THE STUDY OF RESONANT EXCITATION OF CHARGE DENSITY WAVES FOR COHERRENT ACCELERATION OF IONS

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Coherent acceleration of ions by forces originating from space charge of a non-compensated electron bunch[1] can be realized in electron beams with density modulation[2]. But it is known that practical realization of high-level modulation of dense beams is accompanied by considerable difficulties. We have found heavy modulation of an electron beam moving in spatially periodic structures when resonant excitation of charge density waves takes place in the beam[3]. It was suggested to use resonant excitation of large amplitude charge density waves for coherent ion acceleration[4].

The elementary mechanism responsible for resonant excitation of charge density waves is the parametric Cherenkov effect[5]. The effective accelerating electric field strength is[6]:

$$E_{eff} = 4\pi e n_0 \xi \tag{1}$$

where n_0 is the undisturbed beam density and $\xi = x(t,t_0) - v_0(t - t_0)$ is the displacement of those electrons which started their motion at a moment t_0 from their initial position, $x - v_0(t - t_0) = \text{const}$ in the beam.

To get some information about the usefulness of resonant excitation of charge density waves for coherent ion acceleration, it is necessary to know the maximum achieved values of ξ and E_{eff} as functions of beam parameters. Note that in the case considered, the variation of wave phase velocity synchronously with ion velocity variation is achieved by the use of external state electric fields, and no principal obstacles arise.

The maximum possible value of ξ is determined by the solution of the corresponding non-linear problem. In some cases such solutions have been obtained and analysed for an idealized one-dimensional problem of electron fluid flow. These solutions confirm the possibility of obtaining in beams very strong accelerating fields such as 1 to 10 MV/cm. But some factors such as relativistic effects, final cross-section of the beam, thermal velocity scatter of particles, screens in canalization and modulation system, limit resonant excitation and decrease the value of ξ_{max}.

In the present work we summarize the results of experiments on resonant excitation of charge density waves in modulated beams. These experiments were performed in the Physics-Technical Institute of the Ukrainian Academy of Science. Some prospects of further development of this work is discussed together with all the factors mentioned above.

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1. In the theoretical treatment of the process of resonant excitation of high amplitude charge density waves, we shall proceed from i) electron motion equations:

$$\frac{dv}{dt} = -\frac{e}{m}E, \qquad E = E' + E'', \tag{2}$$

where E is the total electric field which consists of the external field E′ and the internal field of the beam E″; and ii) Maxwell equations describing longitudinal waves in the beam:

$$\frac{\partial(\varepsilon E'')}{\partial x} = 4\pi e(n_0 - n), \quad \frac{\partial(\varepsilon E'')}{\partial t} + 4\pi e(n_0 v_0 - nv) = 0. \tag{3}$$

Here v_0 is undisturbed electron velocity.

Resonant excitation of charge density waves takes place even without external field ($E' = 0$) if properties of the medium change periodically along beam direction:

$$\varepsilon(z + L) = \varepsilon(z) \tag{4}$$

A corresponding apparatus can be realized by a system of dielectric disks with central orifice through which the beam can pass.

If the medium is uniform, resonant excitation takes place in an external field:

$$E' = E_0 \sin kz, \tag{5}$$

where $k = \pi/L$ is the parallel wave number, L is the period of the structure. In practice, such a field is realized on the axis of the cylindrical electrodes system if a constant potential difference ϕ_0 is applied between each pair of electrodes.

Other systems in which the motion of modulated electron beams is accompanied by high amplitude, longitudinal charge density waves are also possible, e.g. the succession of weakly coupled cavities whose natural frequencies coincide with a natural frequency of the beam. But it is more convenient to illustrate the main physical regularities, characterizing resonant excitation of oscillations in beams on the two particular systems mentioned above.

2. It is convenient to illustrate the essence of resonant excitation of oscillations in beams by the following example. Let an initially uniform electron beam pass through modulating grids and experience velocity modulation. The velocity modulation transform into density modulation in the drift space. But space charge effects prevent heavy modulation, and to get any considerable effect, high modulation potential must be applied.

