LIMITATIONS ON BEAM QUALITY AND INTENSITY

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Abstract

Transverse spacecharge is the limiting factor on beam quality in some simple situations, but a great variety of other effects are known, and can be classified in several ways. The longitudinal instabilities of bunched beams have received a lot of attention in recent years.

1. Ultimate Limits

When I was first asked to give this paper I began to look whether there were any simple ultimate limits on beam intensity or quality, that could be used as a scale to measure our progress. I cannot claim that I had much success. I had thought that the limitations on electron storage rings were well understood, but Amman [1)] has told us that the limiting linear vertical tune shift no longer seems to be a constant of nature. This limiting $\Delta\nu$ for two strong beams seems now to depend on energy and on the focusing β at the intersection, both at ACO and at ADONE. If this is confirmed, the idea that this limit is a pure nonlinear optics phenomenon will have to be abandoned.

We have Orsay making an e^+-e^- ring with 4-beam crossings, so that beam-beam spacecharge will be compensated [2)], and so we may see what the next limiting process will be.

For proton storage rings we are not at a good moment to discuss ultimate limitations: in the ISR plenty of interesting and some unexpected things happen [3)], but at present it is a vacuum problem that prevents us from stacking more than seven amperes, and this limit increases steadily from week to week. Apart from this vacuum problem I think it is fair to say that the things that trouble us in the ISR are combinations of two or more effects: thus the transverse coherent instability does not bother us in itself, but it obliges us to work with a large Q-spread [4)], so that the stack crosses many high order resonances; these resonances should not bother us either unless some collective effect can feed particles into them, or pockets of neutralisation make them much wider [5)].

At the Panel Session there was considerable discussion about Arnold diffusion, which is a process by which small nonlinearities can cause a slow growth of betatron amplitude. It seems possible that the nonlinearities coming from these pockets of neutralisation may be a reason for the way the loss rate increases with current in the ISR, but we do not really have a practical quantitative theory about Arnold diffusion, so in the present state of knowledge it is impossible to say.

In proton synchrotrons the Laslett spacecharge limit still occupies a key place. For a round beam that is not too relativistic this says

$$\frac{N}{\pi\varepsilon\beta\gamma} < 2/r_0 \quad \beta\gamma^2 \quad B \quad |\Delta Q| \tag{1}$$

Where $|\Delta Q|$ is the tolerable Q change for the transverse oscillations, usually about ¼, B the bunching factor, and $\pi\varepsilon\beta\gamma$ is the invariant transverse phase-space area, r_0 the classical radius of the particle. Expressed this way the formula is machine independant - if you want more particles per unit invariant area you must increase the injection energy, to use the $\beta\gamma^2$ factor.

One of the uses of a slow booster with 4 rings or a fast booster making 12 shots is to make possible a factor of 4 or 12 in this phaseplane density in the main ring without increasing the linac energy. We have a paper from the CERN booster group [6)] to remind us that one does not obtain beam quality just by manipulating scaling laws: a lot of work has to go into keeping down the classical single particle effects of imperfections and aberration.

Continuing backwards, one comes to a linac, where considerations are different. The condition that spacecharge should be less than the transverse focusing must be a guide to where the limit lies. I write this

$$\frac{I}{\pi\varepsilon\beta\gamma} < e/r_0 \quad \beta\gamma^2 \quad B \quad f_\perp \tag{2}$$

where I is the current and $f_\perp$ is the particle frequency in the transverse focusing system. In contrast to the synchrotron case, one can profit by working with a small diameter tightly focused beam, so far as it is technically possible.

Note that these formulae set limits to the number of particles per unit area of <u>one</u> of the transverse phaseplanes: to double the number of particles at constant right-hand-side you must double both transverse phaseplane areas, so the density in 4-space gets halved. And conversely a low-intensity low-emittance beam may be the right thing to aim for, if density in 4-space is the ultimate requirement.

