THE POSSIBILITY OF SELF-FOCUSING OF PARTICLES IN A CYCLIC ACCELERATOR

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Abstract

The possibility of self-focusing of particles in cyclic accelerators is shown. The energy at which self-focusing effects occur is found for some special cases.

1. Introduction

We consider the influence of space charge of accelerating particles on the betatron oscillation frequencies in cyclic accelerators[1]; The possibility of the self-focusing of beam during its motion in a cyclic accelerator is discussed.

2. Derivation of basic relations

The curvature of the vacuum chamber is neglected. It is assumed that the beam is a cylinder of infinite length. If the magnet gap, vacuum chamber and beam have a general symmetry axis of second order, then the components of electric and magnetic fields inside the beam to a first approximation have the form

$$E_x = 2\pi\rho(1-\varepsilon)x, \quad H_x = 2\pi\rho\beta(1+\delta)z,$$
$$E_z = 2\pi\rho(1+\varepsilon)z, \quad H_z = -2\pi\rho\beta(1-\delta)x, \tag{1}$$

where the coordinates X and Z are with respect to the axis of the beam in the horizontal and vertical directions respectively; ρ is the charge density; constants ε and δ depend on the form of the magnet gap, walls of vacuum chamber and form of the beam; $\beta = v/c$, v is the velocity of particles. Asymmetry gives rise to the insignificant constant component in (1).

It follows from Maxwell's equations and the symmetry conditions that for $\rho \neq 0$,

$$\frac{1}{\rho}\frac{\partial E_x}{\partial x} > 0, \quad \frac{1}{\rho}\frac{\partial E_z}{\partial z} > 0, \quad \frac{1}{\rho}\frac{\partial H_x}{\partial z} > 0, \quad \frac{1}{\rho}\frac{\partial H_z}{\partial x} < 0,$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} = 4\pi\rho, \quad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = 4\pi\beta\rho .$$

Therefore, the magnitudes ε and δ must satisfy the conditions $-1 \leq \varepsilon \leq 1$, $-1 \leq \delta \leq 1$. Note that ε is equal to δ for a beam in the free space; in the general case ε is not equal to δ. The force caused by self-field has components

$$F_x = 2\pi\rho e\left[(1-\varepsilon)\eta_x - (1-\delta)\beta^2\right]x,$$
$$F_z = 2\pi\rho e\left[(1+\varepsilon)\eta_z - (1+\delta)\beta^2\right]z, \tag{2}$$

where e is the electron charge; the coefficient η is the fraction of beam neutralized. It follows from equation (2) that focusing in one direction and defocusing in the other are possible (for example $\varepsilon \simeq 1$, $\delta \simeq -1$, $\beta \simeq 1$) when neutralization is absent ($\eta = 1$). The self-field of the beam leads to a change of the index of the magnetic field (n) that is equal to the change of the betatron oscillation frequency. This change is found from the equation of motion of particles in a magnetic field in the presence of the self-field of a beam. Thus, change of index of magnetic field may be presented in the form

$$\Delta n_x = \frac{R^2}{mc^2\beta^2\gamma}\frac{\partial F_x}{\partial x},$$
$$\Delta n_z = -\frac{R^2}{mc^2\beta^2\gamma}\frac{\partial F_z}{\partial z} \tag{3}$$

where R is the radius of the accelerator; m is electron mass, $\gamma = (1-\beta^2)^{-1/2}$. If the influence of beam charge on particle motion in the straight section is neglected, then the change of the betatron oscillation frequency $\Delta\nu$ is found with the help of diagram $\nu(n)$ for the accelerator. The neutralization of beam by ions may give rise to increasing of change of the betatron oscillation frequency when this change is positive in the absence of ions.

