

MATERIALS AND COST ANALYSIS
OF
CONSTANT-TENSION MAGNET WINDINGS FOR TOKAMAK REACTORS*

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Abstract

The successful design of a tokamak fusion reactor for economic power generation depends to a great extent on the costs associated with the magnets required to develop the magnetic fields. In this paper emphasis is placed on the design of constant-tension coils in which mechanical and electrical stability are enhanced by keeping the material stresses elastic. The numerical analysis techniques used are described and the design curves, cost analyses and material requirements are presented for a magnet design for a 1000 MW electrical reactor. The toroidal magnets require approximately 15,000 tons of steel and copper.

I. Introduction

One of the key problems in eventually attaining a commercially feasible fusion reactor is to design an economical superconducting containment magnet. The University of Wisconsin¹ feasibility study has considered the tokamak system in which the superconducting magnet is a very large toroid, 12.5 m major radius and 5 m minor radius. The blanket, shield, walls and supports leave a free and clear region approximately 2.5 m in radius for the plasma. If the central field is 5.148 tesla then a self-consistent operating point at 1000 MW electrical seems possible.²

Two toroidal cross sections are possible: (1) a circular cross section with external reinforcement rings to resist bending and (2) a "D" shaped magnet which provides a constant tension winding region without external rings. Lubell³ has designed a toroidal system based on the circular cross section and has presented relations for extrapolation in size and field within the operating field region of NbTi. File⁴ has suggested the use of constant tension toroids and has outlined some of the design considerations. We at Wisconsin are designing our first reactor with "D" sections in order to provide extra space above and below the circular plasma-shield-blanket region in which to place vacuum pumps and diverters for unwanted particles. Frequent use is made of designs introduced by Purcell and Desportes⁵ for the NAL bubble chamber magnet. In this paper we consider the magnet only, not the total system, and emphasize the design choices and reasons for those

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choices particularly for stress levels, conductor operation, and materials costs. We restrict our study to maximum fields below 8.6 tesla, the upper useable field for NbTi.

II. Stress-Strain Problems

A basic consideration which entered into the magnet design was that the stresses and strains in the copper of a composite copper-NbTi conductor must in no way ever compromise the stability of the superconductive windings. This consideration can be assured if a maximum design stress in the copper is taken as 12,000 psi.⁶ Then copper will not yield or plastically deform beyond $\epsilon = 0.001$. In most large magnets the magnetic load is shared between the copper and interleaved stainless steel. A low stress level in the copper will therefore keep the stress level in the stainless steel quite low since both metals suffer the same elongation. Such a situation results in very poor use of the steel whose stress level could never exceed 24,000 psi. To avoid excessive amounts of steel these stress limits make it mandatory that the copper be prestressed into compression during assembly. If prestressing allows copper to experience a stress change of 24,000 psi and steel 48,000 psi during magnetic loading then obvious reductions in material follow. Prestressing can be accomplished by using solid steel forms which are held in tension during winding.

A second stress problem of major concern is the radial bearing stress on the insulation used between the copper and stainless steel. In the inner portion of a "D" shaped magnet, for example, the conductors are straight and insulation stresses become of prime importance in choosing one design over another. Solid steel formers with grooves for the copper conductor avoid pressure accumulation on the insulation.

III. Superconductor Considerations

The NbTi superconductor in copper is sized to carry the total design current at 5.2 K. The copper will be cooled so that no superconductor filament temperature can ever exceed 5.2 K for either of two cases: (1) all the current is in the copper or (2) one half the current is carried by the superconductor while partially resistive. As Purcell⁵ points out this second case corresponds to maximum power generated in the filaments and determines the number of fila-

ments required to ensure smooth current sharing with temperatures kept below a chosen limit. We select here a 0.4 W/cm^2 surface heat transfer to 4.2 K liquid helium with a copper surface temperature 4.7 K . The copper at the filament location will not exceed 4.95 K (for either case), and the filaments will not exceed 5.2 K . Under these conditions $j = 1.5 \times 10^4 \text{ A/cm}^2$ can be used at 8.58 tesla . In Fig. 1 we have estimated critical currents for NbTi at three different temperatures. The curves are composite idealized design curves using an alloy in each field region which has the highest current density at 4.2 K .^{7, 8} The higher temperature curves result from extrapolations using Hampshire's⁹ measurements of j vs B and T for the Nb-44 w/o Ti system. The amount of superconductor required is minimized by using only the cross section required according to the 5.2 K curve in Fig. 1. This fit is made to the maximum fields per turn which are assumed to decrease linearly to zero away from the maximum field at the central median plane of a toroid.

