HIGH INTENSITY BUNCH SELF-FOCUSING IN PERIODIC WAVEGUIDE SYSTEMS

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Abstract

Some problems of high-density particle bunch movement in periodic waveguide structure are discussed. Such bunches can be used for a collective method of acceleration. Taking into consideration the peculiarities of reaction force in these waveguide systems, it is shown in the elementary examples that we can obtain effective greater longitudinal and tangential focusing of bunches if we make use of periodic waveguide systems.

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Recently in the Moscow Physical Engineering Institute, work on the conceptual design of a 1-10 MeV linear accelerator of heavy particles has been carried out[1-3]. The collective method is used in this accelerator.

One of the main problems in the design of such accelerators is to ensure the stability of a toroidal shape bunch, when the torus moves in the static magnetic field. Assuming that there is no space-charge neutralization, the Coulomb repulsion in the longitudinal direction should be compensated with some external force.

It should be noted that providing longitudinal bunch focusing is the essential problem, not only for the project discussed but also for other designs where high-intensity electron-beam bunches are used for collective acceleration of heavy particles[4,5]. Generally, it is proposed to obtain longitudinal bunch focusing with external RF fields[6], which requires an additional power supply for the system. However, this method cannot be accepted as it may complicate construction of the accelerator, while the basic merit of our accelerator is simplicity of the constructional features.

The purpose of the present report is to prove that efficient focusing can be provided by the self-radiation field of high intensity bunches moving along a periodic waveguide having suitable parameters.

The study should proceed from a determination of the longitudinal electric field component E_z excited by the bunch in an arbitrary periodic waveguide. If a point charge e moves longitudinally with a velocity value of $\beta_{\parallel} c$, this can be found from the formula

$$E_z(\vec{r},t) = -\frac{e\beta_{\parallel}c}{2}\sum_{\substack{l_1=-\infty \\ \alpha=\alpha_i}}^{\infty}\left[\frac{E_{\alpha_i z}^{(l)}(\vec{r}_e^{\perp})\, E_{\alpha_i z}^{(l_1)}(\vec{r}^{\perp})}{P_{\alpha_i}\,|1-\beta_{\parallel}/\beta_g^{(\alpha)}|_{\alpha=\alpha_i}} \times \cos(K_{l_1}z-\omega_{\alpha_i}t+\varphi_{ll_1})\right] \tag{1}$$

where $\vec{E}_\alpha^{(l)}$ is the harmonic amplitude of the intrinsic wave $\vec{E}_\alpha$ propagating in the periodic waveguide system at frequency ω_α; $\beta_g^{(\alpha)}c$ is the group velocity of the corresponding intrinsic wave, and P_α is the energy flux across the waveguide cross-section. Frequencies $\omega_{\alpha i}$ may be found from the following equation:

$$\frac{\omega_{\alpha i}}{v} = K_l = K + \frac{2\pi l}{D}, \tag{2}$$

where K is the propagation constant for the intrinsic wave with index α, D is the waveguide cell length.

Suppose a bunch containing N particles has the shape of a torus with radius R. For simplification it is assumed that the minor cross-section of the torus has a rectangular area with the longitudinal side a and the transverse side b. The force applied to an electron inside the bunch at the point with coordinates z_e and $\vec{r}_e^{\perp}$ can be found by adding fields generated by all particles at point $(z_e, \vec{r}_e^{\perp})$. This sum should be added to the component responsible for the Coulomb interaction of particles.

Suppose that the plasma frequency in the bunch $\Omega_{e,i}$, and the Larmor frequency Ω_H are non-commensurable with the value of $2\pi/T$, where T is the time for the bunch to pass the waveguide cell*). If we use the average-method, the equation for the longitudinal coordinate z_e may be written as

$$\frac{d}{dz}\left(\gamma_{\parallel}\beta_{\parallel}\frac{d\xi_e}{dz}\right) = \frac{4\pi^2}{D^2}\left\{B - F^2/2\gamma_{\parallel}\beta_{\parallel}\right\}\xi_e \tag{3}$$

Here $\xi = (\bar{z}_e - z_{cm})/a$ (z_{cm} is the coordinate of the centre of mass for the bunch particles):

$$B = \frac{Nr_0}{a}\left(\frac{D}{2\pi\lambda}\right)^2 \times \left\{\frac{A^2\beta_g}{2|\beta_{\parallel}-\beta_g|} + \frac{2\lambda^2}{Rb\beta_{\parallel}\gamma_{\perp}^2}\right\}, \tag{4}$$

$$F = \frac{Nr_0}{a}\left(\frac{D}{2\pi\lambda}\right)^2\frac{\varkappa A^2\beta_g}{2(\beta_{\parallel}-\beta_g)}. \tag{5}$$

*) The case when these values are commensurable is studied in detail elsewhere[7].

Equation (3) is given for radiation at frequencies which are within the basic passband of the symmetrical wave. Since the longitudinal velocity of the bunch $\beta_{\parallel}c$ is not high, taking into account the higher frequencies should not have a notable effect on the physical picture of the described phenomenon. In formulae (4) and (5) the dimensionless parameter A is

$$A = \frac{E_{\alpha}^{(\ell)}\lambda}{\sqrt{P_{\alpha}/c}},$$

while the parameter χ, determining the relative content of harmonics in the intrinsic wave, can vary within the range $(0 < \chi \leq 1)$. The maximum value of parameter χ is reached at the passband boundary.

As seen from Eq. (3), focusing can occur provided

$$F^2/2\gamma_{\parallel}\beta_{\parallel}B \geqslant 1 \gg B \qquad (6)$$

The right-hand inequality shows the condition due to application of the average-method; the left-hand one indicates the critical number of particles per bunch, where focusing is carried out. Both inequalities exist simultaneously if

$$F^2/2\gamma_{\parallel}B^2 \gg \beta_{\parallel} \qquad (7)$$

The above condition does not depend upon the number of particles per bunch and is valid for the particle energies considered.

If the number of particles per bunch is given, then the basic parameters of the periodic waveguide can be found from inequality (6). The structure with such parameters ensures longitudinal self-focusing of the bunch. For example, in the particular case when $N \approx 10^{11}$ to 10^{12}, parameter A has a reasonable value (A = 60 to 80).

In conclusion it should be noted that the above-described methods of focusing are quite universal, and enable high-intensity bunch focusing not only longitudinally but also in the transverse direction, if required. The latter assertion can be readily verified on the simplest bunch model of a tubular shape with length a. The equation for the bunch envelope is (3), where B and F are given by:

$$B = \frac{Nz_0}{a\gamma_{\parallel}^2}\left(\frac{D}{2\pi\lambda}\right)^2 \times \left\{\frac{A^2\beta_g}{4(\beta_g-\beta_{\parallel})} + \frac{2\lambda^2}{R^2\beta_{\parallel}}\right\},$$

$$F = \frac{Nz_0}{a\gamma_{\parallel}^2}\left(\frac{D}{2\pi\lambda}\right)^2 \frac{\chi A^2\beta_g}{4(\beta_g-\beta_{\parallel})}. \qquad (8)$$

Simple analysis of formulae (4), (5), and (8) shows that both longitudinal and transverse bunch focusing can be achieved even at $N \gtrsim 10^{11} - 10^{12}$ particles, if use is made of highly dispersive periodic waveguide systems with parameters meeting the requirements of formula (6).

References

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