COLLECTIVE ACCELERATION OF IONS BY SCANNING AND ROTATION OF AN ELECTRON BEAM

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Abstract

The two variants of the acceleration of ions captured by the potential well of dense electron beam are considered:scanning method and "centrifugal" method.In the latter case electron beam rotates in the plane or in the surface of a cone; the ions are being accelerated slipping along the beam and shifting onto a greater radius. The influence of the ions on the depth of the electron beam potential well during acceleration is considered.

For some time a great deal of attention has been paid to collective methods of acceleration. Among these the acceleration of ions captured into rings of relativistic electrons should be noted /1/. But there are some methods of collective acceleration which do not require the use of electron rings. The idea of one such method, based on moving foci, was suggested long ago (see /2/ and also /3/). This idea can be developed further taking into account the latest achievements in the field of intense relativistic electron beams.

Let us consider some features of ion acceleration by transverse motion (scanning) of an electron beam. A stream of electrons I_e having total energy $E_e = \gamma_e mc^2$ passes through a focusing system in the direction OB (Fig.1) and reaches a maximum electron density at the crossover. By means of a special deflection system the beam as a whole is shifted in the transverse direction CD. It is known /4/ that a dense electron beam forms a potential well for ions. The capture of ions takes place at point C; then the ions are carried by the field of the electron beam and accelerated in the direction CD.

One should keep in mind that the electrons injected at different instants form a sort of "spiral" (the curve SAB, Fig.1). The ions move along this spiral as well as in the direction normal to it. In /5/ the case of approximately translational motion of the beam and normal ion motion was considered. We want to draw attention here to the case when the ions are slipping along the beam and being accelerated, shifting to greater radii /6/. In this variant of acceleration which can be called "centrifugal", the case of uniform beam rotation is of particular interest. In this case all times are equivalent and the possibility of achieving continuous (not pulsed) ion acceleration arises. The regime of beam rotation can be realized in principle in two main variants: I) on the surface of a cone (characterized by apex angle 2α), 2) in the plane (this variant can be obtained from the previous one putting $\alpha=\pi/2$). Ion acceleration by uniform electron beam can be schematically described in the following manner (Fig.2). The electron beam from the injector passes the focusing system and enters the resonator which makes the beam rotate along the cone surface. Further deflection of the beam can be carried out by means of a static toroidal magnetic field ("deflector"). As a result the beam is forced to rotate in the plane normal to its initial direction.

Let us consider the beam to be infinitely fine and to form a sufficiently deep potential well so that the captured ions can move only along the beam. The injection of ions into the electron beam can be performed near the center O. At an instant t the ion coordinates r, θ ,z are related as follows

$$\theta=\phi\left(t-\frac{r}{c\beta_e \sin\alpha}\right),\ z=r\,ctg\,\alpha. \quad (1)$$

where the function ϕ defines the scanning law and $c\beta_e$ is the velocity of the electrons. Actually formula (I) expresses the constraint on ion motion due to the electron beam. The velocity of an ion at instant t is

$$\beta=\frac{1}{c}\left[\left(\frac{dr}{dt}\right)^2\frac{1}{\sin^2\alpha}+r^2\phi'^2\left(1-\frac{1}{c\beta_e \sin\alpha}\cdot\frac{dr}{dt}\right)^2\right]^{\frac{1}{2}} \quad (2)$$

where ϕ' is the derivative of ϕ with respect to $t-r/c\beta_e \sin\alpha$. The equation of ion motion in terms of variables r, $\frac{dr}{dt}$ considered as the generalized coordinates of an ion can be derived from Lagrangian

$$L=-Mc^2\sqrt{1-\beta^2},\quad \frac{d}{dt}\frac{\partial L}{\partial\left(\frac{dr}{dt}\right)}-\frac{\partial L}{\partial r}=0 \quad (3)$$

where M is the rest mass of an ion. In terms of dimensionless coordinates x,y the Lagrangian is given by

$$L=-Mc^2\left\{1-\beta_e^2\left[\dot{y}^2+y^2\varphi'^2\left(1-\dot{y}\right)^2\right]\right\}^{\frac{1}{2}} \quad (4)$$