I mentioned ¼, the Q-shift limit that is traditionally used with the Laslett formulae. It is difficult to give this any very precise justification. On the one hand the half integer resonances are not uncrossable barriers, on the other hand crossing the higher order resonances is certainly not harmless if beam quality is a consideration.

2. The Instabilities of Bunched Beams

First some classification. There are of course transverse and longitudinal, but there are also many ways of classifying that cut across both. I have two tables that show different types of motion:

Table I

LONGITUDINAL TYPES OF MOTION

Bunch relationship:

- Only one bunch
- Independant motion of bunches
- Orderly motion of bunches:
  - symmetrical modes
  - other modes
  - open-loop instabilities

Longitudinal phaseplane:

- Lowest mode
- Higher modes round the phaseplane
- Higher modes radially in the phaseplane
- Higher modes longitudinally
- Incoherent effects

Table II

TRANSVERSE TYPES OF MOTION

Bunch relationship:

- Only one bunch
- Independant motion of bunches
- Orderly motion of bunches:
  - symmetrical modes
  - other modes
  - open-loop instabilities

Longitudinal phaseplane:

- Lowest mode
- Higher modes round the phaseplane
- Higher modes radially in the phaseplane
- Higher modes longitudinally
- Incoherent effects

Transverse phaseplane:

- Dipole modes
- Higher envelope modes
- Density modes
- Incoherent effects

And I have another that shows the types of force:

Table III

Local and instantaneous

Brief transient wake: - bunch on itself, same turn
- bunch on next one or few

Long transient wake: - bunch on all bunches
- bunch on itself, later turns

Beam-beam versions of the above

Bunch-stack interaction

Propagating waves and radiation

Lastly, I can try to list the mechanisms of interaction that may come into play

Table IV

Magnet imperfections (static and dynamic)

Direct spacecharge

Images (well conducting smooth wall)

Resistive wall

Nonresonant structures and layers

Resonant structures

Ions and electrons (static and dynamic)

Non-passive devices

Transverse effect of longitudinal forces

Longitudinal effect of transverse forces

At the risk of being obvious, let me remark that for each type of motion one should consider all the types of force and mechanisms, in order to find which are important. It is not sufficient to take them one at a time, because some instabilities involve in an essential way two or more types of force. I should also remark that in putting things into these tables I have been rather indiscriminate. The aim is a tool that will help you to think of everything, rather than a tidy classification of what is known.

By way of example let us look at the longitudinal dipole oscillations of many equal and equally spaced bunches. This has recently been attracting some attention at the Brookhaven AGS [7] and at the CERN PS [8] in connection with parasitic resonances of the RF cavities; earlier the effect was observed, analysed and cured at ADONE [9]

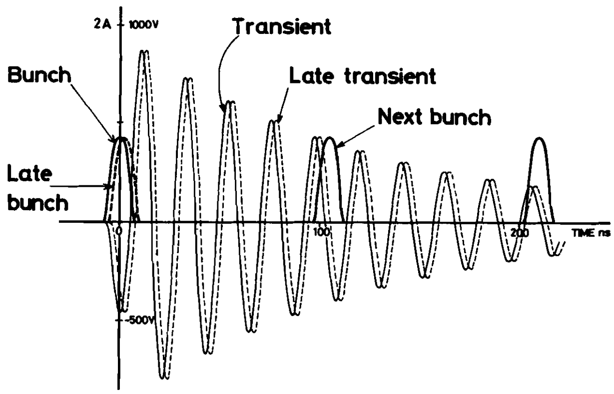

Fig. 1 Bunch current and volts per turn from one bunch for the 46 MHz cavity transient in the CPS.

Figure 1 shows a short high-frequency transient such as one may get from a higher resonance in a cavity: the important thing about this is that the force on a bunch is a function of the phase of a preceding one, as one may see from the solid and the dotted curves. Specially simple results apply if the transient only lasts until the next bunch, or at least we start by neglecting what it does to further ones. Then we take a model with just each bunch driving the next, the equation of motion for the m'th bunch is

$$\ddot{\phi}_m + \Omega_o^2 \phi_m = \beta_1 (\phi_m - \phi_{m-1}) \tag{3}$$

with (m-1) interpreted cyclically.