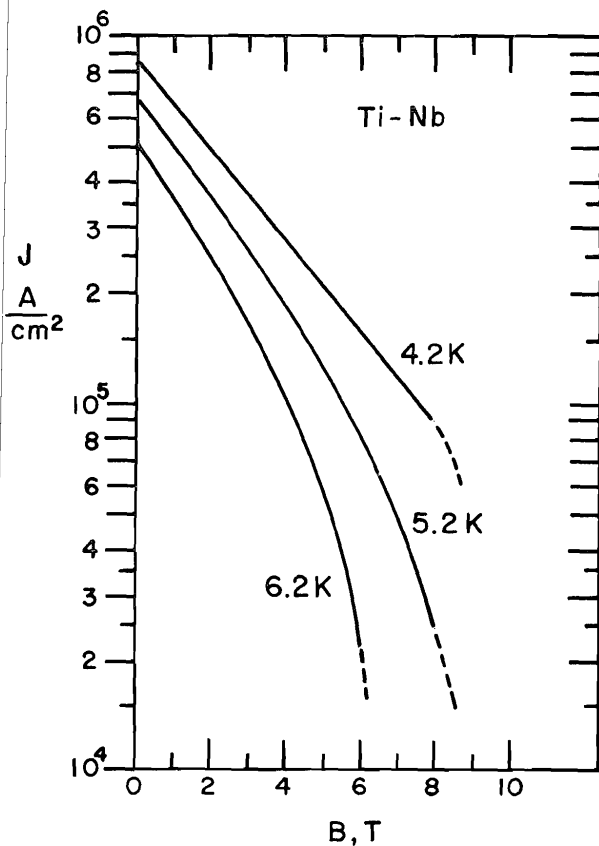


Fig. 1. Optimum design currents.

Edge cooling only on one edge will be used for the composite copper-superconductor so that the conductor can be firmly epoxied into the grooves in the stainless steel discs. This procedure dictates that the conductor grooves should be fairly shallow since copper too far from the cooling surface will not be adequately cooled and therefore of less value. For this first example we choose a stainless steel disc which is 5 cm in thickness and grooved from both sides to form a double pancake. Superconductor filaments will be positioned in the copper matrix within 0.5 cm from the cooled surface and that location will be held less than 4.95 K as mentioned above. The two thermal conditions which are satisfied are (1) the total stability criteria which is set at 0.4 watts/cm^2 when all the current is in the copper and (2) the temperature profile within the copper shall be such that the superconductor filament location is always colder than 4.95 K . In establishing these limits we use:

$$\rho = 10^{-8} (1 + 0.455 B) \Omega \text{ cm} \quad (1)$$

$$k = 0.6 \left(\frac{T}{1 + 0.455 B} \right) \frac{\text{W}}{\text{cmK}} \quad (2)$$

for copper resistivity and thermal conductivity where T is in degrees K and B in tesla. Equation (1) applies to high conductivity copper which has been stress cycled several times¹⁰ and Eq. (2) is obtained by assuming that k is equal to 2.5 W/cmK at 4.2 K in zero field and by assuming that for small temperature excursions $k \propto T/\rho$. With these equations we get the usual results

$$\frac{w}{I} = \sqrt{\frac{\rho}{q h_c}} \quad \text{cm/A} \quad (3)$$

and

$$T(x) = T_0^2 + 1.67 \times 10^8 j^2 \rho^2 (2h_c x - x^2) \quad (4)$$

where

w = edge cooled width, cm

I = current, A

q = 0.4 W/cm^2 , surface heat transfer

h_c = copper depth, cm

x = distance from copper surface, cm

$T(x)$ = temperature at x , K

T_0 = copper surface temperature, K

j = current density, A/cm^2

If $h_c = 2 \text{ cm}$ then w/I is determined from Eq. 3 below 7.8 tesla and by Eq. 4 between 7.8 and 8.6 tesla . A simple form for w/I