where $t\sin\alpha=\varkappa x$, $r=\varkappa\beta_e cy$, $\dot{y}=dy/dx$, φ'-

is the derivative of $\Phi \sin\alpha$ with respect to x-y, and $\varkappa$ is some scale factor; for example, in the case of constant beam rotation frequency ν we have $\varkappa = (2\pi\nu)^{-1}$. The equation of ion motion (3) for Lagrangian (4) is

$$\ddot{y}\left(1+\frac{1+y^2\varphi'^2}{\gamma_e^2-1}\right)+y\varphi'(1-\dot{y})^2\Big[y^2\varphi'^3(1-\dot{y})-\varphi'(1+2\dot{y})-y\varphi''(1-\dot{y})\Big]-\frac{y\varphi'(1-\dot{y})}{\gamma_e^2-1}\Big[\varphi'(1+\dot{y})+y\varphi''(1-\dot{y})\Big]=0 \qquad (5)$$

where $\gamma_e = (1-\beta_e^2)^{-\frac{1}{2}}$. The law of motion of an ion depends on the type of function $\varphi = \varphi(x-y)$: e.g., $\varphi = x-y$ in the case of uniform electron beam rotation or $\varphi = (x-y)^2$ in the case of approximately constant ion acceleration etc. The ion trajectories for different rates of scanning and different beam rotation frequencies are presented in Fig. I (CD_1, CD_2, CD_3) and in Fig.3 (OD_1, OD_2, OD_3).

An important parameter characterizing the ion motion in the electron beam is the force component G acting on the ion in the direction normal to the electron spiral (the reaction of constraint). Knowledge of this parameter gives us the possibility to go over to the problem of a real beam which is characterized by initial beam cross-section radius ρ_{in}, crossover radius ρ_c, electron current I_e, electron relativistic factor γ_e, initial angular convergence τ. Using the equation of motion (5) we can obtain the expression for the "normal" force G in the form

$$G=\frac{Mc\beta_e}{\varkappa}\gamma_e^2\gamma\frac{(1-\dot{y})\sqrt{1+y^2\varphi'^2}}{\gamma_e^2+y^2\varphi'^2}\Big[2\dot{y}\varphi'-y^2\varphi'^3(1-\dot{y})+y\varphi''(1-\dot{y})\Big], \qquad (6)$$

$$\gamma=\left\{1-\frac{\gamma_e^2-1}{\gamma_e^2}\Big[\dot{y}^2+y^2\varphi'^2(1-\dot{y})^2\Big]\right\}^{-\frac{1}{2}} \qquad (6a)$$

Let F be the force component normal to electron spiral and due to the beam self-field. For an ion to move together with electrons, the value F must satisfy the inequality

$$F \gtrsim G \quad or \quad e\mathcal{E} \gtrsim G \qquad (7)$$

if we neglect the ion current action ($\mathcal{E}$ is the electric field). We can obtain for $\mathcal{E}$ the expression

$$\mathcal{E} \simeq \frac{60 I_e}{\rho}\frac{\cos\delta}{\cos^2\frac{\delta}{2}} \quad \frac{volts}{cm} \qquad (8)$$

where δ is the angle between the instaneous beam direction and the electron spiral direction.

The relation between the radius of the beam envelope and its longitudinal extent is given by the formula

$$\ell=\frac{\rho_{in}}{\sqrt{A}}e^{-\eta^2}\int_{-\eta}^{\sqrt{\eta^2+\ln\rho/\rho_{in}}} e^{\xi^2}d\xi \qquad (9)$$

where l is the length of the beam in centimeters. $A=6.10^{-5}I_e/\gamma_e^3$, $\eta^2=\tau^2\cdot\gamma_e^3/2.4.10^{-4}I_e$. This formula takes into account the space charge interaction in the case of zero phase space volume.

Let us consider now the particular case of uniform beam rotation. In this case, the quantities γ, G and also β_r (the ion radial velocity) may be obtained in explicit form as functions of the dimensionless radius $y = r/R\beta_e$, where $R= c/\omega$ is the cyclotron radius:

$$\gamma=\frac{y^4\beta_e^2+\gamma_{in}^2(y^2+1)}{\left[(1-\beta_e^2)y^2+1\right]^{1/2}}\left\{\gamma_{in}\left(\frac{y^2}{\gamma_e^2}+1\right)^{1/2}+y^2\beta_e^2\left(y^2\beta_e^2+\gamma_{in}^2-1\right)^{\frac{1}{2}}\right\}^{-1}, \qquad (10)$$

$$\beta_r=\gamma^{-1}(y)\left[\frac{y^2\beta_e^2+\gamma_{in}^2-1}{y^2(1-\beta_e^2)+1}\right]^{\frac{1}{2}}\cdot\sin\alpha \qquad (11)$$