This β_1 is a real coefficient which can be calculated from the shape of the transient of Figure 1, and ϕ_m stands for the longitudinal barycentre of bunch m, with its stationary equilibrium value taken as origin. I can assimilate the $\beta_1\phi_m$ term into the left hand side, and get

$$\ddot{\phi}_m + \Omega^2 \phi_m = - \beta_1 \phi_{m-1} \tag{4}$$

In such a system fastest growth occurs if the oscillations of each bunch are in quadrature with the next, so that each receives an acceleration in phase with its velocity. In fact I can consider the general symmetrical mode

$$\phi_m = A \exp j(\omega t + m\psi) \tag{5}$$

in which all bunches do the same thing, with a phase difference of ψ between each and the next. Put (5) into (4) and find

$$\omega \simeq \Omega + \frac{\beta_1}{2\Omega^2} (\cos \psi - j \sin \psi) \tag{6}$$

corresponding to an efolding rate of $\frac{\beta_1}{2\Omega^2} \sin \psi$. This ψ is not unrestricted; to have a phase shift of ψ between each bunch and the next all round a ring of h bunches, $h\psi$ must be a multiple of 2π. But in fact this is not an important restriction in multi-bunch machines; provided h is more than two, we can find modes with $\sin \psi$ of the order of unity.

Notice that we cannot stabilise this system by choosing the sign of β_1. If β_1 is positive it is the modes with ψ around 90° that grow, if we make β_1 negative the modes with ψ around -90° take their place, and the system remains unstable. It is not difficult to see that this unpleasant symmetry is still there for a more general transient wake, that lasts long enough to affect more than just the next bunch. For that case the efolding rate becomes

$$\frac{1}{2\Omega^2} (\beta_1 \sin \psi + \beta_2 \sin 2\psi + \beta_3 \sin 3\psi \ldots) \tag{7}$$

where β_2, β_3 are the coefficients calculated from the transient after 2, 3, etc. bunch intervals. If this sum should be negative, it will be positive for the corresponding ψ of opposite sign.

The Landau damping of this instability is worth a few remarks [8) 10)]. We must write down an equation of motion for an individual particle, e.g. one of the particles of the m'th bunch

$$\begin{aligned} \ddot{\phi} + \Omega_o^2 \phi + \text{N.L.T.} = \;& \beta_o (\phi - \phi_m) \\ &+ \beta_1 (\phi - \phi_{m-1}) \\ &+ \ldots\ldots\ldots\ldots \end{aligned} \tag{8}$$

As before, I can assimilate all the ϕ terms on the right into Ω^2. This does not mean we should forget about them too quickly; if they are not small they may influence the equilibrium bunch length, and their associated nonlinearities may not be negligible. But there remains on the right

$$-\beta_o \phi_m \; -\beta_1 \phi_{m-1} \; -\beta_2 \phi_{m-2} \; - \ldots \tag{9}$$

The nonlinearity on the left makes a frequency spread, and by the Vlasov equation or other method [11)] one gets in such a case a dispersion equation with the gradient of the phasespace density in the integral

$$1 = (U + jV) \Omega_o \int \frac{h'(a) a^2 \, da}{\Omega^2 - \omega^2} \tag{10}$$

By now it is known that finally there is an instability threshold which depends on the shape of the distribution and the phase of the coupling coefficient U + jV, but is roughly given by [12)]

Instability if

$$\textit{full } \Omega \textit{ spread} \lesssim 4\ |U + jV| \tag{11}$$

where U + jV is the effect of all the terms in (9) expressed as a complex change in frequency:

$$U + jV = (\beta_o + \beta_1 \exp j\psi + \beta_2 \exp 2j\psi + ..)/2\Omega_o \tag{12}$$