which satisfies both Eq. 3 and 4 for $h_c = 2$ cm and $T - T_0 \leq 0.25$ K at $x = 0.5$ cm is

$$w/I = 1.3 \times 10^{-4} (1 + 0.125B) \text{ cm/A} \quad (5)$$

Instead if w is to be constant then we get for $w = 2.55$ cm

$$h_c/I = 4.0 \times 10^{-5} (1 + 0.466B) \text{ cm/A} \quad (6)$$

which was the form chosen for detailed design since a more shallow conductor is better cooled and uses less copper. Using either Eq. (5) or (6) the copper will be minimized by the indicated linear taper.

IV. Design-Mechanical

The need for space for particle diverters inside the toroidal field magnet led to the choice of the constant tension "D" shaped toroidal magnet described in this paper. It should be recalled that $Br = \text{constant}$ for a toroid, where B is the local field and r is measured from the

major axis and that the tension in a curved conductor is $T = BIR$, where B is the applied field, I the current and R the radius of curvature. Then the rationale for the constant tension design is simply to match the radius of curvature at any point along the conductor to the magnetic field at the same point such that the total tensile force in the conductor remains a constant. While this implies a constant average stress in the conductor, if the cross section remains constant, it does not imply a constant value of the maximum stress in a given cross section. An examination of the formula for maximum hoop stress in a thick-walled cylinder under internal pressure p

$$\sigma_h = p \frac{a_2^2 + a_1^2}{a_2^2 - a_1^2} \quad (7)$$

shows the dependence of hoop stress on the ratio of the external radius a_2 to the internal radius a_1 .

