AUTOACCELERATION IN ELECTRON BEAMS

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Abstract

Possibilities of particle acceleration in proper fields of an intense electron beam are discussed. A self-consistent steady state is investigated where the necessary field configuration and the longitudinal stability of the bunched beam are provided by a passive retarding system.

1. Introduction

Recent progress in the physics of intense electron beams 1,2) has advanced two new tasks. The first is post-acceleration of such beams to high energies to overcome limitation imposed by high-voltage techniques. The second involves using the large proper fields and stored energy of such beams to accelerate other particles, for example, protons. From this standpoint, such a method of acceleration may be readily qualified as a collective method.

Various ingenious schemes of particle acceleration by beams have been proposed recently 3,4,5). However their realisation as well as the realisation of the classical collective method 6) is hampered by insufficient understanding of the complicated collective particle motion. Therefore, a search for physically simpler methods of acceleration is warranted.

Considering the problems mentioned above, one can see that the proper fields of an intense electron beam, including the longitudinal electric field, can be much larger than practical values of external accelerating fields. Therefore, the possibility of achieving beam particle acceleration in the proper fields appears attractive. The necessary field configuration may be provided in principle by boundary conditions, i.e., by passive structural elements. Although only a small part of the beam may be accelerated in this manner at the expense of the energy of the other particles, accelerated currents unachievable by usual methods may be obtained if the initial current is of order of several tens of kiloamperes.

The simplest feasible schemes of such "autoacceleration" have been proposed in 7);. In the first case (Fig. 1), the front portion of a monochromatic intense electron beam radiates into a short-circuited line, being decelerated to the energy $W_o - \Delta W$. The excited pulse U_o is reflected at the short-circuited end, returns to the beam with opposite polarity and accelerates the trailing portion of the beam to the

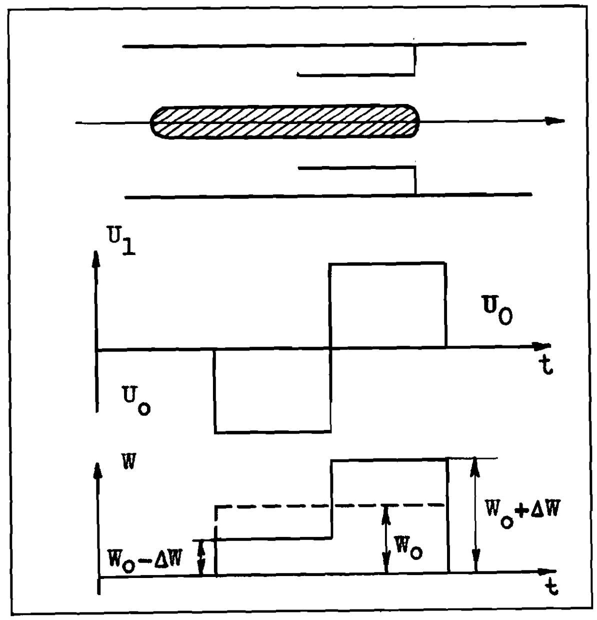

Fig.I

energy $W_o+\Delta W$. The process may be repeated if one sweeps away the electrons which have lost energy. The energy of the remaining particles increases while the pulse beam current remains constant in a relativistic case.

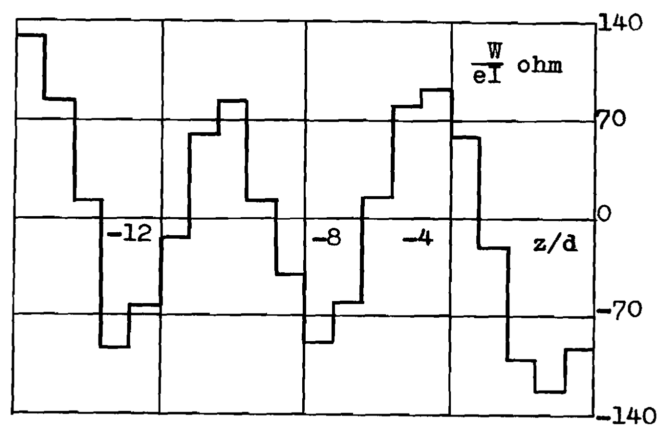

Fig. 2

Fig.2 shows the redistribution of energy in an initially monochromatic beam after passing through a cylindrical passive cavity. The ratio of the energy gain of the "luckiest" particles to the total current turns out to be as large as ~ 10^2

ohms. This means that the effective energy gain can be sufficient for currents of the order of $\sim 10^4$A. To cascade the acceleration process one has to remove low-energy particles and maintain complex phase relations in a chain of cavities. This means using a distributed system excited by an intense electron beam, a part of which is accelerated in the excited field. The efficiency of the mechanism will be high if the phase velocity of a propagating wave in the system is close to the beam velocity. In this sense, one can speak of particle acceleration in fields of Čerenkov radiation.

