BEAM ROTATOR*

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Abstract

A magnetic system has been designed which in TRANSPORT notation effects the transformation $x \to -y$ and $y \to -x$ or $x \to y$ and $y \to x$ on the coordinates of the beam. This is in effect a 90° rotation of the mirror image of the beam cross section.

The system consists of a symmetric arrangement of five quadrupole magnets whose symmetry axes are at 45° to their normal orientation. Longitudinal disposition of the quadrupoles along the beam axis is variable and may be adjusted to a configuration whose optical length corresponds to its physical length.

The most obvious use of such a system is in conjunction with energy-loss spectrometer arrangements. In this case, for spectrometers arranged to analyze scattered particles in a vertical plane, the beam rotator converts the normal horizontal dispersion of the typical beam handling system into a vertical dispersion required for the energy-loss technique. Such a system has been designed for the MIT Energy-Loss Spectrometer and similar systems have been proposed for the LASL high resolution proton spectrometer and also the DARMSTADT electron linac.

I. Introduction

The use of energy-loss spectrometer systems[1,2] requires that a variable dispersed beam be presented on the target. This dispersion plane will normally be either vertical or horizontal. In most cases it is necessary to have the scattering plane horizontal with subsequent analysis in a vertical plane. The latter requirement allows the independent measurement of momentum, in first order, to be decoupled from the measurement of scattering angle.

A vertical analysis system specifies that the dispersion on the target be vertical also. One way to achieve a dispersion plane which is vertical is to analyze the accelerator beam with dipoles having vertical bends. This can be cumbersome for large bend angles and may lead to different floor elevations along the beam analysis system.

A well-known method for rotating the image plane through 90° is by means of a solenoidal field. In this case we must satisfy the condition

$$\frac{1 + \cos kL}{2} = 0$$

where,

L = effective length of solenoid

$k = B_0/(B\rho)$ where B_0 is the axial field inside the solenoid and $(B\rho)$ is the momentum of the charged particle.

As a practical example, for electrons of 0.5GeV/c momentum, a solenoid having a length of 4 meters and a central field value of 13.1 kG is required. Apart from being uneconomical, at least in this case, the resulting solution also introduces into the magnetic optics strong focussing properties for which compensation must be made.

A third method, which we will discuss in detail, is a unique solution consisting of a symmetric arrangement of five quadrupole magnets whose symmetry planes are at 45° to their normal orientation. This solution, as we will show, has several interesting properties. It is also economical and simple to design into new systems or to retrofit into existing magnetic systems.

II. Theory

We shall use the standard TRANSPORT[3] notation for beam transport optics to describe the operation of the device. In this notation, the charged particle is represented by a vector

$$\vec{x}(x_1, x_2, x_3, x_4, x_5, x_6) = \vec{x}(x, \theta, y, \phi, \ell, \delta)$$

whose six components are the differential coordinates of the particle with respect to a reference particle. This reference particle (central trajectory) has a given position and direction at the input (origin) and a given momentum. The coordinate system is defined with origin on the central trajectory, t-direction along the ray, x-direction perpendicular to it and in the median plane of the bends, and y-direction perpendicular to this plane (Fig. 1). The following define our symbols with standard units:

- x, radial displacement of an arbitrary ray (cm)
- θ, angle this ray makes in the radial plane with respect to the central ray (mr = 10^{-3} rad)
- y, transverse displacement of an arbitrary ray (cm)
- ϕ, angle this ray makes in the transverse plane with respect to the central ray (mr)
- ℓ, path length difference between arbitrary ray and the central ray (cm)
- δ, the momentum deviation of the arbitrary ray from the central ray (%).

With the above definitions, any magnetic system may be represented to first order, by the matrix R such that at any point along the central

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trajectory the differential coordinates are given by

$$x_i(t) = \sum_{j=1}^{6} R_{ij}\, x_j(0)$$

The transfer matrix R is a product matrix of individual matrices for each magnetic element from the origin to the point in question. To avoid confusion later we will define two coordinate system which we shall use in our discussion. The laboratory system will be that in which the x-axis is horizontal and the y-axis is vertical. The quadrupole axes system will be that in which the x- and, y-axes will lie along the symmetry planes of the quadrupole system. This latter system will in general be rotated through some angle α with respect to the laboratory system.