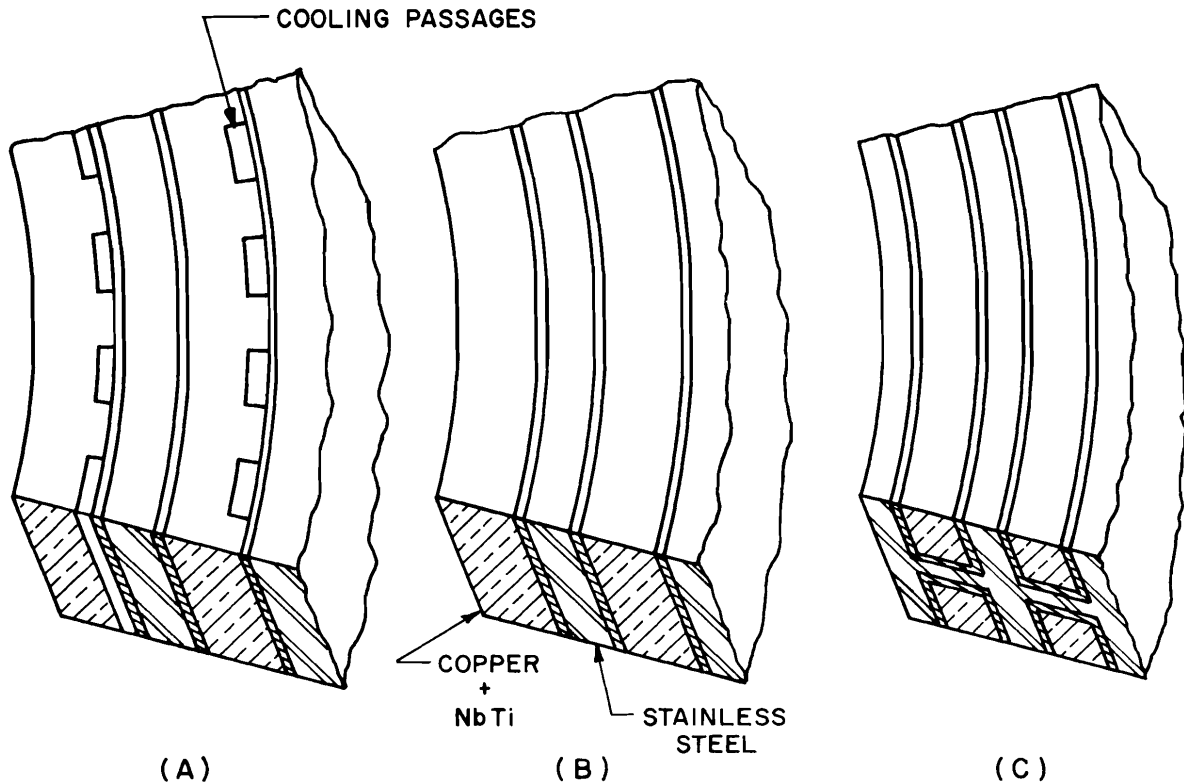


Fig. 2. Pancake designs.

The thick-walled cylinder formula, while adequate to indicate dependency on the ratio a_2/a_1 cannot be used directly for the non-isotropic and non-homogeneous structure being designed.

Three basic conductor designs were considered in detail all of which were based on magnet coils constructed by stacking individual pancakes.⁵ The first design (see Fig. 2a) used interleaved stainless steel bands and face cooled copper bands such as the NAL conductor.⁵ The second (see Fig. 2b) considered interleaved copper and stainless steel with edge cooling and the third and ultimately chosen design consisted of a "D" shaped forged stainless steel pancake with spiral grooves of constant width and varying depth on each face into which the copper conductor will be inserted and bonded with fiberglass reinforced epoxy as insulation (see Fig. 2c).

All the designs were carried out in the following steps.

- (a) A simple finite element analysis treated the equilibrium and compatibility of each layer of copper, insulation and stainless steel in a cylindrical geometry. The six equations developed for each layer related the following.

1. Equilibrium of a layer of copper under the internal and external radial pressures from the stainless steel layers pressing through the insulation, from the magnetic body forces on the conductor and from the resulting hoop or circumferential stresses in the copper.
2. A similar equilibrium equation for a layer of stainless steel.
3. Relationship between radial displacement of a copper layer to the radial and circumferential stresses in the copper.
4. A similar expression for a steel layer.
5. A compatibility expression relating the radial displacements of adjacent steel and copper to the thickness changes from the center of a copper layer, through the cooling lugs used in the face cooled design, through the insulation and to the center of the next outer steel layer.
6. A similar expression from the center of a steel layer to the center of the next outer copper layer.

The output gave the hoop stresses in each layer of copper and steel, the radial stress in each

layer of insulation and the radial displacement of each layer.

This analysis was carried out for a variety of pancakes in which the following variables were considered.

1. Ratio of total conductor thickness to inside radius.
2. Volume fraction of copper, stainless steel and insulation.
3. Linear tapers in copper thickness.
4. Linear tapers in stainless steel thickness.
5. Varying sizes of cooling passages for the face cooled conductors.
6. The addition of an outer reinforcing band of stainless steel.

With so many variables no purpose would be served trying to show all of the interrelated effects but a few significant facts are worth noting. All of the following comments assume a constant field on the inside of the cylinder and a constant product of current flow and number of turns in a pancake.

1. The maximum copper stress varies linearly with the ratio a_1/t where a_1 is the inside radius and t the coil thickness, over the range of a_1/t from one upward.⁶
 2. If a given amount of stainless steel is to be used, the maximum stress in the copper is reduced if the greater amount of steel is used near the inside where the fields are the largest.
 3. The maximum copper stresses are increased by the radial softness created by deeper cooling lugs in the face cooled design, by thicker insulation or as noted in 2 above by placing the steel more to the outside.
- (b) From the output of section (a) just described, design curves were fitted to relate the maximum copper stress, maximum insulation pressure, total circumferential load and volume fractions of materials to the variables in the several designs. A numerical analysis then computed the precise shape and size as well as the quantities of materials required in order to produce "D" shaped magnets with the required clear space inside to accommodate the diverter and associated pumping equipment.

