POSSIBILITIES FOR CYCLOTRON ACCELERATION OF HIGH-ENERGY PARTICLES

V.P. Dzhelepov, V.P. Dmitrievsky, B.I. Zamolodchikov V.V. Kolga and N.I. Polumordvinova
Joint Institute for Nuclear Research, Dubna, USSR.

(presented by V.P. Dmitrievsky)

1. Introduction

The cyclotron method of particle acceleration for generating accelerated beams of maximum intensity is beyond competition nowadays for its perfection. The cyclic accelerators of other types are not competitive owing to their duty-factor and linear accelerators because of the great losses of RF power in the continual regime of operation. However, the absence of dynamic orbit similarity in the cyclotron method of acceleration limits considerably the maximum kinetic energy of proton cyclotrons which are either under investigation[1] or under construction at present[2,3]. The limitation is a result of shifting the working point to the region of integral resonance on radial oscillations at the kinetic energy close to E_0.

Here a possibility of developing a circular cyclotron is considered, which has -- along with the isochronism of closed orbits -- the dynamic similarity (constancy of natural frequencies) in the mirror-symmetric spiral structure of magnetic fields when the kinetic energy of the injected proton or electron beam exceeds E_0.

2. Dynamic Properties of Orbits in the Spiral Structures of Magnetic Fields

The properties of orbits in fields of the type

$$H_z = H(r)\left\{1 + \varepsilon(r)\sin\left[N\phi - K(r)\ln\frac{r}{r_0}\right]\right\} \quad (1)$$

have been described in other works[4-6]. They give the dynamic similarity of orbits, provided that (ε, n, K) = constant, where $n = (r/H)/(dH/dr)$, $K = N/\mathrm{tg}\,\alpha$, α is an angle between the tangent to the spiral and the circumference, N is structural periodicity. From the above-mentioned conditions, only the condition n = constant is sufficiently strict for the field (1), since the frequency of radial oscillations is determined mainly by "n" and is weakly dependent on ε and K. The relations ε, K = constant are a condition for the geometric similarity of orbits. However, this similarity is a sufficient but not a necessary requirement for the constancy of the frequencies of natural oscillations during acceleration. Taking into consideration this fact, let us consider in more detail the effect of the variation parameter of the magnetic field upon the isochronism of closed orbits.

Denoting the mean radius of the closed orbit for the particle of the momentum "p" by r_c, one takes into account the change of H(r)r compared to the azimuthally symmetric field by the factor λ, then

$$pc = eH(r_c)\cdot r_c\cdot\lambda\ , \quad (2)$$

where λ depends upon the quantities describing the field variation $(\varepsilon, d\varepsilon/dr, N)$. The elongation of the closed orbit due to forced oscillations in the presence of variation can be taken into consideration by the factor σ, i.e.:

$$L = 2\pi r_c\cdot\sigma\ . \quad (3)$$

With these notations, for particle motion in the magnetic field (1), where $H(r) = H_0(r/r_c)^n$, the variation of rotation frequency of closed orbits can be written as

$$\frac{f_0}{f} = \frac{\beta_0}{\beta}\left(\frac{\beta\gamma}{\beta_0\gamma_0}\right)^{1/(1+n)}\cdot\left(\frac{\lambda_0}{\lambda}\right)^{1/(1+n)}\cdot\frac{\sigma}{\sigma_0}\ , \quad (4)$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $H_c = H_0(r_c/r_0)^n$.

From Eq. (4) it follows that in order to keep the isochronism of closed orbits it is necessary to hold the conditions

$$\left(\frac{\lambda_0}{\lambda}\right)^{1/(1+n)}\cdot\frac{\sigma}{\sigma_0} = \left(\frac{\gamma_0}{\gamma}\right)^{1(1+n)}\cdot\left(\frac{\beta}{\beta_0}\right)^{n/(1+n)} =$$

$$= C_0\,\frac{\beta^{n/(1+n)}}{\gamma^{1/(1+n)}}\ . \quad (5)$$

The diagram of the right-hand side of Eq. (5) is shown in Fig. 1.

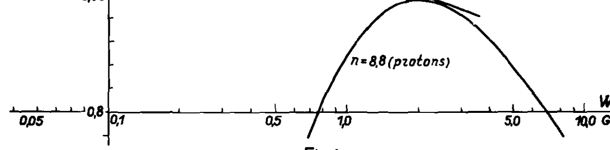

Fig.1.