You will notice that I include the autonomous self-bunch term β_o in this threshold calculation. This can have important consequences, first of all because these short transient wakes can often have as much effect on the bunch that generates them as on all the others, secondly because it means we must go back to our table of mechanisms, and see whether, for example, wall effects or ordinary longitudinal spacecharge make an important contribution to β_o. On the ordinary longitudinal spacecharge I can mention that in the CERN PS we have, a little after transition, a spacecharge parameter η_{sc} in the region of 0.2 so the associated U/Ω is like 10%, and is not completely easy to overwhelm with the nonlinear spread.

In passing, autonomous $x - \langle x \rangle$ terms can also be important in transverse instabilities, perhaps especially radially [13], and have the property of not showing up as a shift of the RFKO frequency [14].

Besides the Landau damping of this bunch instability we have papers on the effect of bunch-to-bunch frequency spread and the influence of beam control [8], stabilisation by feedback [15) 8], and a more rigorous calculation of the dynamics [16]. The coefficient β, that I introduced in equation (3) can be regarded as the slope of the transient, averaged over the second bunch; but if you want to know the dependance on bunch length you have to do this averaging properly, and that amounts to going much deeper into the detailed mechanics. The effect of missing bunches can be interesting, and in this connection I would like to show you something very simple.

I take many bunches, each making a wake that affects the next, but with a big enough gap that the first is unaffected:

$$\begin{aligned}
\ddot{x}_o + \Omega^2 x_o &= 0 \\
\ddot{x}_1 + \Omega^2 x_1 &= \beta x_o \\
\ddot{x}_2 + \Omega^2 x_2 &= \beta x_1 \\
&\cdots\cdots \\
&\cdots\cdots \\
\ddot{x}_n + \Omega^2 x_n &= \beta x_{n-1}
\end{aligned} \tag{13}$$

I do not have a closed loop, neither Nyquist's criterion nor a dispersion equation will tell me it is unstable. A solution can be calculated line by line from the top. I write down only the leading terms:

$$\begin{aligned}
x_o &= \exp j\Omega t \\
x_1 &= \left(\frac{\beta}{2j\Omega}\right) t \exp j\Omega t \\
x_2 &= \tfrac{1}{2}\left(\frac{\beta}{2j\Omega}\right)^2 t^2 \exp j\Omega t + \ldots\ldots \\
&\cdots\cdots \\
&\cdots\cdots \\
x_n &= \frac{1}{n!}\left(\frac{\beta}{2j\Omega}\right)^n t^n \exp j\Omega t + \ldots\ldots
\end{aligned} \tag{14}$$

and you are immediately struck by the fact that most of these x increase to infinity with t — though not exponentially. With a bit of manipulation I can even show that the biggest amplitude does increase approximately exponentially for the first n efoldings, like

$$\left(\frac{\Omega}{\pi\beta t}\right)^{\frac{1}{2}} \exp \frac{\beta t}{2\Omega} \tag{15}$$

with the same timeconstant as the closed loop case.

One of these "open-loop" or cumulative instabilities first showed up as the multi-section beam breakup in the Kharkov [17] and SLAC [18] linear accelerators, and another transverse example seems to have been seen in the Cambridge stored beam [19) 20]. Longitudinal cases, more like my simple example, have been calculated for the CPS by Mary Bell [21]. So we must not assume that a closed feedback loop is needed to make a dangerous instability, and be cautious about assuming that a few gaps will stabilise the beam in a synchrotron.