The numerical analysis started at the outside of the magnet with a given design

stress in the copper, a given radius of curvature of the inside surface of the magnet, a given thickness for a single pancake, the required copper dimensions to assure proper cooling and a given distribution of stainless steel in the pancake. With this information and the design curves, a total pancake width is determined as well as the total circumferential load. With the circumferential load and the total width now held constant, new radii of curvature are calculated to match the changing magnetic fields. The radii of curvature are of course used to determine the magnet shape which permits the correct fields to be calculated.

When this incremental procedure indicates that the inside dimension of the torus has been reached, a straight vertical section is added to complete the design. The printout gives the coordinates of the discrete points calculated for the magnet shape as well as the maximum copper and insulation stresses.

- (c) The unbalanced radially inward magnetic loading on each pancake is resisted by providing a thick-walled cylinder interior to the toroidal windings. A final choice of material for this cylinder will depend upon factors involving the magnetic programming of the poloidal fields so that stainless steel was arbitrarily chosen and 60,000 psi used as a maximum compressive stress in the design.

V. Design Details and Comparisons

Figure 3 shows the shape and dimensions of the pancake chosen for the most economical design considered to date. In this design the maximum stresses calculated for magnetic loading as well as the prestress values proposed and the thermal stresses due to cool down from room temperature to 4.2 K are shown below. For steel and copper the moduli of elasticity used were 30×10^6 and 15×10^6 psi and Poisson's ratios were taken as 0.30 and 0.35 respectively.

TABLE I. Maximum Stresses

Loading	Copper psi	S.S. psi
Prestress	-12,000	6,000
Cool Down	3,600	-1,800
Magnetic	20,400	40,800
Total	12,000	45,000

The maximum insulation pressure was found to be 611 psi.

A study of the stability of the individual steel pancakes under loading necessary to produce the prestress for copper winding shows a need for clamping the outer edge while the inner edge is loaded radially. Consideration of stability also indicates that the 0.46 meter wall thickness of the inner core under an average external

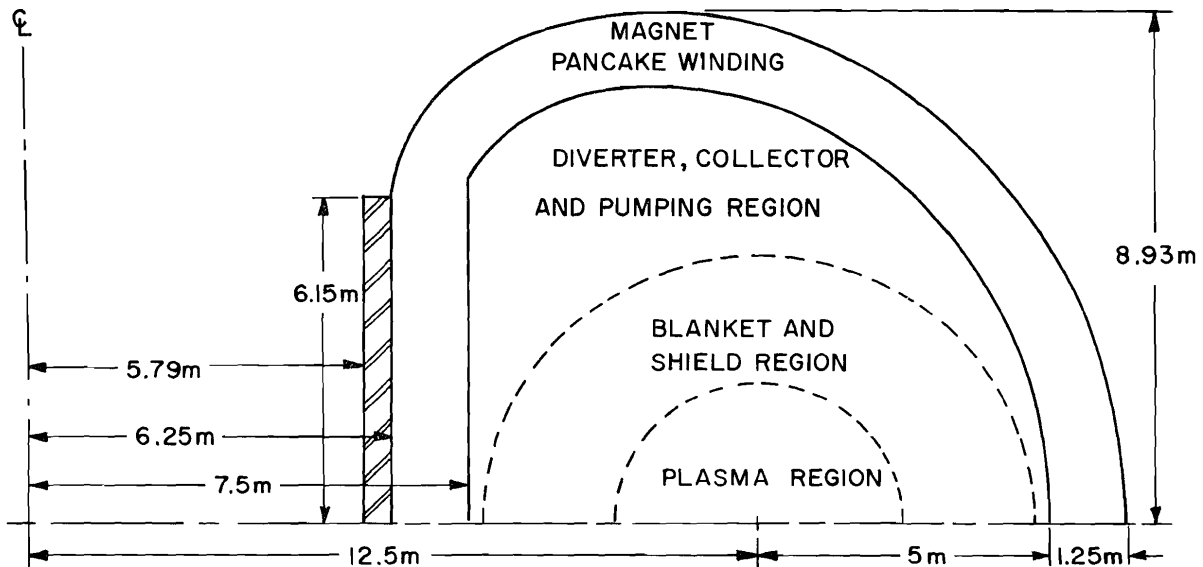


Fig. 3. Toroidal magnet cross section.