2. Beam-excited fields in a retarding system

For qualitative investigation we shall use the well-known model of a real retarding system - a cylindrical waveguide (radius b) filled with a dielectric medium the permeability ϵ of which is independent of frequency [8]. Let us suppose that the modulated beam is monochromatic and moves along the Z-axis with a constant velocity $\beta_0 c$. In a steady state all values depend on the radial (r) and phase ($x=z-v_0 t$) coordinates only. For a scalar potential ϕ, one has the equation

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\phi}{\partial r} - s\frac{\partial^2\phi}{\partial x^2} = -4\pi\epsilon^{-1}\rho(r,x); \qquad s = \beta_0^2\epsilon - 1 \tag{I}$$

with the boundary condition $\phi=0$ at $r=b$. Representing the beam charge density ρ and the potential ϕ as Fourier-Bessel series and looking for a periodic in x solution of Eq. (I), one obtains for the electric field component $E_{zn}=s\partial\phi/\partial x$

$$E_{zn} = -\frac{2\pi}{\epsilon}\int_{x-L}^{x}\frac{\cos(\pi s_{wn}^{+1/2}(x/L - x'/L - 1/2)s^{-1/2})}{\sin(\pi s^{-1/2}s_{wn}^{1/2})}\rho(x')dx' \tag{2}$$

$$s_{wn} = \mu_n L/(2\pi b) = (\beta_{wn}^2\epsilon - 1)^{1/2},$$

where L is the modulation period, β_{wn} - is the phase velocity of the wave whose length is equal to L, and μ_n is the n-th root of the Bessel function $J_0(\mu_n)$. One can see from (2) that in a "superlight" case ($\beta_0>\epsilon^{-1/2}$, $s>0$) waves with $s_{wn}= sk^2$ ($k=\pm1, \pm2$ ...) are rezonantly excited. The physical meaning of this condition is the approximate equality of the beam velocity and the phase velocity of the waveguide mode with k wavelengths per modulation period L. Let us note that for $s_{wn}> s^{1/2}k$ the wave overtakes the beam and vice versa. The electric field pattern as a function of $s^{-1/2}s^{1/2}_{wn}$ is presented in Fig. 3 for the simplest case of k=1 and a density which is uniform over the interval $|x|<\ell/2<L/2$. It should be noted that the field amplitude strongly depends on the difference between the beam and wave velocities and for

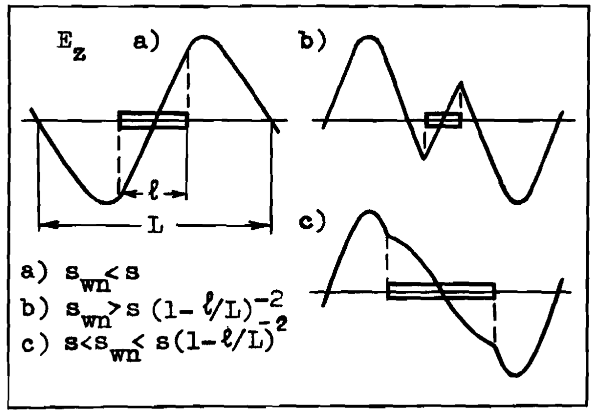

Fig.3

relatively small current reaches the breakdown limit. Moreover, the whole pattern depends strongly on the detuning if s>0 (see Fig.3). If the wave velocity is larger than $\beta_0 c$ the waveguide has a capacitive impedance, in the opposite case inductive*). Correspondingly, the longitudinal electric field inside the bunch appears to be focusing (Fig. 3a) or defocusing (Fig.3c). In the first case, one can foresee the possibility of self-phased steady bunches in the retarding system (a similar effect in cyclic accelerators was discussed in ref.[11]). A longitudinal electric field of such bunches can be used for acceleration of particles.

Let us note also that $\partial A_z/\partial r=\beta_0\epsilon\partial\phi/\partial r$. Thus, for charge and current densities being provided by the beam only the total transverse force acting on the particles is

$$F_r = es\partial\phi/\partial r \tag{3}$$

In the "superlight" case, the radial force is opposite to the radial electric field (i.e., Lorenz force prevails over electrostatic repulsion). This opens an interesting possibility of three-dimensional equilibrium of bunches in proper fields.