The expression for $\gamma = \gamma(y)$ is identical for both variants of the regime of beam rotation: along the surface of a cone and in a plane. The apparent difference for the corresponding $\beta_r = \beta_r(y)$ expressions is the appearance of $\sin\alpha$ in the case of a cone. It should be noted that in the nonrelativistic region

$$\gamma=\gamma_{in}+y^2\beta_e^2, \quad \frac{Mv^2}{2}\simeq M\omega^2r^2 \qquad (12)$$

where γ_{in} is the initial value of γ. In the extreme case $r \gg R$ we have $\gamma \to \gamma_e$. This means that the limiting ion velocity is equal in principe to the velocity of electrons in the beam, which is the basic idea of collective ion acceleration in general. In this same limit, ions tend to move in the radial direction, that is, $\beta \to \beta_r \to \beta_e$

The function $G = G(y)$ in the case of uniform rotation is expressed as follows:

$$G(y)=Mc\omega g(y)$$

$$g(y)=\gamma(y)\frac{\sqrt{1+y^2}\,(\beta_e-\beta_r)}{\beta_e[1+y^2(1-\beta_e^2)]}\left[2\beta_r-y^2(\beta_e-\beta_r)\right] \tag{13}$$

where $\gamma=\gamma(y)$, $\beta_r=\beta_r(y)$ are given by eqns. (10) and (II). Using eq.(13), we can express the inequality (7) in another form

$$n_e \gtrsim \frac{M}{2m}\,\frac{\rho}{R}\,g(y)\,\frac{\cos^2\frac{\delta}{2}}{\cos\delta} \tag{14}$$

where n_e is the number of electrons in the beam along a length equal to the classical electron radius. The dependence of G on y for uniform beam rotation is shown in Fig.4.($\nu = 10^8$ 1/sec; γ_e = 10, 20, 30). In the same figure the dependence $\gamma=\gamma(y)$ is shown for γ_e = 10. As the energy of the ion (or its radius) increases the reaction G increases initially as y (in nonrelativistic case) reaches a maximum value and then tends to zero for large radii.

An ion could be actually captured and accelerated by the beam in accordance with relations (10) and (II) if inequality (7) or (14) is fulfilled. Therefore curve $e\mathcal{E}(y)$) calculated on the basis of focusing condition (9) must be located above curve G. This requirement restricts the choice of beam parameters. Let us present some examples of sets of parameters and ion energies attainable: I) R=50 cm, γ_e=10, I_e = 170 kA, ρ_{in} = 5 cm, ρ_c = 2 cm, $\tau \simeq .3$, W $\simeq$ 120 Mev; 2) R=50 cm, γ_e = 20, I_e = 300 kA, ρ_{in} = 3 cm, ρ_c = 1.2 cm, $\tau \simeq .1$, W $\simeq$ 300 Mev.

We shall also present briefly the results obtained for nonuniform beam scanning according to the law $\theta = (x-y)^2$. The scanning magnetic field change rate was chosen to be of the order of 10^{10}-10^{11} gauss/sec and two variants were considered: a) ion injection at r_{in}=125 cm, b) ion injection at r_{in}=0. Example of parameter sets are the following: I) r_{in}=125 cm, r_{max}=140 cm, γ_e = 15, I_e=10 kA, ρ_{in}=5 cm, ρ_c=3 cm, $\tau \simeq 0.5$, W $\simeq$ 100 Mev; 2) r_{in} = 0, r_{max} = 80 cm, γ_e = 15, I_e= 20 kA, ρ_{in} = 4 cm, ρ_c= .4cm, $\tau \simeq .1$, W $\simeq$ 150 Mev.

All four examples show that for system of sufficiently small dimensions the simplest scanning laws $\varphi \sim x-y$, $\varphi \sim (x-y)^2$ correspond to large values of pulsed beam power. Therefore thefuture of the methods considered is completely dependent on the possibilities of effective beam energy recuperation.