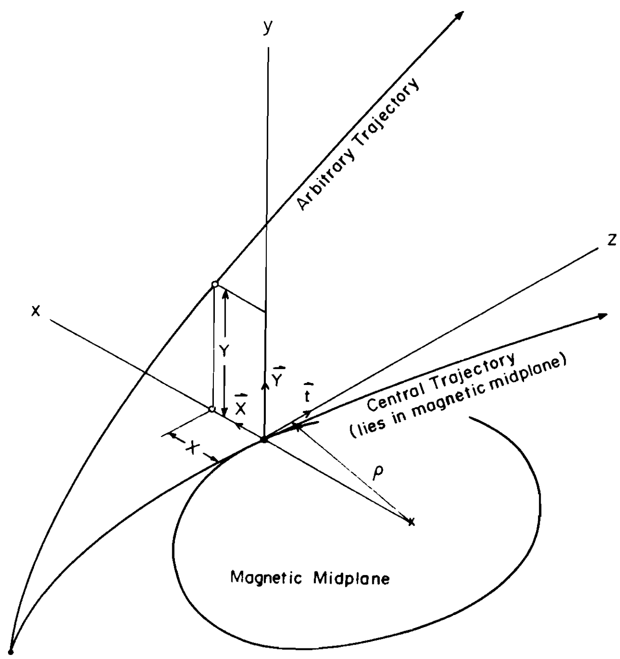


Fig. 1. Beam optics coordinate system.

Let us now consider an arbitrary magnetic system with midplane symmetry consisting of drifts, dipoles and quadrupoles. It will have a transfer matrix of the form

$$R \equiv \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Our objective is to find a new sub-system matrix T such that the product matrix M,

$$M = TR$$

has the form

$$M \equiv \begin{bmatrix} 0 & 0 & M_{13} & M_{14} & 0 & 0 \\ 0 & 0 & M_{23} & M_{24} & 0 & 0 \\ M_{31} & M_{32} & 0 & 0 & 0 & M_{36} \\ M_{41} & M_{42} & 0 & 0 & 0 & M_{46} \\ M_{51} & M_{52} & 0 & 0 & 1 & M_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Such a form for M would clearly interchange the radial and transverse planes. The final coordinates of the ray in the radial plane are dependent only on the initial coordinates in the transverse plane. Similarly the final coordinates of the ray in the transverse plane are a function only of the initial coordinates in the radial plane.

To achieve the desired result, the matrix T must have the following form.

$$T \equiv \begin{bmatrix} 0 & 0 & T_{13} & T_{14} & 0 & 0 \\ 0 & 0 & T_{23} & T_{24} & 0 & 0 \\ T_{31} & T_{32} & 0 & 0 & 0 & T_{36} \\ T_{41} & T_{42} & 0 & 0 & 0 & T_{46} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In designing a magnetic system whose transfer matrix will have the form of T we will look at simple configurations of quadrupoles and drift spaces. For such a system, its transfer matrix S has the following form.

$$S \equiv \begin{bmatrix} x/x & x/\theta & 0 & 0 & 0 & 0 \\ \theta/x & \theta/\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & y/y & y/\phi & 0 & 0 \\ 0 & 0 & \phi/y & \phi/\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

If we rotate the system S, through some angle α with respect to our original coordinate system we will have the desired matrix T.

$$T = \text{Rotate}\ (\alpha)\ S\ \text{Rotate}\ (-\alpha)$$

$$T \equiv \begin{bmatrix} (x/x)C^2 + (y/y)S^2 & (x/\theta)C^2 + (y/\phi)S^2 & [-(x/x) + (y/y)]CS & [-(x/\theta) + (y/\phi)]CS & 0 & 0 \\ (\theta/x)C^2 + (\phi/y)S^2 & (\theta/\theta)C^2 + (\phi/\phi)S^2 & [-(\theta/x) + (\phi/y)]CS & [-(\theta/\theta) + (\phi/\phi)]CS & 0 & 0 \\ [-(x/x) + (y/y)]CS & [-(x/\theta) + (y/\phi)]CS & (x/x)S^2 + (y/y)C^2 & (x/\theta)S^2 + (y/\phi)C^2 & 0 & 0 \\ [-(\theta/x) + (\phi/y)]CS & [-(\theta/\theta) + (\phi/\phi)]CS & (\theta/x)S^2 + (\phi/y)C^2 & (\theta/\theta)S^2 + (\phi/\phi)C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where,