3. Self-phased bunches.

Here we shall investigate the self - consistent steady state of electron bunches. To simplify the problem we shall neglect the transverse motion of particles, assuming, for example, that the beam moves in the strong longitudinal magnetic field. Then the longitudinal motion in the steady elec-

*) As a matter of fact, this is a capacitive impedance which provides the suppression of the negative mass instability in a circular pipe with a dielectric layer [9]. Moreover, one can easily trace the close relationship between the described phasing mechanism and longitudinal instabilities in circular accelerators [10].

tric field $E_z(r,x) = s\partial\phi/\partial x$ is described by the Hamiltonian

$$H = (m_o^2c^4 + p^2c^2)^{1/2} - \beta_o pc - es\phi \ , \quad (4)$$

where p is the longitudinal momentum canonically conjugated to the phase variable x. A distribution function in the phase plane (p,x) in a steady state depends on H only. Thus, the charge density may be obtained from the distribution function f as $\rho = e\int f(H)dp$. For an arbitrary dependence f(H) (within the obvious physical limitation) the substitution of ρ into (1) gives an equation for the potential ϕ under zero boundary conditions.

Let us consider a uniform distribution in the phase plane ($f=f_o=$const) within a phase trajectory $H=H_o(r)$ and $f=0$ elsewhere. Then one gets from (4):

$$\rho = \begin{cases} \dfrac{2f_oe}{c(1-\beta_o^2)}\left[(es\phi + H_o)^2 - m_o^2c^4\gamma_o^{-2}\right]^{1/2}, & (es\phi + H_o) > m_oc^2\gamma_o^{-1} \ , \\ 0 \ , & (es\phi + H_o) < m_oc^2\gamma_o^{-1} \ , \end{cases} \quad (5)$$

where $\gamma_o=(1-\beta_o^2)^{-1/2}$.

For relativistic particles and large current densities one may neglect the term $m_o^2c^4\gamma_o^{-2}$ because the charge density is comparatively small in those regions where this term is of importance. Under this assumption, equation (1) becomes linear. For simplicity we shall assume also that the beam fills all the space inside the waveguide and has no discontinuities but all the particles are inside the bucket and do not slip relative to the field. These conditions actually mean that the boundary phase trajectory $H_o(r)$ is a separatrix, i.e. $H(r)=-es\phi_{min}(r)$, where ϕ_{min} is the minimal value of $\phi(z,r)$ at a radius r.

A zero harmonic of the potential $\phi_o(r)$ is related to a zero harmonic of the current density by:

$$\frac{d\phi_o}{dr} = -\frac{4\pi}{\epsilon\beta_o cr}\int_0^r r' j_o(r')dr'$$

so for our boundary conditions

$$\phi_o(r) = \frac{4\pi}{\epsilon\beta_o c}\int_r^b \frac{dr'}{r'}\int_0^{r'} r'' j_o(r'')dr'' \quad (6)$$

and

$$f_o = \frac{(1-\beta_o^2)\ j_o(r)}{2e\beta_o(e\phi_o s + H_o(r))} \ .$$

An alternating component of the potential, generally speaking, is a superposition of harmonics $\phi_n(r)\cos(q_nx/s)$, where $\phi_n(r)$ and $-q_n^2$ are eigenfunction and eigenvalues of the operator

$$\frac{1}{r}\frac{d}{dr} r \frac{d}{dr} + \frac{4\pi j_o(r)}{\beta_o c\epsilon(\phi_o-\phi_{min})}$$

But a periodic solution must be represented by a single (but arbitrary) harmonic, because q_n are not multiples. Therefore $\phi - \phi_{min} = \phi_n$, $q_n=2\pi s^{1/2}/L$, where L is a period and

$$\frac{1}{r}\frac{d}{dr} r \frac{d}{dr}\phi_n + q_n^2\phi_n = -\frac{4\pi j_o(r)}{\beta_o c\epsilon} \ . \quad (7)$$

Hence, for the modulation period and the distribution $j_o(r)$ being fixed a self-phased state is defined. The equilibrium velocity $\beta_o c$ is still arbitrary and is to be determined from the energy balance.