Transition

We have an interesting situation, and several papers, on the longitudinal spacecharge effect at transition. At the transition energy the effective mass of the particles changes sign, and one switches the sign of the RF focusing to accommodate this, but the longitudinal spacecharge does not switch, and so one gets a mismatch of bunch length which ultimately results in a dilution. In recent years people have worked on different ways of eliminating this mismatch [22) 23], but have not worried much about the possibility of a negative-mass instability, because it has been known that longitudinal spacecharge, in the negative-mass region, has the effect of shortening the equilibrium length of bunches until their energy spread is on the stable side of threshold [24]. Now we have the paper of Lee and Teng [23] which shows that, even so, one does have negative-mass instability just after transition, the reason being that for a short period of time the bunch is longer than its equilibrium length, with less than the corresponding energy spread, and so unstable.

In this instability the fastest growth occurs for rather short wavelength perturbations, which may make it difficult to get convincing detail out of computer studies [25]. It may be that one should attack it by fluid dynamics techniques, where one puts the infinite discontinuities into the lowest order of approximation. I believe it is possible [26] to derive a hydrodynamic equation, in which the pressure would consist of a kinetic term and a spacecharge term of the opposite sign above transition, which would show that it is the actual local energy spread that determines stability, not the equilibrium value or the overall value.

Several years ago it was suggested [27] that one could suppress the negative-mass instability by a helical insert. This would compensate the capacitative longitudinal spacecharge by making some inductive wall. It might be of interest for curing this transition problem, but in principle one must not overcompensate, because a net inductive wall might give positive-mass instability before transition. Also one must compensate over a very broad frequency band, because wavelengths ranging from about a bunch length down to $1/\gamma$ of the chamber diameter are at risk [28]. But it is sure that if we could compensate two-thirds or three-quarters of the longitudinal spacecharge with a helix, the matching of the remainder by Q-jump etc. would be a lot easier.

4. Conclusion

I wonder whether one can say anything in conclusion, something that is true and useful over a wide range. Perhaps the work in this field can be divided into three kinds:

- Calculating known effects
- Explaining experimental observations
- Predicting the limitations of future machines

The first of these seems to me fairly straightforward, though the amount of work in some of these problems can be an obstacle. The second is a sort of detective work, you need both patience and luck, and good diagnostic instrumentation. On the last, I believe we are just beginning to have enough knowledge to attack it systematically.

References

1) Amman; Electron and positron storage rings: present situation and future projects. This Conference.

2) The Orsay compensated colliding beam rings. Presented by H. Zyngier. This Conference. See also 29) and 30).

3) Schnell; Experimental observation of beam behaviour in the ISR. This Conference.

4) Hübner, Keil and Zotter; Coherent space charge phenomena in the ISR. This Conference.

5) Keil; Incoherent space charge phenomena in the ISR. This Conference.

6) Bovet, Guignard, Reich and Schindl; Potential beam blow-up in the CERN PS booster due to unavoidable magnet imperfections and corrective measures provided. This Conference.

7) Barton and Raka; USSR 2nd National Conference on Particle Accelerators.

8) Boussard and Gareyte; Damping of the longitudinal instability in the CERN PS. This Conference.

9) Pellegrini; Linear theory of RF cavity, bunched beam interaction. Private communication 1969.

10) Karliner; Dependance of the phase motion stability of a bunch of particles in a storage ring on form of particle momentum distribution function. This Conference.

11) Hereward; Landau damping by non-linearity. Internal report CERN/MPS/DL 69-11. 1969.

12) Hübner and Vaccaro; Dispersion relations and stability of coasting particle beams. Internal report CERN-ISR-TH/70-44.

13) Gendreau; Coherent radial instability due to horizontal resistive walls. Application to Saturne. This Conference.

14) Möhl and Sessler; The use of RF-knockout for determination of the characteristics of the transverse coherent instability of an intense beam. This Conference.

15) Tazzioli; Damping of phase oscillations in the Frascati $e^+ e^-$ storage ring. These Proceedings.

16) Gumowski; Determination of an equipment design criterion for the CERN PS booster from stability considerations of coherent synchrotron motion. This Conference.

17) Valter et al; 2 GeV travelling wave electron accelerator. Vth International Conference on High Energy Accelerators, Frascati 1965.