$$C = \cos\ \alpha$$

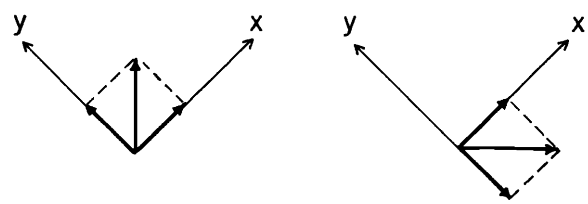
$$S = \sin\ \alpha$$

The outlined matrix elements of T must be zero if we are to satisfy our initial requirements.

III. Design

A. 3-Quadrupole System

To gain further insight into the qualitative behavior of the proposed rotator we shall start with the description of a simple symmetric configuration of three quadrupole lenses. The system utilizes the fact that quadrupole lenses are strongly astigmatic; therefore, it is possible to design a system that has widely different ion-optical properties in two perpendicular planes. Although the beam twister described here is not limited to operate between gaussian image planes only, it is most easily described in such terms. Consider the object in Fig. 2a, a vertical arrow. We desire to rotate the image of this by 90° to the horizontal arrow depicted in Fig. 2b. This can be accomplished as illustrated on the figure by having an optical system with symmetry planes rotated by 45° from the vertical and horizontal and otherwise having such properties that in the x z plane an upright image is formed as shown in the figure, and in the y z plane an upside down image is formed. The simplest such system is shown in Fig.3. It consists of three quadrupole singlets, symmetrically displaced, two focussing in the x plane, and one focussing in the y plane. Clearly, with appropriate choice of parameters, an upright image can be formed in the x z plane and an upside down image in the y z plane.



(a) Object Plane (b) Image Plane

Fig. 2.

More, formally, some of the important matrix elements of this system relative to the quadrupole axes coordinate system, are

$$x/x = 1$$
$$x/\theta = 0$$
$$\theta/\theta = 1$$

$$y/y = -1$$
$$y/\phi = 0$$
$$\phi/\phi = -1$$

with $\alpha = 45°$, the corresponding T matrix becomes

$$T_{3\text{-Quad}} \equiv \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ \dfrac{(\theta/x) + (\phi/y)}{2} & 0 & \dfrac{-(\theta/x) + (\phi/y)}{2} & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ \dfrac{-(\theta/x) + (\phi/y)}{2} & -1 & \dfrac{(\theta/x) + (\phi/y)}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