To simplify the mathematics let us consider the distribution of the current

$$j_o(r) = I\mu_1 J_o(\mu_1 r/b)/(2\pi b^2 J_1(\mu_1)),$$

where I is the total current through the waveguide. Then one gets from (6)

$$\phi_o = \frac{2IJ_o(\mu_1 r/b)}{\beta_o c\ \mu_1 J_1(\mu_1)} \ ; \quad \phi_n = \frac{\phi_o s_w}{(s_w-s)} \ ; \quad (8)$$

Bearing in mind that $\phi_n>0$ the modulation period in the self-phased π state can not be too small (i.e. $L > 2\pi b s^{1/2}/\mu_1$). In other words the phase velocity of the wave has to exceed the equilibrium beam velocity. The longitudinal electric field reaches its maximum value at the waveguide axis where

$$E_{z\ max} = \frac{2I}{b\beta_o\epsilon\ cJ_1(\mu_1)}\frac{ss_w^{1/2}}{(s_w-s)} \ . \quad (9)$$

The space charge distribution is

$$\rho = \frac{j_o(r)}{\beta_o c}\ (\ 1 + \cos(2\pi x/L)) \ . \quad (10)$$

Now, let us calculate the total power flux W through the waveguide cross-section S. The flux is the sum of the power transported by the particles

$$W_{part} = \int dS\int \overline{pc^2 f(r,x,p)}dp = \frac{3}{2}\ s\,\gamma_o^2\int\phi_n j_o dS$$

and the flux of the Pointing vector

$$W_{field} = \frac{c\beta_o\epsilon}{4\pi}\int dS\ \overline{(\partial\phi/\partial r)^2}$$

(the bar signifies averaging over the period L). For the chosen current distribution $j_o(r)$ we obtain following relation between β_o, β_w and W:

$$\frac{Wc\beta_0\epsilon}{I^2} = \frac{3}{2}\frac{ss_w\gamma_0^2}{(s_w-s)} + 1 + \frac{s_w^2}{2(s_w-s)^2} \; . \qquad (11)$$

Since the introduced power is determined by the mean injection energy $\bar{\gamma}m_0c^2 = We/I$, one gets readily from (9) and (11) the values of $\bar{\gamma}$ and β_w (or L) which provide a self-phasing for fixed β_0 and I;

$$\frac{m_0c^3\bar{\gamma}}{I_0e} = \frac{1}{4\beta_0\epsilon}((3(s\gamma_0^2-1)+2)I/I_0 + (3(s\gamma_0^2-1)-2)(I^2/I_0^2+\beta_0^2/s)^{1/2}+\beta_0^2I_0/(2sI)), \qquad (12)$$

$$(\mu_1L/(2\pi b))^2=s_w=s^2(I/I_0+(I^2/I_0^2+\beta_0^2/s)^{1/2})/\beta_0^2,$$

where $I_0=\epsilon bJ_1(\mu_1)E_{z\ max}$. The functions $\bar{\gamma}(I,\beta_0)$ and $\beta_w(I)$ are shown in Fig.4.

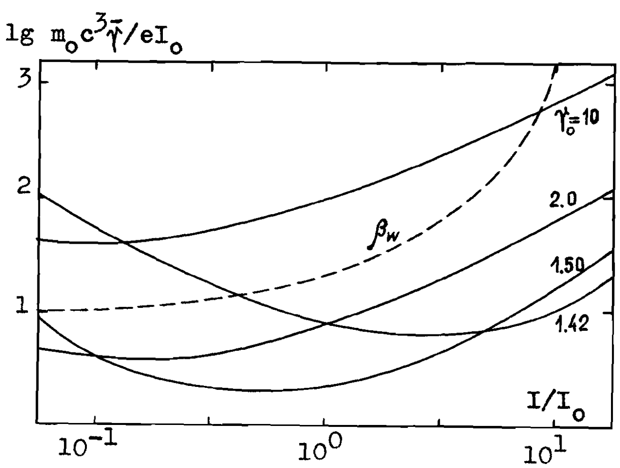

Fig.4

One can see that for fixed injection energy $\bar{\gamma}$ and field level I_0 there are two possible steady states. The first one corresponds to large currents and large differences between β_0 and β_w. The second state can take place for relatively small currents when the corresponding free wave is almost synchronous with bunches. In this case the modulation period L is comparable with b.

The self-phasing effect appears to be possible if the injection energy exceeds some threshold value depending on the field level and total current. This threshold value shown in Fig.5 has in turn a minimum at some $I \lesssim I_0$. The equilibrium energy in this case appears to be close to the mean injection energy.