18) Loew et al; Linac beam instabilities. VIIth International Conference on High Energy Accelerators, Yerevan 1969. And references therein.

19) Morton, Rees and Spencer; An attempt to explain the coherent bunch motion in the Cambridge electron positron ring. Internal report SPEAR 110, 1971.

20) Averill et al; Synchrotron and betatron instabilities of stored beams in the CEA. This Conference.

21) Bell; Two programs for studying the coupling, caused by excitation of the resonator, between bunches in the CPS. Internal report CERN/MPS/DL/71-3, 1971.

22) Hardt, Schönauer and Sørenssen; Passing transition in the future CPS. This Conference.

23) Lee and Teng; Beam bunch length matching at transition crossing. This Conference.

24) Lebedev; Longitudinal instability in the presence of an RF field. VIth International Conference on High Energy Accelerators. Cambridge 1967.

25) Sørenssen; An expansion method for calculating space-charge dynamics in accelerators. This Conference.

26) Hereward; Longitudinal spacecharge effects in circular accelerators. Unpublished note Bev-524, 1960.

27) Sessler and Vaccaro; Passive compensation of longitudinal spacecharge effects in circular accelerators: the helical insert. CERN 68-1, 1968. Also Neil and Briggs; Stabilisation of nonrelativistic beams by means of inductive walls. LRL Report UCRL 70133, 1966.

28) Keil and Zotter; The coupling impedance of corrugated vacuum chambers for cyclic particle accelerators, CERN-ISR/TH/70-30. And Zotter, private communication.

29) Jejcic and Le Duff; Non linear beam-beam effect. This Conference.

30) Piwinski; Beam break-up due to the space charge effect. This Conference.

DISCUSSION

E. D. COURANT: On the problem of unequal spacing of bunches: in the work of Sessler and myself a few years ago, we proposed the conjecture that with equal bunches unequally spaced there would always be both stable and unstable modes. This has now been proved, at least for two bunches with arbitrary spacing.

H. G. HEREWARD: Intuitively I would expect that, if the wake reaches from each bunch to the next, with more than two bunches and maybe also with just two bunches one would always find instability, with about half the modes unstable, unless some damping is introduced. Such damping could be unequal frequencies, perhaps coming from unequal populations of the bunches, but unequal spacing would not do it. So what you say goes to confirm this.

P. MORTON: Are you completely satisfied with the present theory of Landau damping with a frequency spread due to nonlinearities, when you always neglect the nonlinear interacting forces?

H. G. HEREWARD: I believe that one can put in the nonlinear forces and finite incoherent amplitudes and do the calculation properly, perhaps not with complete mathematical rigour but quite convincingly, working in the limit of small perturbations. That means small coherent amplitudes. The linearised Vlasov equation, from first order perturbation theory, is for the limit of small changes in the distribution function; it does not oblige you to neglect the nonlinear terms in the particle motion. On the other hand, you would be right to be sceptical about applying the results of this theory to any perceptible coherent amplitude. If you stimulate the beam, or consider build-up from some given level of noise, it may be good to check whether the levels are low enough for this theory[11].

C. NIELSEN: There are cases in which an instability grows only after an external signal has produced a finite amplitude perturbation. Could you comment on the relation of linear theory to this situation?

H. G. HEREWARD: A situation can arise where this sort of behaviour is theoretically possible. Sometimes a mode may be stable provided the phase-space density is a falling function of amplitude. Give the beam a kick and let the oscillation filament away, and you may have a distribution with a dip in the density in the middle of the phase plane, so there is a region of rising density, and the mode becomes unstable.

On the other hand, it can be argued that under these conditions it is not really the beam that goes unstable, it is the (partial) hole in the middle of it; and maybe the hole can go unstable, break into oscillations and escape, without any great harm being done!

More seriously, I do not think it is known how such a situation would evolve, but it is a possible theoretical explanation of the phenomenon that you describe.