PARTICLE MOTION NEAR A SURFACE ON WHICH TOTAL INTERNAL REFLECTION OF ELECTROMAGNETIC WAVES TAKES PLACE

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Abstract

The motion of a particle in vacuum in the field of an elliptically polarized electromagnetic wave undergoing total internal reflection, is considered. Conditions for the stability of the motion are found. The motion of an equilibrium particle, as well as the change of the small deviations, is considered. Expressions for the frequencies of the small oscillations are obtained. The possibility of using the effect for particle acceleration by means of a laser beam is analysed.

1. Introduction

The use of the strong fields of intense laser beams for particle acceleration is quite alluring. However, up to now, none of the proposed methods appeared to be sufficiently realizable. The main difficulties are the transverse nature of the field oscillations and the lack of suitable methods for decreasing the wave phase velocity. In this report the results of an investigation of the particle motion near a surface on which the electromagnetic wave undergoes total internal reflection, are presented. In the case of such a reflection, in a thin layer near the surface, the electromagnetic field has a longitudinal (along the surface) component, and the phase velocity of the wave propagation along the surface is less than c . The thickness of the layer in which the field has a noticeable magnitude is of the order of wavelength, but it might be of the order of several thousands of wavelengths if the wave incidence angle is close enough to the limit angle of the total internal reflection, and the angular divergence of the beam is $\sim 10^{-6}$.

At the same time, the condition H > E which is valid for the electromagnetic wave in dense media, remains valid in the surface layer in the vacuum too. In virtue of this, in some conditions the particle motion in this layer appears to be stable. This phenomenon, in principle, can be used for particle acceleration.

2. Fields

Let an elliptically polarized plane wave fall from a medium with refractive index n > 1 under an angle θ onto the boundary between the vacuum and this optically transparent medium. Let us take the boundary surface as the y = 0 plane, the incident plane as the xz plane, and direct the x axis towards the propagation of the refracted beam. In such a frame the components of the electric vector of the incident arbitrary elliptical wave will be of the form

$$E_{x\,in} = -E_2 \cos\theta\, e^{i(\Phi+\psi_2)}$$

$$E_{y\,in} = E_2 \sin\theta\, e^{i(\Phi+\psi_2)}$$

$$E_{z\,in} = E_1 \cdot e^{i(\Phi+\psi_1)}$$

$$\Phi = \vec{k}\vec{r} - \omega t$$

If $E_2 = 0$, then one would obtain a wave linearly polarized in a plane parallel to the boundary plane. If $E_1 = 0$, then the electric vector would lie in a plane perpendicular to the boundary plane. The phases Ψ_1 and Ψ_2 of the corresponding components, generally speaking, are arbitrary. Using the Fresnel formulae separately for the components polarized in parallel and perpendicularly to the boundary plane, it is not difficult to find the expressions for the field components in the vacuum. Let us write out the expressions for the case of total internal reflection, i.e. when $n \sin\theta \gtrsim 1$. In this case the wave phase is of the following form:

$$\Phi = \frac{\omega}{c} n \sin\theta\, x - \omega t + i \frac{\omega}{c}\sqrt{n^2\sin^2\theta - 1}\, y,$$

corresponding to the solution which decreases for y > 0. For short, let us introduce the following notations:

$$F_1 = \frac{2E_1 n \cos\theta}{\sqrt{n^2-1}} \exp\left\{-\frac{\omega}{c}\sqrt{n^2\sin^2\theta-1}\, y\right\}, \tag{1}$$

$$F_2 = \frac{2E_2 n^2 \sin\theta\cos\theta}{\sqrt{n^2-1}\sqrt{n^2\sin^2\theta-\cos^2\theta}} \exp\left\{-\frac{\omega}{c}\sqrt{n^2\sin^2\theta-1}\, y\right\} \tag{2}$$

$$\Phi_1 = \frac{\omega}{c} n\sin\theta\, x - \omega t + \psi_1 - \varphi_1, \quad \left(tg\,\varphi_1 = \frac{\sqrt{n^2\sin^2\theta-1}}{n\cos\theta}\right) \tag{3}$$

$$\Phi_2 = \frac{\omega}{c} n\sin\theta\, x - \omega t + \psi_2 - \varphi_2, \quad \left(tg\,\varphi_2 = \frac{n\sqrt{n^2\sin^2\theta-1}}{\cos\theta}\right) \tag{4}$$