In the case when the drift space is filled with a stratified medium, and transit time of the period is comparable with natural oscillation period of the beam, the situation changes considerably. Under conditions which we call resonant ones:

$$\tilde{K}_{res} \simeq \frac{\omega_0 L}{\sqrt{\varepsilon_{eff}}\, v_0} = n\pi \qquad \cos\psi \geq 1, \tag{6}$$

where $\cos\psi$ is found from

$$\cos\psi = \cos\tilde{K}\beta \cdot \cos\tilde{K}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}(1-\beta) - \frac{1}{2}\frac{\varepsilon_1+\varepsilon_2}{\sqrt{\varepsilon_1\varepsilon_2}}\sin\tilde{K}\beta\,\sin\tilde{K}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}(1-\beta), \tag{7}$$

ω_0 is electron plasma frequency. The current after the modulation grid at the start of ν^{th} period is ($x = \nu L$):

$$j(x,t) = j_0 + 2j_0\sum_{n=1}^{\infty} J_n\Big[n\frac{eV_1\omega}{mv_0^3}\times \left(A_0\nu - \frac{C_0}{2}\nu(\nu+2)\right)\Big]\cos n\omega\left(t-\frac{x}{v_0}\right). \tag{8}$$

Here A_0 and C_0 are constants which depend on dielectric parameters and beam density; $L = a + b$ is the period of the system; a is the depth of a layer with dielectric constant ε_1; b is the depth of a layer with dielectric constant ε_2, $\beta = b/L$. It is seen from formula (8) that under resonant conditions an electron beam with arbitrarily large current density can be deeply modulated by even a small modulation potential V_1.

The defect of the klystron method of particle bunching is that the region where an electron bunch exists is small and localized in the focus of the klystron. This region can be considerably increased if a periodic force affects the beam actively over all its length rather than at some local plane. Such an effect can be realized, for example, by a succession of electrodes, the potentials on each pair being antiphased.

3. Some preliminary estimates of ξ_{max} in a system with external modulating field (2) were made in non-relativistic approximation[7]. For ξ one obtains the following equation when relativistic factors are taken into account:

$$\frac{d^2\xi}{dt^2} + \left\{\omega_0^2\xi + \frac{eE_0}{m}\sin(k\xi + kv_0t)\right\}\times\left[1 - \frac{\left(v_0 + \frac{d\xi}{dt}\right)^2}{c^2}\right]^{3/2} = 0, \tag{9}$$

It is more convenient to consider this equation with dimensionless variables $y = k\xi$, $\tau = \omega_0 t$.

$$\frac{d^2y}{d\tau^2} + \left[y + \varepsilon\sin(y+\nu\tau)\right]\times\left[1-\beta_0^2\left(1+\frac{1}{\nu}\frac{dy}{d\tau}\right)^2\right]^{3/2} = 0, \tag{10}$$

where $\beta_0 = v_0/c$, $\nu = kv_0/\omega_0$ and $\varepsilon = eE_0k/m\omega_0^2$.

In general, ν should be considered as a parameter. In non-relativistic approximation integers $\nu = 1, 2, \ldots$ give the conditions of resonant excitation of the oscillations. An analytical analysis of Eq. (10) has been made[8] for small values of the parameter $\varepsilon \ll 1$. The shape of the resonant curve has been obtained as a function of beam velocity (Fig. 1), and it has been shown that the curve being symmetrical in non-relativistic approximation is distorted when relativistic effects are taken into account. It is seen from the graphs that the maximal value of ξ_{max} in the centre of resonant region decreases when β_0 increases. To retain ξ_{max} unchanged when $\beta_0 \to 1$ it is necessary to alter the parameters of the system smoothly (see dotted line on Fig. 1).

More detailed information of the behaviour of ξ at arbitrary values of ε can be obtained by numerical methods. Typical curves $\xi = \xi(\tau)$ characterizing resonant excitation of charge density waves in a beam are represented on Fig. 2. It is important that resonant values of ν are not integers in the relativistic region. The dependence of the maximal value ξ_{max} on ν at various β_0 is represented on Fig. 3. It is seen from this figure that the maximal value ξ_{max} decreases when β_0 increases. This agrees qualitatively with data shown in Fig. 1. The physical nature of this decrease lies in the deformation of the resonant curve. Together with β_0 increase, not only ξ_{max} decreases but w_0 increases, so the effective accelerating field calculated in accordance with Eq. (1) increases too (see Fig. 4). The data represented by Fig. 4 confirm the possibility of forming charge density waves with very high E_{eff} ($\sim$ 1 MeV/cm) though the beam is modulated by considerably smaller fields. It follows from the graph of Fig. 1 that these values of the accelerating field can be increased several times if together with excitation of oscillations the process follows the dotted line on Fig. 1.

Some preliminary study has been made of the influence of final cross-section of the beam and thermal velocity scattering of the particles on resonant excitation of the oscillations. These factors affect the shape of the resonant region if a sufficiently strong longitudinal focusing magnetic field is applied. A metal wall in canalization or modulation systems can also affect the value of ξ_{max} considerably, provided these walls are close to the beam. The effect can be ignored if $kR \gg 1$, $R/z \gg 1$, where r is radius of the beam, R is radius of the screen.

