two different manufacturers). Computer calculations indicate that alterations of a synchrotron magnet cycle can have profound effects on the residual field at injection.

In short, residual fields may be important in all kinds of superconducting magnets. Residual fields and other circulating current fields should not be ignored during the design of any. Superconducting magnet which must produce predictable high uniformity field over a wide range of current excitations.

REFERENCES

1. M.A. Green, IEEE Transactions on Nuclear Science NS-18(3) p. 664 - 668, June 1971.
2. Rutherford measurements of residual fields in AC-3, "Towards a European Superconducting Synchrotron" a report of the work of the GESSS Collaboration, GESSS-1 p.17(May 1972).
3. E.U. Haebel, F. Wittgenstein, Proc. International Conference on Magnet Technology Hamburg 1970, p. 874 - 895 (1970).
4. K. Halbach, Nuclear Instruments and Methods, 78 (1970) p. 185 - 198.
5. R.A. Beth, Proc. of the 1968 Summer Study on Superconducting Devices and Accelerators, BNL 50155, part III, p. 843 - 859 (1968).
6. L.M. Milne-Thomson, Theoretical Hydronamics, (MacMillan Company, N.Y., 1960) pp. 204 - 205.
7. M.A. Green, "The Theory of Residual Fields in Two Dimensional Dipoles and Quadrupoles without Iron Shields" Lawrence Berkeley Laboratory report UCID 3508 (1971).
8. H. Brechna and M.A. Green, Proc. 1972 Applied Superconductivity Conference, Annapolis Md. (to be published in the Journal of Applied Physics).
9. M.A. Green, "A simplified theory for residual fields generated by superconducting dipoles and quadrupoles with unsaturated iron shields" Kernforschungszentrum Karlsruhe report Ext. Ber. 3/71-10 (1971).
10. "The Karlsruhe Pulsed Superconducting Magnet Programme" Status report by the Karlsruhe IEKP Magnet Group (to be published in these conference proceedings) 1972.
11. R.G. Hamphire, J. Sutton, M.T. Taylor, Conference on Low Temperature and Power London 1969.
12. W.R. Smythe, Static and Dynamic Electricity (McGraw Hill, New York, 1950).
13. M.A. Green, "Field Aberrations in Superconducting Dipole and Quadrupole Magnets for a Superconducting Synchrotron", Kernforschungszentrum Karlsruhe report Ext. Ber. 3/71-7 (revised in June 1972).
14. Ferd Voelker LBL, private communication.