3. Examples

Let us consider the following examples. Conductive walls of vacuum chamber and the surface of magnet poles are parallel and have infinite extent in the horizontal plane, the magnetic penetration of iron is infinite, the beam is unbunched and all fields are time independent. In this case using conformal mapping of the half-band on the half-plane, one may obtain an expression for scalar and vector[2] potentials respectively inside the vacuum chamber of an accelerator in the form

$$\phi(x,z) = \iint \rho(x',z') \ln S \, dx'dz',$$

$$S = \frac{\mathrm{ch}\frac{\pi}{h}(x - x') + \cos\frac{\pi}{h}(z + z')}{\mathrm{ch}\frac{\pi}{h}(x - x') + \cos\frac{\pi}{h}(z - z')}, \tag{4}$$

$$A(x,z) = \beta \iint \rho(x',z') \ln G \, dx'dz',$$

$$G = \left(\sin\frac{\pi}{q}z' - \mathrm{ch}\frac{\pi}{q}(x - x')\sin\frac{\pi z}{q}\right)^2 + \mathrm{sh}^2\frac{\pi}{q}(x - x')\cos^2\frac{\pi}{q}z,$$

where A is the vector potential component along the axis of the beam, coordinates x,x' and z,z' are calculated from the axis of the beam in the horizontal and vertical directions respectively; q is the separation between magnetic poles; h is the vertical aperture of the vacuum chamber. The coefficients of proportionality in expressions (1) $2\pi\rho(1-\mathcal{E})$ and $-2\pi\rho(1-\delta)$ are equal to the derivatives

$$\frac{\partial E_x}{\partial x} = -\frac{\partial^2 \phi}{\partial x^2}\Bigg|_{\substack{x=0\\z=0}},$$

$$\frac{\partial H_z}{\partial x} = \frac{\partial^2 A}{\partial x \partial z}\Bigg|_{\substack{x=0\\z=0}}$$

respectively. (For the sake of convenience differentiation is performed in the integrals (4) in calculating $\partial E_x/\partial x$ and $\partial H_z/\partial x$ and then letting x=0, z=0.) The constants $\mathcal{E}$ and δ are then determined. In the three special cases of practical interest calculations are performed.

3.1 Circular beam

In the case of a circular beam of radius $r \ll h$ and of uniform density we have

$$\mathcal{E} = \frac{\pi^2}{6}\frac{r^2}{h^2}, \quad \delta = -\frac{\pi^2}{3}\frac{r^2}{q^2} \tag{5}$$

The focusing in the radial direction begins from $\gamma = 1.7$ for the beam dimension $r/h = r/q = 0.3$.

3.2 Elliptical beam

In the case of an elliptical beam of major radius a_x and minor radius a_z ($a_x \ll h$, $a_z \ll h$) of uniform density we have

$$\mathcal{E} = \frac{\pi^2}{6}\frac{a_x a_z}{h^2} + \frac{a_x - a_z}{a_x + a_z}, \tag{6}$$

$$\delta = -\frac{\pi^2}{3}\frac{a_x a_z}{q^2} + \frac{a_x - a_z}{a_x + a_z}$$

In this case it is worth while to present coefficients $\mathcal{E}$ and δ in the form $\mathcal{E} = \mathcal{E}' + \mathcal{E}''$, $\delta = \delta' + \delta''$. The coefficients $\mathcal{E}'$ and δ' are due to the presence of the medium and the coefficients $\mathcal{E}''$ and δ'' are due to the non-circular form of the beam. It is obvious for a circular beam that we obtain $\mathcal{E}'' = \delta'' = 0$. For an elliptical beam a simple calculation gives $\mathcal{E}'' = \delta'' = (a_x - a_z)/(a_x + a_z)$. For elliptical beam $a_x > a_z$ the self-focusing in the radial directions begins at a lower energy than that in the case of a circular beam.

3.3 Rectangular beam

In the case of a rectangular beam of dimensions a_x and a_z ($a_z = h = q$ large beam) and of uniform density we have

$$\mathcal{E} = 1 - \frac{4}{\pi}\,\mathrm{arctg}\left(\mathrm{sh}\frac{\pi a_x}{2h}\right)^{-1}, \quad \delta = -1 \tag{7}$$

If $a_x = 2h$ the self-focusing of the particles in the radial direction begins for $\gamma \simeq 1$ ($\beta = 0.24$).

4. Conclusion

Thus, the effect of self-focusing in higher current beams may have significant influence on particle dynamics in a cyclic accelerator. Moreover, this effect may be used in designing beam transport systems as was indicated in reference[3].

References

1. A.M. Sessler, Proc. 5-th Int. Conf. on High Energy Accelerators, Frascati (1965); CNEN 319.
2. W.R. Smythe, Static and Dynamic Electricity (New York, Toronto, London, 1950).
3. A.G. Bonch-Osmolovski et all. Trudy Vsesouznogo Sovetshaniya po uskoritelyam. Moskva 1970, v.2, p.507.