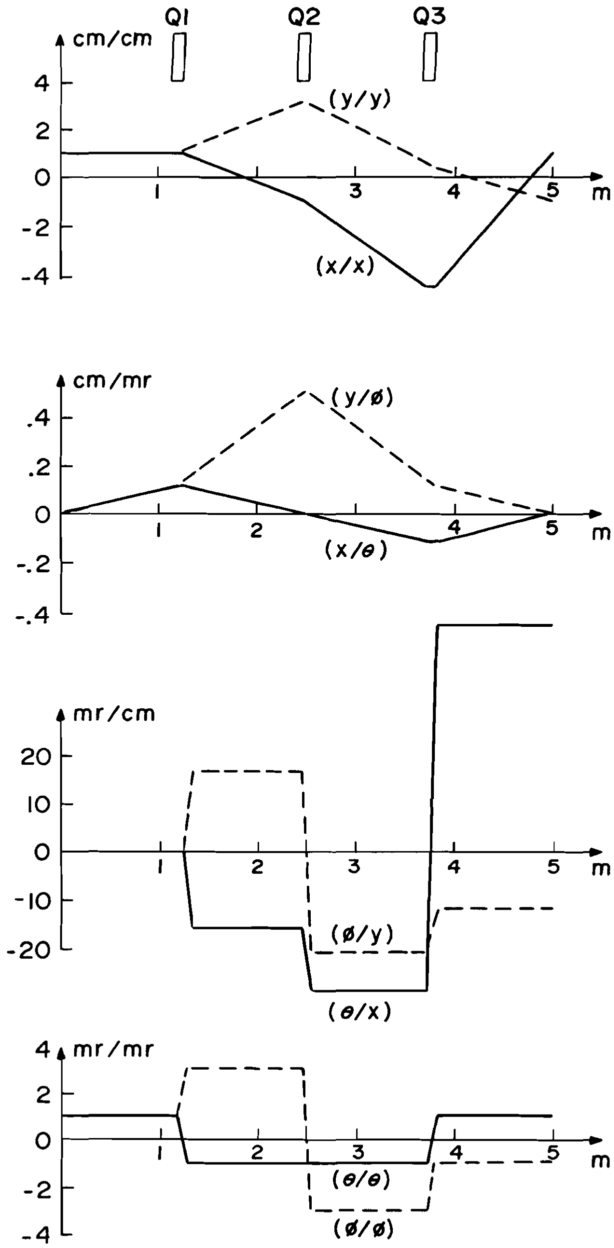


Fig. 3. 3-Quadrupole system. Matrix elements are shown plotted along the quadrupole axes system coordinates. Quadrupole gradients for a particle momentum of .5 GeV/c are: Q1 = Q3 = 2.74 kG/cm, Q2 = -2.03 kG/cm.

Since the elements T_{21} and T_{43} are not zero this system does not completely meet our requirements and has some serious deficiencies. Although on a plane immediately following the physical end of this system, the radial and transverse displacement coordinates will be interchanged, giving an effective 90° rotation, the corresponding angular displacements will not be completely interchanged. The effect of this is that at further points along the system following the rotator the beam will continue to deform and twist. Other points to be noticed are that the system has the properties of a thin lens of equal focal length in both planes and a total optical length of zero in both planes. The addition of an additional lens at the end of the system can correct some of the problems. In the approximation of thin lenses the strength of this lens can be adjusted to give $(\theta/x) = -(\phi/y)$.

B. 5-Quadrupole System

In order to achieve exactly our requirements for the beam rotator we now go to a system of five quadrupole singlets, symmetrically displaced. Three of the quadrupoles are focussing in the x plane and two are focussing in the y plane. The one additional variable field strength along with a variable physical arrangement which may go from a doublet-singlet-doublet configuration to a singlet-triplet-singlet configuration gives us all the necessary freedom to meet the conditions. The important matrix elements of this system are

$$x/x = -1$$
$$\theta/x = 0$$
$$\theta/\theta = -1$$

$$y/y = 1$$
$$\phi/y = 0$$
$$\phi/\phi = 1$$

$$x/\theta = -y/\phi$$

With $\alpha = 45°$, the corresponding T matrix becomes

$$T_{5\text{-Quad}} \equiv \begin{bmatrix} 0 & 0 & 1 & -(x/\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & -(x/\theta) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Such a system has all the required properties of a 90° beam rotator. The image planes are completely interchanged and non-mixing at any point following the rotator. It has no focussing properties external to itself and its equivalent optical length as given by $-(x/\theta)$ is variable and may be made equal to its physical length by choosing a suitable physical layout of the lenses. Figures 4,5,and 6 show examples of typical systems for different physical layouts. The examples chosen are for arrangements 5 meters long with gradients calculated for particle momenta of .5 GeV/c and effective quadrupole lengths of 10 cm. For each of the examples we show the matrix elements plotted along the quadrupole axes system.