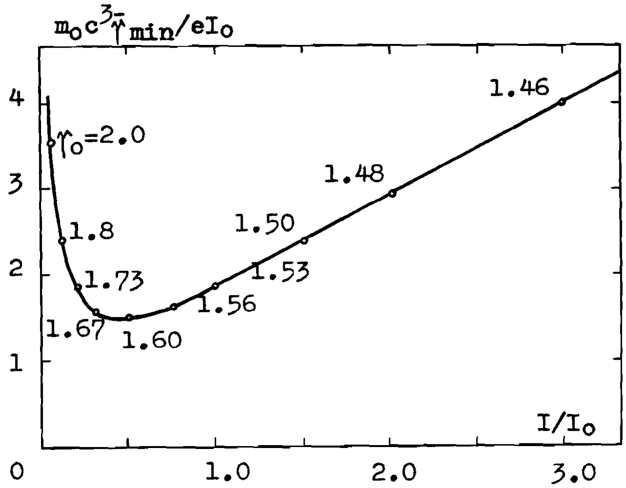

Fig.5

4. Particle acceleration in self-phased bunches

From the standpoint of particle acceleration the main parameter of the investigated system is the amplitude of the longitudinal electric field. As follows from the previous sections the amplitude readily achieves a level which is limited by breakdowns in the waveguide. One should expect this because the power transported by a high current beam is much greater than the power which might be transmitted in an empty waveguide*). Taking as an example $E_{z\ max}$=0.2 Mv/cm and b=5 cm one obtains for I_0 a value about 20-40 kA (for various $\epsilon \gtrsim 1$). This is comparable with total currents now attainable.

Let us discuss first the perspectives of electron acceleration. The most attractive situation is when the steady state corresponds to a large value of $\gamma_0=(1-\beta_0^2)^{-1/2}$, the injection energy being kept at a low level ($\bar{\gamma} \gtrsim 1$), i.e., when the main part of the beam radiates energy and only the trailing portion absorbs it. Unfortunately, it follows from Fig.5 that very small values of I_0 are necessary in this case, i.e., the effective accelerating field appears to be too small. The situation might be improved by removing low energy particles from each bunch as mentioned in the Introduction. This leads naturally to gradual decrease of total current. Supposing for simplicity that the removed electrons have zero velocity, i.e., that the total power flux remains unchanged, we obtain from (11) and (12) for $\gamma_0 \gg 1$ and $I \ll I_0$

$$\gamma_0^2 \approx \frac{4}{3I_0I(\epsilon-1)^{1/2}}\left(Wc\epsilon - \frac{I_0^2}{8(\epsilon-1)}\right) . \qquad (13)$$

*) This is why the field level in the system could be practically independent of RF losses in walls.

Thus, for a current of about 10^2 A, total power 10^{11} Wt (i.e., 2 Mv×50 kA), I_0=10 kA and ϵ=2, we obtain $\gamma_0 \approx 100$. Note that energies up to several Gevs may be present in the bunch for this equilibrium value of γ_0.

Acceleration of protons and other heavy particles requires a low velocity $\beta_0 c$ of bunches combined with the condition $\beta_0^2 \epsilon > 1$, so a large retardation is necessary. For $I \ll I_0 \beta_0$ Eq.(12) gives the power of the electron beam

$$cW \approx \frac{\beta_0 I_0^2}{8 s \epsilon} \quad . \tag{14}$$

For the same field level, the parameter I_0 is to be much greater than for acceleration of electrons because of a large value of ϵ. Supposing s=1, β_0=0.1, $I_0 = 3.10^5$ A and $I \ll I_0$, the power estimated from (14) is about 200 MW. Apparently, a more realistic estimation, taking into account wall losses could increase this value, but it still would appear reasonable.

The equilibrium velocity β_0=0.1 corresponds to 5 Mev protons captured by an electron bunch. Further acceleration may be provided by an adiabatic variation of parameters along the waveguide. In the case under consideration, $(I \ll I_0 \beta_0)$ $\beta_W \approx \beta_0$ and the bunch length (i.e., $(\beta_W^2 - 1)^{1/2}$) varies $\sim b\beta_0^{-1}$. Therfore, β_0 increases with decreasing ϵ as ϵ^{-y} where y is somewhere between ½ and 1/3. The maximum proton energy is determined by the system length and by the electric field level. As for a possible proton current, it has to be much less than the electron current which is evidently large enough.

These estimates are not claimed to be rigorous or prove the practical realizability of acceleration in self-phased bunches. In the first place the model of a real retarding system is too rough and the question is open as to what extent a dielectric medium can simulate the real properties of irised waveguides or other practical systems. Among other essential problems, one should mention transverse motion of the particles, possibilities of removing slow electrons, uncertain practical level of the electric field, possible instabilities, transient processes etc. We should like merely to direct attention here to one possibility of field generation directly in the vicinity of accelerated particles.

Large transported power fluxes, the possibility of self-phasing, the flexibility of the excited field pattern and field level can alter significantly customary ideas concerning possible accelerated beam intensities and requirements for accelerating systems.

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