$$\beta_0 = \frac{1}{n\sin\theta} \lesssim 1. \tag{5}$$

Then the field components in the vacuum will be of the form

$$E_x = F_2\sqrt{1-\beta_0^2}\sin\phi_2;\quad H_x = -\frac{F_1}{\beta_0}\sqrt{1-\beta_0^2}\sin\phi_1;$$
$$E_y = F_2\cos\phi_2;\qquad H_y = -\frac{F_1}{\beta_0}\cos\phi_1;\qquad (6)$$
$$E_z = F_1\cos\phi_1;\qquad H_z = F_2\beta_0\cos\phi_2;$$

3. Equations of motion

The equations of the particle motion in the field (6) are satisfied by the following equilibrium values of velocities, phases, and energy:

$$\begin{aligned}
&\beta_{xs} = \beta_0, \\
&\beta_{ys} = 0, \\
&\beta_{zs} = -\beta_0\sqrt{1-\beta_0^2}\,F_2/F_1, \\
&\phi_{1s} = \phi_s - \pi/2; \\
&\phi_{2s} = \phi_s = const; \\
&\mathcal{E}_s = mc^2/\sqrt{(1-\beta_0^2)(1-\beta_0^2F_2^2/F_1^2)}.
\end{aligned}\qquad (7)$$

Such a form of the equilibrium energy makes one think that two methods of particle acceleration are possible:

i) Increase of the parameter β_0. The particle energy will increase if either the incident angle θ or the refractive index n depends on the coordinate, so that $\beta_0 \to 1$.

ii) Increase of the parameter F_2/F_1. The increase of the parameter β_0 can appear impossible, since the thickness of the layer in which the field has a noticeable magnitude depends on this parameter. In this case the increase of the energy is possible if the ratio of the amplitudes of the two components in the elliptically polarized wave depends on the coordinates x and z.

In order to study the problem of the stability of the solution (7), let us consider the behaviour of the change of the small deviations from the equilibrium values. Let us denote

$$\begin{aligned}
&P_x = P_{xs}(1+\rho), \\
&P_y = P_{xs}\cdot\tau, \\
&P_z = P_{zs}(1+\sigma), \\
&\mathcal{E} = \mathcal{E}_s(1+\epsilon), \\
&\epsilon = \left(\frac{P_{xs}c}{\mathcal{E}_s}\right)^2\rho + \left(\frac{P_{zs}c}{\mathcal{E}_s}\right)^2\sigma.
\end{aligned}$$

Using the smallness of the magnitudes ρ, τ, and σ, one can obtain the following system of three linear equations which they obey:

$$d\rho/dt = -\rho\frac{ec\beta_{zs}}{\mathcal{E}_s}F_1\sin\phi_s + \tau\frac{ec\beta_0}{\mathcal{E}_s}F_2\cos\phi_s + \sigma(1-\beta_{zs}^2)\frac{ec\beta_{zs}}{\mathcal{E}_s\beta_0^2}F_1\sin\phi_s;$$
$$d\tau/dt = -\sigma\frac{ec}{\mathcal{E}_s\beta_0}F_2\cos\phi_s\,(1-\beta_0^2-\beta_{zs}^2);$$
$$d\sigma/dt = -\rho(1-\beta_0^2)\frac{ec}{\mathcal{E}_s\beta_{zs}}F_1\sin\phi_s - \tau\frac{ec}{\mathcal{E}_s\beta_{zs}}\sqrt{1-\beta_0^2}\,F_1\cos\phi_s + \sigma\frac{ec\beta_{zs}}{\mathcal{E}_s}F_1\sin\phi_s$$

The system secular equation, the roots of which determine the proper frequencies of the small oscillations ρ, τ, and σ, in this case is of the following form:

$$\lambda = (\lambda^2 + \Omega^2) = 0$$

where

$$\Omega = \frac{ecF_1}{mc^2\beta_0}\left(\frac{mc^2}{\mathcal{E}_s}\right)^2. \qquad (8)$$

Thus in the field (6), stability occurs with respect to the two degrees of freedom with equal frequencies (8), and an indifferent equilibrium with respect to the third one ($\Omega_3 = 0$).