Although we have discussed only a few examples of symmetric five quadrupole systems- particularly those equivalent to a simple drift section- it is clear that a multitude of other solutions exist. Similarly, the rotations of the

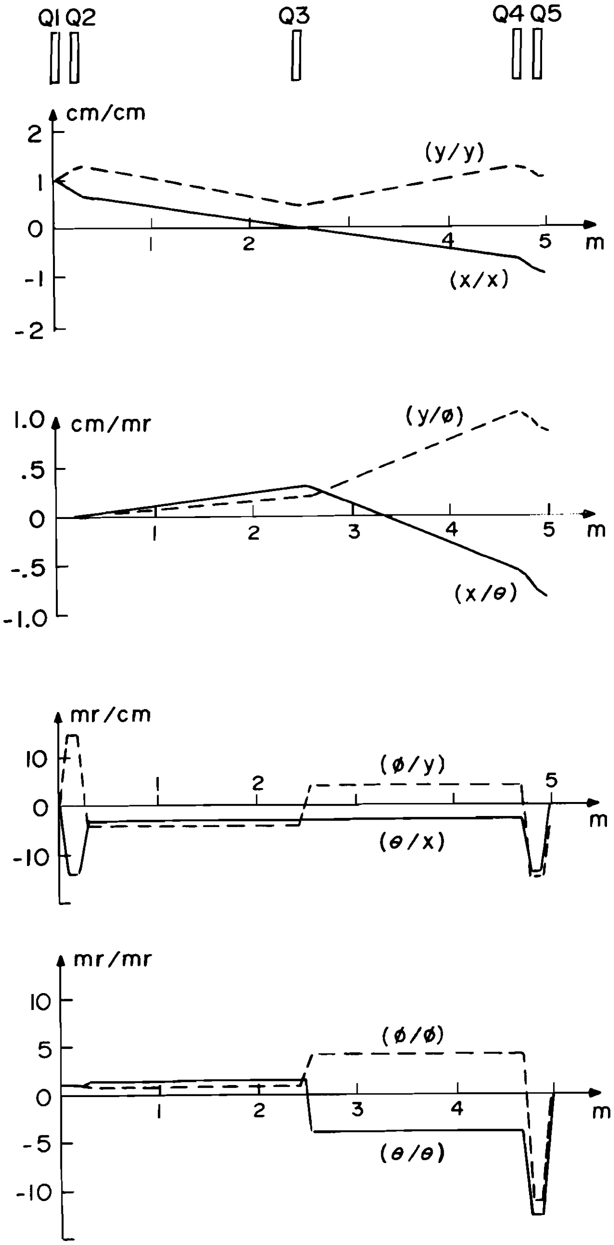


Fig. 4. 5-Quadrupole system- doublet-singlet-doublet arrangement. Matrix elements are shown plotted along the quadrupole axes system coordinates. Quadrupole gradients are: Q1 = Q5 = 2.40kG/cm, Q2 = Q4 = -2.45kG/cm, Q3 = 2.90kG/cm. Doublet spacing is 10 cm.