The acceleration processes considered are based on the use of dense electron beam. But since accelerated ions also contribute to electromagnetic fields and forces we shall discuss briefly this phenomenon.The total force due to electrons and ions acting on a given electron at a distance r from the spiral axis is equal to

$$F_e=\frac{2mc^2 I_e}{\rho^2\beta_e I_o\cos^2\frac{\delta}{2}}\left[1-\beta_e^2\cos^2\delta-\frac{I\beta_e}{I_e\beta}\,\frac{f_e(\delta)}{\cos\delta}\right]\zeta \tag{15}$$

where I is the ion current, I_o = 17 kA, $f(\delta) = 1-\beta\beta_e\cos\delta\cos(\delta+\chi)$ and χ is the angle between $\vec\beta_e$ and $\vec\beta$. The presence of ion current also alters the force acting on a given ion

$$F=\frac{2mc^2 I_e}{\rho^2\beta_e I_o\cos^2\frac{\delta}{2}}\left[-f(\delta)+\frac{I\beta_e}{I_e\beta}\,\frac{1-\beta^2\cos^2(\delta+\chi)}{\cos\delta}\right]\zeta \tag{16}$$

The trajectory of an individual electron in the electron beam is described by the equation

$$\zeta''+\frac{\psi}{\rho^2}\,\zeta=0 \tag{17}$$

ψ(y) is a slowly varying function describing the action of electron charge and its compensation by ions. In particular some sort of laminar motion is possible when an electron at the beam edge will stay there continuously, i.e. $\zeta=\rho\sin\delta$ and equation (17) takes the form

$$\zeta''\zeta+\Psi=0\,,\qquad \Psi=\psi\sin^2\delta \tag{18}$$

For the case Ψ= Const, the solution of equation (18) is

$$\int_0^{\sqrt{\ln \zeta_{in}/\zeta}} e^{-\xi^2}\,d\xi=\frac{\sqrt{\Psi}}{\sqrt{2}\,\zeta_{in}}\,z \tag{19}$$

$$\text{or}\quad z=\frac{\zeta_{in}\sqrt{\pi}}{\sqrt{2\Psi}}\,\Phi\left(\sqrt{\ln(\zeta_{in}/\zeta)^2}\right)$$

where Φ is a probability integral. In the general case,

$$\Psi=-\frac{2I_e}{\gamma_e\beta_e^3 I_o}\sin2\delta\,\mathrm{tg}\frac{\delta}{2}\left[1-\beta_e^2\cos^2\delta-\frac{I\beta_e}{I_e\beta}\,\frac{f(\delta)}{\cos\delta}\right] \tag{20}$$

and a numerical solution of (18) is required.

Selfconstriction of the beam i.e. decrease of the value of ρ corresponds to the condition of positivity of Ψ

$$1-\beta_e^2\cos^2\delta - \frac{I\beta_e}{I_e\beta}\,\frac{f(\delta)}{\cos\delta} < 0 \tag{21}$$

This is fulfilled when the ion density is sufficiently large. On the other hand, for selfconstriction to be accompanied by an increase in the depth of the potential well for ions, their density must not be too large.

$$\frac{I}{I} \ll \frac{\beta\cos\delta\cdot f(\delta)}{\beta_e\left[1-\beta^2\cos^2(\delta+\chi)\right]} \tag{22}$$

The upper limit of the ion density is determined also by the validity of the approximation of the given electron beam motion, An analysis of the eq.(18) and inequalities (21), (22) is cumbersome and is beyond the scope of this paper. A rough estimate can be obtained for the simplest case of parallel electron and ion motion. In such a case the mentioned inequalities can be reduced to

$$\frac{\beta}{\beta_e\gamma_e^2(1-\beta\beta_e)} < \frac{I}{I_e} \ll \frac{\beta\gamma^2(1-\beta\beta_e)}{\beta_e} \tag{23}$$

In conclusion it should be noted that in this paper we have considered only certain physical principles of ion acceleration by the centrifugal method and scanning of an electron beam in the simplest case. Other more complicated variants are possible, e.g. the useoof two intersecting scanning beams forming moving foci. In all variants the basic problems are providing proper electron beam recuperation and beam scanning and focusing equipment.