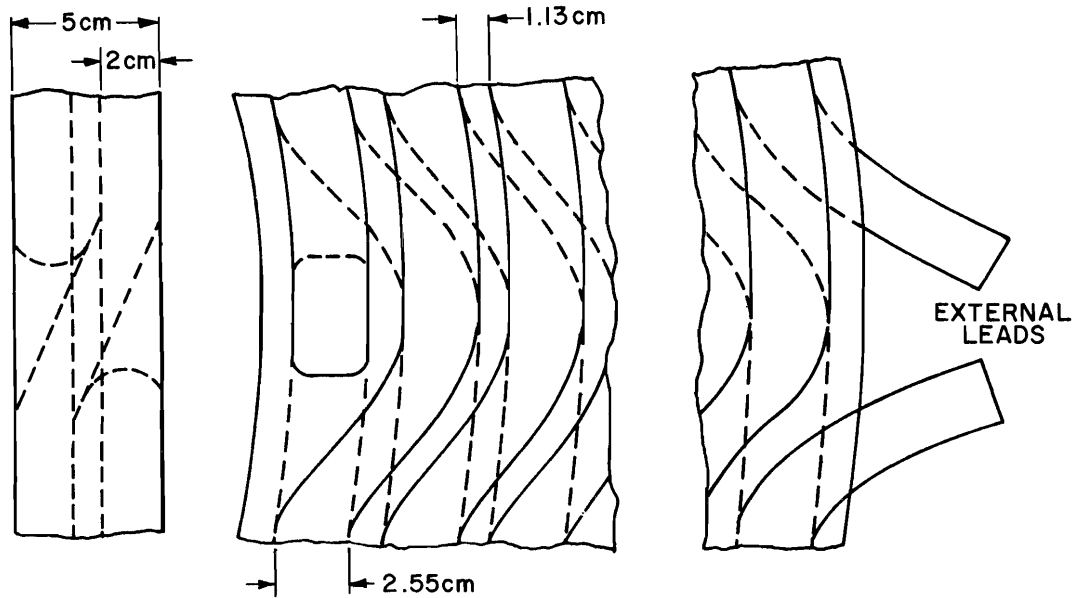


Fig. 4. Details of winding stabilized NbTi Superconductors into stainless steel pancake.

pressure of 4,250 psi is not sufficient to prevent buckling so a ribbed design will be required but the added materials for the ribs will be compensated for by a reduction in the general wall thickness.

A listing of advantages and disadvantages of the three pancake designs shown in Fig. 2 includes the following more important factors.

1. Prestressing is possible only with the design using the solid stainless steel pancake or disc.
2. No interpancake electrical connections are necessary on the inside with the solid pancake design and the external electrical connections are extremely short and easily made. See Fig. 4.
3. Internal heating due to relative motion of conductors and reinforcement should be minimized in the solid pancake design.
4. Bolting through the pancakes to assemble them into a magnet is possible only with the solid pancake design.
5. Compressive stresses in the insulation are lowest in the solid pancake design. Insulation stresses were found to be as high as 8,000 psi in design (b) and 24,000 psi in design (a).
6. While the best cooling is possible with design (a) it must be noted that this advantage is somewhat reduced when the magnets are used in a vertical position and the coolant channels become horizontal.
7. Coolant channels between pancakes are more stable in size and shape for the solid pancake design due to the bonding of the conductors.
8. Although winding techniques are well documented for designs (a) and (b) and the forging and/or machining costs are unknown for the solid pancake design, the immense size of this construction makes accurate cost analyses of all of the designs very difficult.