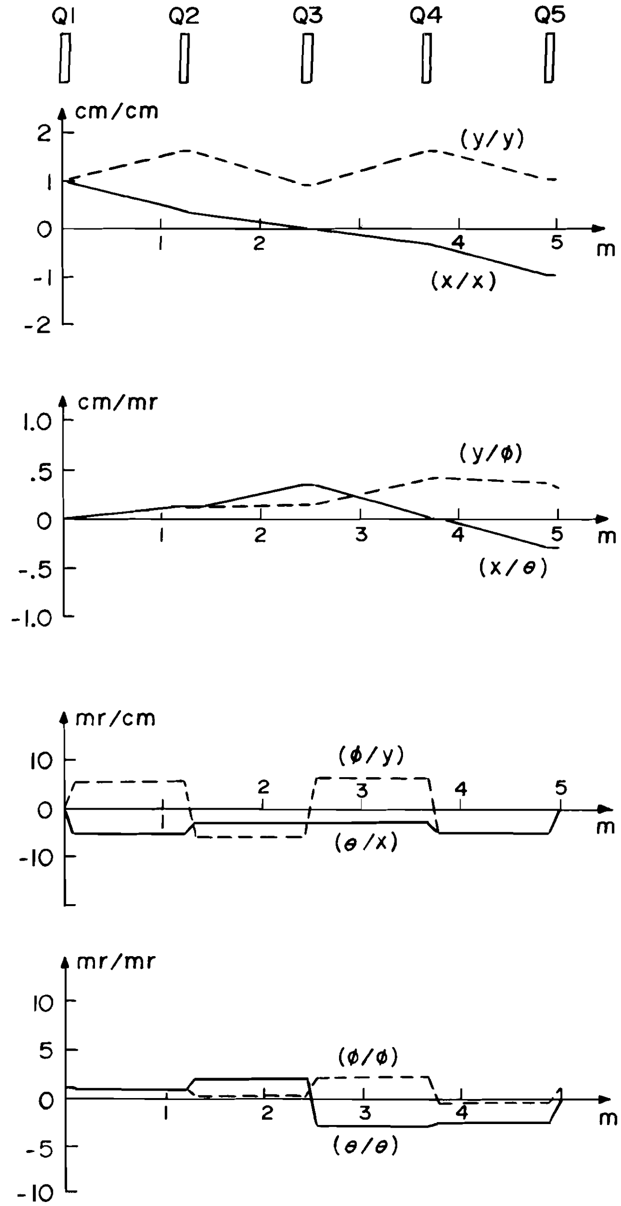


Fig. 5. 5-Quadrupole system - singlets configuration. Matrix elements are shown plotted along the quadrupole axes system coordinates. Quadrupole gradients are: Q1 = Q5 = .90 kG/cm, Q2 = Q4 = -1.18kG/cm, Q3 = 2.24 kG/cm.

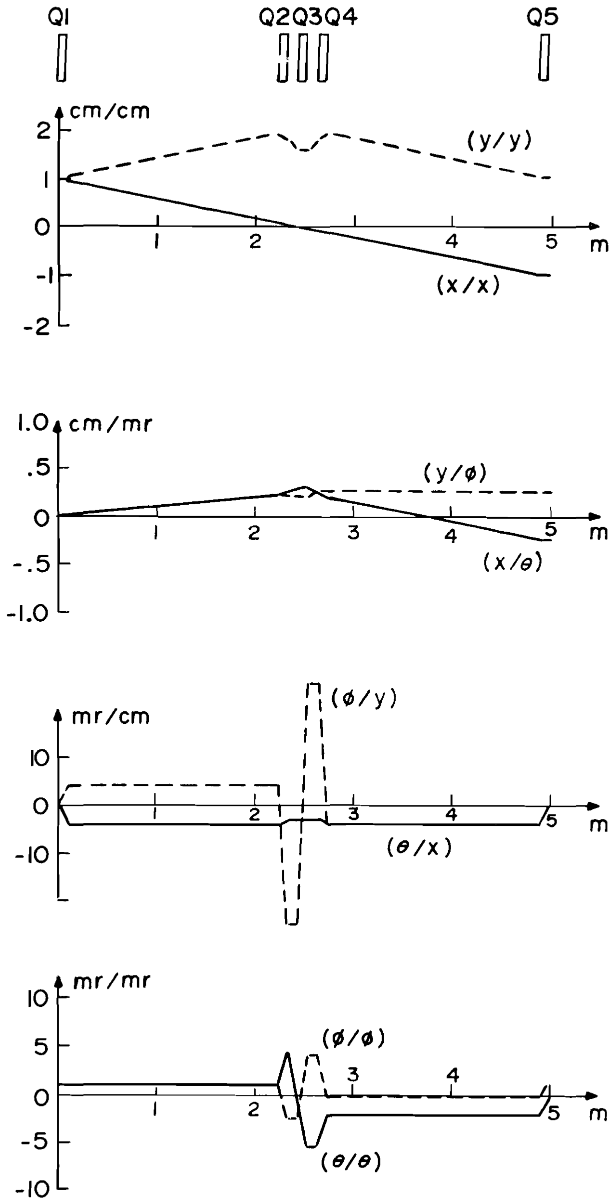


Fig. 6. 5-Quadrupole system- singlet-triplet-singlet arrangement. Matrix elements are shown plotted along the quadrupole axes system coordinates. Quadrupole gradients are: Q1 = Q5 = .70 kG/cm, Q2 = Q4 = -2.60 kG/cm, Q3 = 5.54 kG/cm. Triplet spacing is 10 cm.

image (actually reflection about half the rotation angle) are not restricted to 90° and equivalent solutions exist for any desired rotation.

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