3. Determination of the λ and σ Coefficients

The λ coefficient characterizing the radial packing of closed orbits is found from the following equation:

$$r'' - \frac{2r'^2}{r} - r = \frac{e}{pc}\,\frac{(r^2 + r'^2)^{3/2}}{r} \cdot H_z(r,\phi) \;. \quad (6)$$

The solution corresponding to the closed orbit with the mean radius can be found in the form

$$r = r_c[1 + \xi(\phi)] \;, \quad (7)$$

where

$$\xi(\phi) = \xi\left(\phi + \frac{2\pi}{N}\right) \;, \quad \int_0^{2\pi} \xi(\phi)\, d\phi = 0 \;.$$

The linear approximation (of ξ) from Eq. (6) is the Mathieu equation having the following right-hand side

$$\xi'' + \left[\frac{2}{\lambda} - 1 + \frac{n}{\lambda} + \frac{\varepsilon}{\lambda}(2 + n + \ell)\sin N\phi - \frac{\varepsilon K}{\lambda}\cos N\phi\right]\xi =$$

$$= 1 - \frac{1}{\lambda} - \frac{\varepsilon}{\lambda}\sin N\phi, \quad (8)$$

where

$$\ell = \frac{r_c}{\varepsilon}\frac{d\varepsilon}{dr}\bigg|_{r=r_c} \;.$$

A particular periodical solution of Eq. (8) may be found in the form

$$\xi = \sum_{\nu=1}^{\infty} \xi_\nu \sin \nu N\phi \;. \quad (9)$$

In the first approximation the quantity λ from Eqs. (8) and (9) is obtained from the equation

$$1 - \frac{1}{\lambda} - \frac{\varepsilon^2}{N^2}\,\frac{2 + n + \ell}{\lambda^2} = 0 \;. \quad (10)$$

Equation (10) was checked by determining the parameter by means of computer calculations of Eq. (6).

The coefficient σ, corresponding to the elongation of the closed orbit, is calculated from the relation

$$\sigma = \frac{1}{2\pi r_c}\int_0^{2\pi} \sqrt{r^2 + r'^2}\, d\phi \quad (11)$$

and equals

$$\sigma = 1 + \chi\varepsilon^2 \;, \quad \chi \approx \frac{N^2}{4(N^2 - n - 1)^2} \;. \quad (12)$$

4. Numerical Illustrations

As an illustration of the method described above, the proton and the electron relativistic cyclotrons of 7 and 2 GeV, respectively, have been calculated.

The possible parameters of such accelerators are presented in Table 1. The accelerator parameters of Table 1 have been obtained by using the approximate expressions of the linear theory and need, undoubtedly, additional precise determination by means of the computer.

Table 1

Parameters	Protons		Electrons	
	Injection	End of acceleration	Injection	End of acceleration
W (MeV)	2000	7000	50	2000
r (m)	32.05	33.33	7.55	8.15
H (rc)	2820	4000	74	8000
ε (r)	1	4	18.2	1
dε/dr cm^{-1}	0	0.045	-0.48	0
λ	1.02	1.97	3.0	1.02
Spiral angle α^o	17	58	50	7
n	8.8		61.3	
N	14		34	
Q_r	3.1 to 3.45		8.1 to 8.45	
Q_z	1.4		1.4	

The emittance of the beam for which the violation of isochronism does not affect the phase mode of acceleration (isochronous emittance), exceeds, as a rule, the corresponding space emittances.

5. Conclusion

Some theoretical investigations are given which point to the possibility of applying the cyclotron method of particle acceleration in the relativistic region of velocities, the dynamic similarity and mirror-symmetric structure of the leading magnetic field being conserved. This possibility may prove optimal in designing powerful K-meson generators.

References

1) V.N. Anosov et al., Atomnaya Energiya 25, 539 (1968).

2) H.A. Willax, The International Cyclotron Conference, Oxford, 1969.

3) R.P. Haddock, K.R. MacKenzie, S.R. Richardson and B.T. Wright, Proc. International Conference on High Energy Accelerators, Dubna, 1963 (Atomizdat, Moscow, 1964), p. 568.

4) P. Dunn, L. Mullett, T. Pickavance, W. Walkinshaw and S. Wilkins, Proc. CERN Symposium on High Energy Accelerators and Pion Physics, Geneva, 1956 (CERN, Geneva, 1956), p. 9.

5) D.P. Vasilevskaya et al., Atomnaya Energiya 8, 189 (1960).

6) A.A. Kolomensky and A.N. Lebedev, Theory Cyclic Accelerator (in Russian) (Fiz. Matizd., Moscow, 1962).

DISCUSSION

T. K. KHOE: What is the weight of the cyclotron?

V. P. DMITRIEVSKY: So far we have only been considering the idea and no weight has been calculated yet. But dimensions are given in the Table in the paper. We envisage a Δr about 1.2 m.