4. The main statements of the theory were verified on an experimental device, with the scheme shown in Fig. 5. A spatially periodic electric field was produced by an assembly of plates. The radii of the vacuum chamber R, plates R_1, central orifices of the plates r_0, and the beam r were 5, 4.3, 1, and 0.6 cm, respectively, the period of the system being

L = 3.1 cm, total number of periods = 10. It has been shown by numerical calculations and experiments that vacuum potential between the plates in the beam region is (0.3 to 0.4)ψ_0, where ψ_0 is a voltage applied to the plates from an external source. A stationary electron beam passed through the central orifices of the plates and hit the collector. The leading magnetic field strength was about 800 Oe.

An initial level of HF oscillations was generated by the beam itself. It was checked experimentally that at some definite values of current and voltage, a certain spectrum in the regions 50-200 and 3000-4000 MHz was excited inside the beam when no plates were present. A relative level of HF oscillations and their spectrum at the output were measured with a type C4-5 analyser, whose input was connected to a probe located near the surface of the beam. More than a hundredfold increase of the initial level of the oscillations has been found. At the present time these results have been confirmed in a wide band of frequencies with the use of initial modulation of the beam. It has been shown that the dependence of oscillation amplitude on current, accelerating voltage, and electric field between the plates has a resonant character. Experimental values of J_{cr} and V_{beam} which correspond to the maximum of the oscillation amplitude are represented with dots in Fig. 6. Calculated values J_{cr} and V_{beam} obtained from resonance conditions are represented with solid curves. The scatter of points on this figure characterizes the reproducibility of the results, the accuracy of measurements, and how much a one-dimensional theoretical model corresponds to real experimental conditions. We think that the results which have been obtained agree satisfactorily with theory and confirm the possibility of resonant excitation of large amplitude charge density waves.

References

1) V.I. Veksler, Atomnaja Energiya 2, 427 (1957).

2) M.S. Rabinovich, Trudy Vsesoyuznogo soveszania po uskoritelam zarjazhennykh chastits, Moskva (1968) (VINITI edit. Moscow, 1970), Vol. 2, p. 473.

3) N.S. Repalov, N.A. Khizhnjak, Radiotekhnika i electronika 10,334 (1965).

4) N.A. Khizhnjak, N.S. Repalov, Trudy Vsesoyuznogo soveszania po uskoritelam zarjazhennykh chastits, Moskva (1968) (VINITI edit., Moskow, 1970), Vol. 2, p. 522.

5) Ya.B. Fainberg, N.A. Khizhnjak, Zh. Eksp. Teor. Fiz. 32, 1882 (1957).

6) N.S. Repalov, N.A. Khizhnjak. Zh. Eksp. Teor. Fiz. 37, 471 (1967).

7) N.S. Repalov, N.A. Khizhnjak, Vysokochastotnyie svoistva plasmy (Coll. High-Frequency Properties of Plasma) ("Naukova dumka" edit, Kiev, 1968), Vol. 3, p. 90.

8) A.M. Korsunskii, N.S. Repalov, Zh. Tekh. Fiz. 41, 826 (1971).

9) G.G. Aseyev, G.G. Kuznetsova, N.S. Repalov, B.G. Safronov, N.A. Khizhnjak, Atomnaja Energiya 28, 513 (1970).

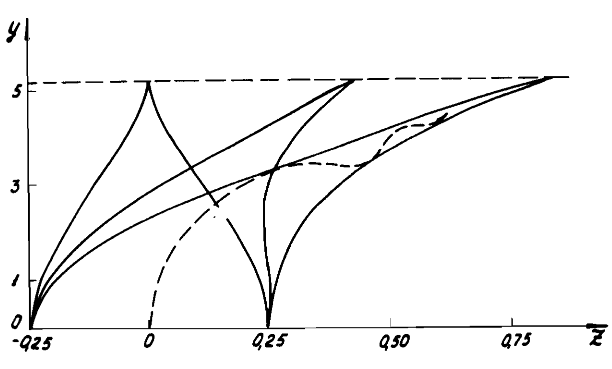

Fig. 1 : The shape of a resonant curve for various relative velocities of the electron beam: 1) $\beta = 0$, $\varepsilon = 0.1$; 2) $\beta = 0.25$, $\varepsilon = 0.1$; 3) $\beta = 0.4$, $\varepsilon = 0.1$; $Z = 1 - (\nu + \frac{3}{4}\beta_0^2)/\varepsilon$.

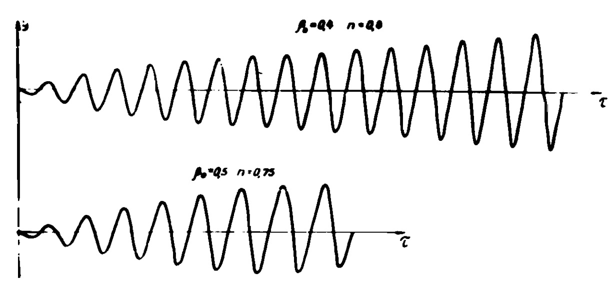

Fig. 2 : Typical functions $\xi(\tau)$, characterizing resonant excitation of charge density waves in the system: 1) $\beta = 0.4$, $\nu = 0.8$; 2) $\beta = 0.5$, $\nu = 0.75$.