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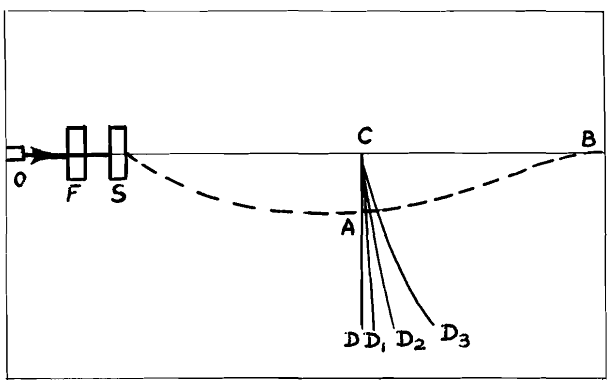

Fig.I. Schematic diagram of ion accelera tion by electron beam scanning.

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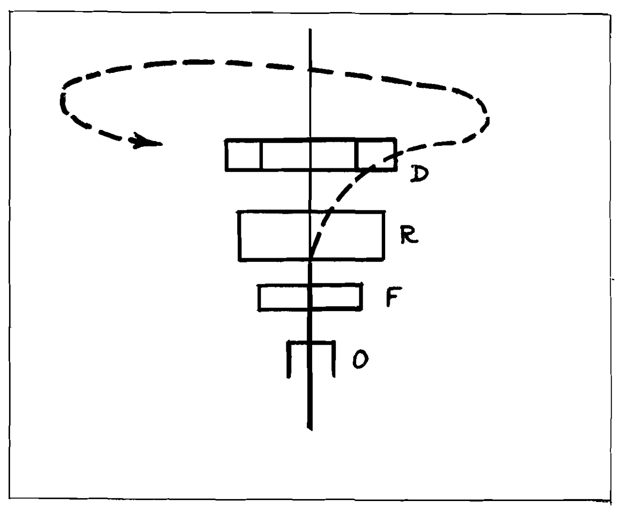

Fig.2. Schematic diagram of ion acceleration by electron beam rotation

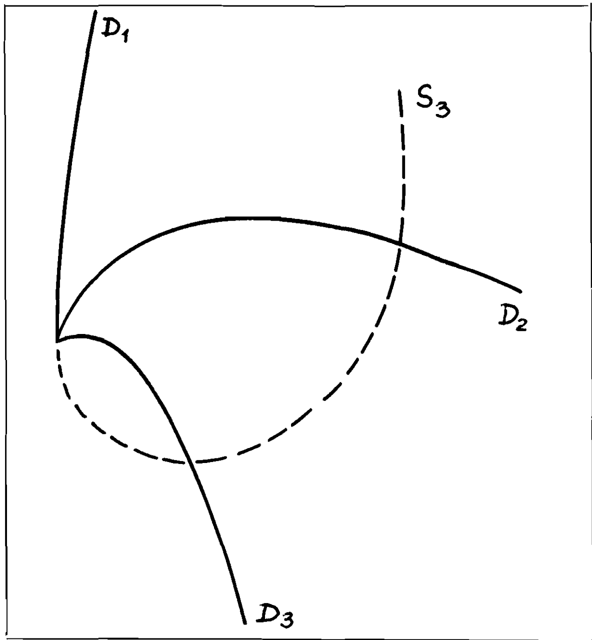

Fig.3. Ion trajectories OD_1, OD_2, OD_3 in the process of electron beam rotation ($\nu_3 \gg \nu_2 \gg \nu_1$); OS_3 - the instantaneous "spiral" position.

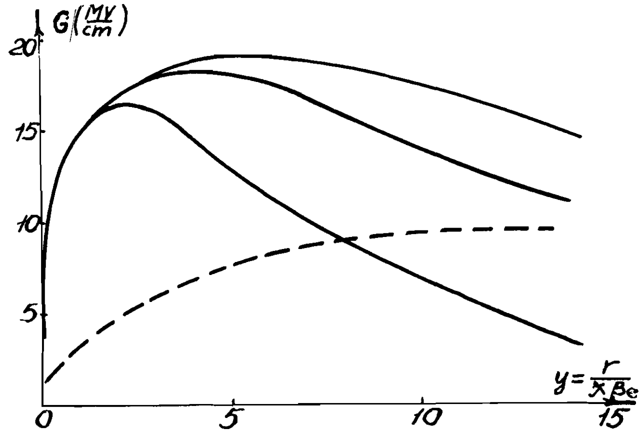

Fig.4. Dependence of G_i on y (the case of electron beam uniform rotation $\nu = 10^8$ I/sec; $\gamma_e = 10, 20, 30$).