VI. Specifications

Twelve separate magnets will be arranged in a torus. The total stored energy is approximately 1.3×10^{11} J. If connected in series the inductance is approximately 2600 H at 9980 A. A magnet consists of 42 discs each 5 cm thick separated from each other by 0.635 cm micarta spacers to allow for edge cooling. The stack is compressed by aluminum alloy bolts which are prestressed mechanically and by differential thermal contraction so that magnetic forces cannot completely relieve the tension.⁵ Each disc has 32 turns on each side with the 64 turns

double wound from a single length of conductor. A single conductor for a double pancake is 3080 m long and tapers from 2.55 x 0.4 cm at each end to 2.55 x 2 cm at the center and weighs 8410 kg. The NbTi filaments will be twisted 1 turn per ft in order to limit circulating current decay times to about 1 minute.⁵ The conductor dissipates 1.02 W/cm with all the current in the copper and 0.51 W/cm with half the current in the copper and the other half in the NbTi. Therefore there must be at least 41 filaments to limit the filament temperature to 5.2K and to ensure current sharing. If the NbTi is ideally tapered from 0 to 8.6T, for example 0.0147 cm² at B = 0, 0.0526 cm² at B = 4 T and 0.667 cm² at B = 8.6 T, then we require 208 kg per conductor of which 97 kg is required to provide the last tesla. The stainless steel disc will weigh 15,000 kg.

Estimates for the magnet material follow.

TABLE II. Magnet Materials Estimates

	Weight 10 ⁶ lb	Cost \$/lb	Totals \$10 ⁶
1. NbTi	0.23	15.00	3.5
2. Copper	0.72	1.00	0.7
3. Fabrication of 1 and 2		1.50	1.4
4. Copper Stabilizer	8.4	1.50	12.6
5. (Conductor)	(9.35)	(1.95)	(18.2)
6. 304 SS Formers	16.7	2.00	33.4
7. Winding and Assembly	26.0	1.00	26.0
8. Central Core	3.5	2.00	7.0
9. Total	29.5	(2.90)	84.6

The present NbTi cost for 2 inch rods is about 27\$/lb for small production quantities.⁷ A near term optimistic projection at 15\$/lb is taken, while Powell¹¹ predicts an eventual best price of 6.50\$/lb. Item 2 includes enough copper to process a 2:1 Cu to NbTi conductor; the copper cost at 1\$/lb is taken for a drilled billet. The fabrication cost for 10⁶ lb of composite conductor at 1.5\$/lb includes all extrusion, drawing, rolling and heat treatment steps and is approximately 1/3 present-day costs. The cost for assembled copper stabilizer at 1.5\$/lb seems realistic compared to recent equivalent costs for soldered conductors at 3\$/lb and the base cost for copper. The steel cost for a large forging 40 ft x 60 ft with grooves has not been properly estimated; however 2\$/lb must be a lower price limit.

An analysis of the costs show that the structure cost is dominant. Replacing copper by aluminum, for example, will probably save

very little cost since more steel will be required to absorb the stress. Although the steel cost is largely unknown one could expect that for \$33 x 10⁶ an efficient process could be developed. The other costs are based on firmer evidence and experience.

VII. Conclusions

1. Overly optimistic conclusions regarding the cost of large superconducting magnets for tokamak fusion reactors should be examined very carefully from the standpoint of the permissible stress in the system.
2. Additional research into the physical and electrical properties of the conductor materials under cyclic loading must be undertaken to obtain improved data.
3. This paper reports on a non-optimized design. According to Levy¹² the minimum amount of structure required to hold together a 1.3 x 10¹¹ J magnet is about 1.5 x 10⁷ lb while we have used 2.6 x 10⁷ lb. Toroids are never efficient in this regard but improvements could be expected particularly if higher stress levels are achieved. Further studies should be made into such possibilities as tapering the conductor on both width and thickness and tapering the thickness of the grooved steel pancake form to produce magnets with trapezoidal cross sections.
4. Consideration might also be given to replacing the stainless steel by an aluminum alloy.
5. An obvious long range goal is to develop structural materials which can be purchased, formed, and assembled for less than 2\$/lb.
6. In comparison with Lubell's³ more complete design and cost analysis we note that the items ignored in the Wisconsin design account for 10 percent of Lubell's costs and therefore that our \$85 x 10⁶ + 10 percent should be compared with his \$70 x 10⁶ for similar toroids. His system stores only 4 x 10¹⁰ J but at today's superconductor prices.
7. Finally the cost per kW seems to be tolerable; the above costs account for 85\$/kW which is less than 200 \$/kW for present generating plants.

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