Let us note that the stability is provided by the wave component, the electric vector of which ($\sim F_1$) is parallel to the boundary plane xz. However, this component has no projection along the x axis and, therefore, it may provide an acceleration only in the case when the particle velocity has a projection along the electric field. The wave ellipticity is important only for the acceleration by the second method.

4. Acceleration

Let us consider the possibility of the particle acceleration by the second method. For this purpose, we shall assume that the parameter F_2/F_1, as well as the phases Ψ_1 and Ψ_2, are slow functions of x and z.

The equilibrium values of the velocities β_{xs} and β_{zs} are in this case determined by the condition of the constancy of the phases ϕ_{1s} and ϕ_{2s} (3,4):

$$d\phi_{1s}/dt = 0\,; \quad d\phi_{2s}/dt = 0.$$

Let us, for simplicity, limit ourselves to considering those particles with phases equal to $\phi_{1s} = 0$; $\phi_{2s} = \pi/2$. Then the changes of the equilibrium values P_{xs}, P_{zs}, and ε_s will be described by the following equations ($P_{ys} = 0$):

$$dp_{xs}/dt = eF_2\sqrt{1-\beta_0^2} + eF_1\beta_{zs}/\beta_0\,,$$

$$dP_{zs}/dt = eF_1\left(1-\beta_{xs}/\beta_0\right)\,, \tag{9}$$

$$d\mathcal{E}_s/dt = ec\left(F_2\sqrt{1-\beta_0^2}\,\beta_{xs} + F_1\beta_{zs}\right).$$

Let us consider the most interesting ultrarelativistic case when $\beta_0 \approx 1$. In this case Eqs. (9) have two integrals of motion:

$$\mathcal{E}_s - cP_{xs} = K\,,$$
$$\mathcal{E}_s - c^2p_{zs}^2/2K = L\,.$$

Introducing the notation $cp_{zs}/\sqrt{2kL} = U$ we obtain the following equation for U:

$$(U^2+1)\,du/dt = a\,; \quad a = eF_1cK^{1/2}/2^{1/2}L^{3/2}$$

the solution of which is of the form

$$U = \sqrt[3]{\sqrt{\left(\tfrac{3}{2}at\right)^2+1} + \tfrac{3}{2}at} - \sqrt[3]{\sqrt{\left(\tfrac{3}{2}at\right)^2+1} - \tfrac{3}{2}at}\,.$$

The energy and the longitudinal momentum can be expressed by means of U and the integrals of motion

$$\mathcal{E}_s = L\left(1+U^2\right),$$
$$cP_{xs} = L\left(1+U^2\right) - K.$$

Let us now consider the change of the small deviations from the equilibrium values:

$$q_x = P_x - P_{xs}\,; \quad q_z = P_z - P_{zs}\,; \quad q_y = P_y\,;$$
$$\varphi_1 = \phi_1 - \phi_{1s}\,; \quad \varphi_2 = \phi_2 - \phi_{2s}\,.$$

The changes of q_x and q_z are in this case connected with each other by the equations

$$\frac{dq_x}{dt} = -q_x\frac{ecF_1}{\mathcal{E}_s\beta_0}\beta_{xs}\beta_{zs} + q_z\frac{ecF_1}{\mathcal{E}_s\beta_0}\left(1-\beta_{zs}^2\right),$$
$$\frac{dq_z}{dt} = -q_x\frac{ecF_1}{\mathcal{E}_s\beta_0}\left(1-\beta_{xs}^2\right) + q_z\frac{ecF_1}{\mathcal{E}_s\beta_0}\beta_{xs}\beta_{zs}, \tag{10}$$

As the study shows, the system (10) describes the damping stable oscillations only for the values $U^2 \le 2$.

5. Conclusion

The use of the field of the totally reflected wave for particle acceleration is, at least in principle, possible. It provides all the conditions necessary for this purpose. The lack of stability with respect to one of the degrees of freedom (perpendicular to the boundary plane of the medium) can be filled by an additional external field (e.g. a magnet). However, such a method will hardly be practically reasonable, since it provides only slow acceleration with only small energy gain at each stage.

Acknowledgement

The author would like to thank all the participants of Yerevan Physical Institute accelerator seminar for useful discussions on the results of this work.