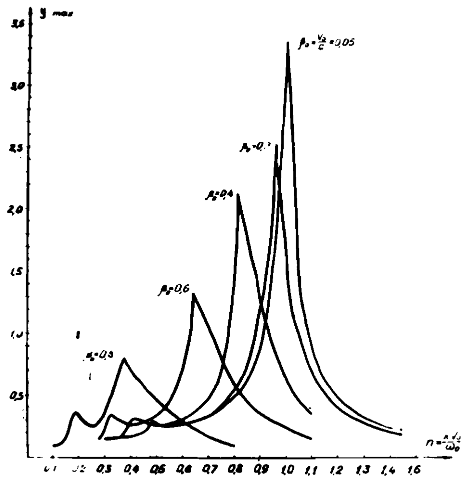

Fig. 3 : Dependence y_{max} on the parameter ν for various values of relative beam velocity and modulation parameter $\varepsilon = 0.1$.

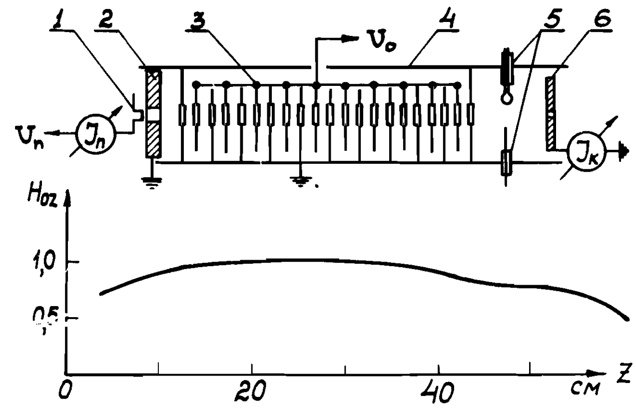

Fig. 5 : Diagram of the apparatus and relative position of the electrodes, producing spatially periodic field. 1: cathode; 2: anode; 3: plates producing electric field; 4: vacuum chamber; 5: probes; 6: collector.
Focusing magnetic field distribution along the axis of the system is shown below.

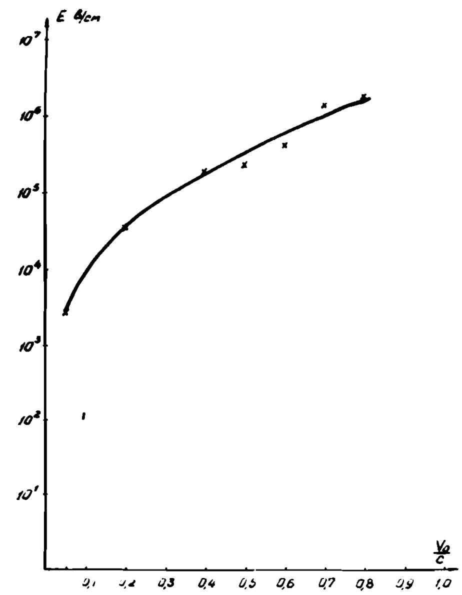

Fig. 4 : Maximal electric field produced by charge density waves versus relative beam velocity at modulation parameter $\varepsilon = 0.1$.

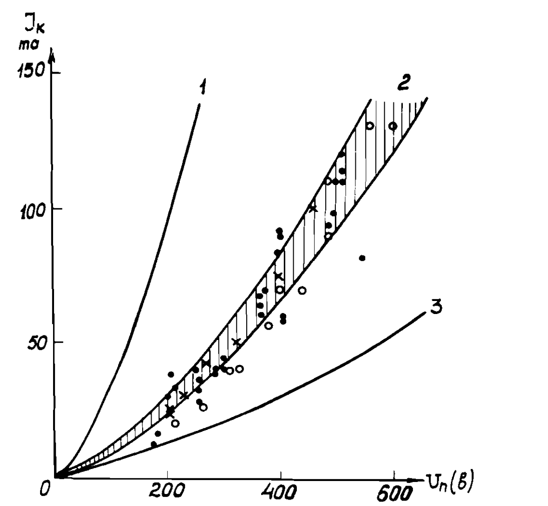

Fig. 6 : Experimental and theoretical functions of beam current versus accelerating voltage $J_k = f(V_{beam})$ in resonance conditions. Curves 1, 2, 3 correspond to the resonance with 1st, 2nd, and 3rd Langmuir frequency harmonics. The bottom curve for 2nd harmonic was calculated for a cathode, whose area is 20 percent less than the geometrical one.