RADIATION REACTION FORCE ON ELECTRON RING PASSING THROUGH ACCELERATOR

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Abstract

The radiation reaction force distribution along a traveling path of the electron ring uniformly moving through a single cavity with drift tubes is investigated by means of numerical methods. The total radiation losses of moving source per one structure period for a model of periodic structure with the channel are calculated using the factorization technique.

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The main task of this work is to clear up some features of electron ring interaction with an element of accelerating system (single cavity) and to carry out a rigorous estimation of energy losses in a closed periodic structure model.

I. The radiation reaction force in the single cavity

The calculation of energy losses arising when a uniformly charged ring moves through a cylindrical cavity (radius b, length 2d) with entrance and exit tube of radii a (see Fig.1) is described in paper[1]. The radiation spectrum consists of a discrete set of structure modes and a continuous frequency region.

The structure resonant frequencies are within $\nu_1 c/b < \omega_{ms} < \nu_1 c/a$ where ν_n are the roots of equation $J_0(\nu)=0$. The frequencies are slightly higher then those of a similar closed cavity. Calculations show that the lowest spectral lines excited by electron ring can be determined by simple expressions for closed cavity[2] with good accuracy; the intensity of higher structure resonant frequencies is well below then that for a closed cavity. The energy losses on discrete line excitation very quickly achieve the limit value and become independent of the relativistic factor $\gamma = 1/(1-\beta^2)^{1/2}$ ($\beta = v/c$).

At relativistic energies ($\gamma \gg 1$) the continuous spectrum radiation ($\omega > c\nu_1/a$) into the waveguides makes the main contribution to the losses. The upper limit of the radiation spectrum is by the order of magnitude equal to

$$\omega_{max} \sim c\beta\gamma/(a-\rho) \qquad (1)$$

(ρ - radius of the electron ring) and increases with , which gives energy dependence of radiation losses close to linear. Some numerical data on radiation for various resonator forms are given in Table.

It is interesting to find the distribution of radiation reaction force along the source trajectory. The total force applied from the radiation field to all elements of the ring with overall charge Q is determined by the field value of the ring

$$T(z)=Q\int_{-\infty}^{\infty} E^1_{\omega z}(\rho,z)\,e^{i\frac{\omega}{v}z}\,d\omega=\int_{-\infty}^{\infty} T_\omega(z)\,d\omega \qquad (2)$$

The spectral amplitude of electrical field induced by the moving ring in the structure is taken as

$$E^1_{\omega z}(r,z)=\begin{cases}\sum\limits_{n=1}^{\infty} J_0(\nu_n r/b)\left(B_n e^{i g_n z}-C_n e^{-i g_n z}\right), & |z|<d\\[2mm] \sum\limits_{n=1}^{\infty} J_0(\nu_n r/a)\begin{bmatrix}A_n\\ D_n\end{bmatrix} e^{\mp i h_n z}, & \begin{bmatrix}z<-d\\ z>d\end{bmatrix}\end{cases}$$

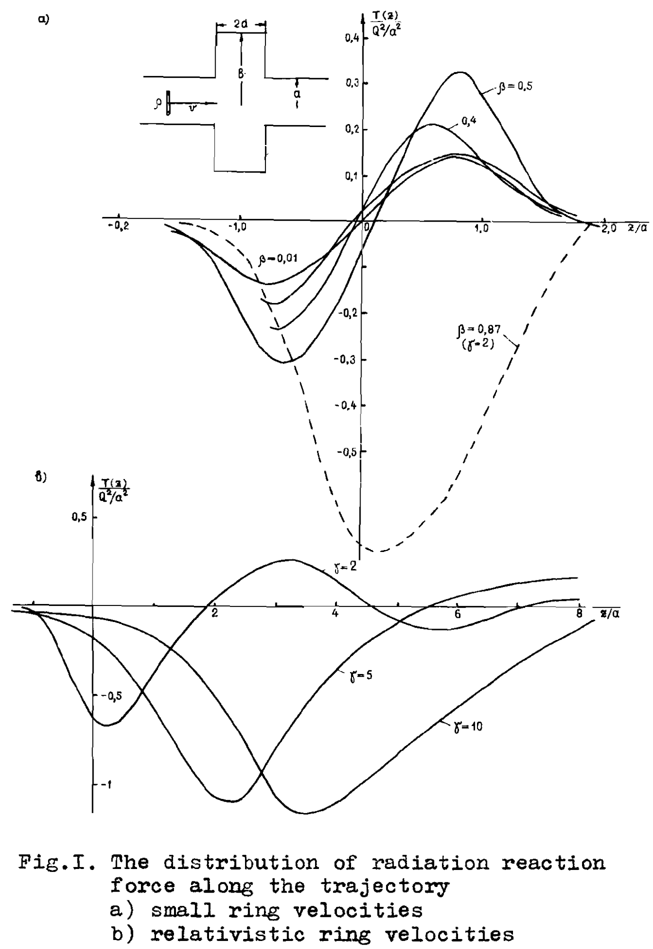

Fig.I. The distribution of radiation reaction force along the trajectory
a) small ring velocities
b) relativistic ring velocities

where $h_n=[(\omega/c)^2-(\nu_n/a)^2]^{1/2}$, $g_n=[(\omega/c)^2-(\nu_n/b)^2]^{1/2}$ – longitudinal wave numbers, and A_n, B_n, C_n, D_n – the unknown amplitudes of the waveguides modes in different regions of the structure. Numerical analysis of T(z) involves solution of a linear system of equation for those amplitudes, presented in paper[1], and computation of the integral (2).

It is worth-while to mention some specific features of spectral component of retarding force. It may be shown from the analysis of the set of equations for the coefficients that in a frequency region $\omega<\nu_1 c/a$ $T_\omega(z)$ is an odd function of variable z. As a function of frequency $T_\omega(z)$ has simple poles. If the dissipation is taken into account those poles are replaced into the lower half-plane of the complex argument . The integration path in (2) goes on real axis and bypasses the poles on semicircles of small radii above them. Then the integral (2) may be expressed as the principal value of the integral minus $\pi i \sum \mathrm{Res}\, T_{\omega_{ms}}(z)$ (here the direction of poles turning is taken into account).

One can see from (I) that the main contribution to (2) for small β is determined by the low frequency region. In this case the function T(z) is antisymmetric about the central plane of the structure and presents the successive ring retardation and acceleration by the surface charges induced on the wall of the structure (see Fig.Ia). For larger γ contributions from resonant frequencies and from continuous high-frequency spectrum into (2) become essential. In Fig.Ib the distribution of the retarding force vs z for three values of γ is presented. It may be seen that the retarding force in the entrance waveguide and in the first part of the trajectory in the cavity is negligible. The length of path without retarding was found to increase with γ . For the successive part of trajectory the retarding force acting on the ring is oscillatory, the amplitude of oscillation decreasing with the distance from the cavity. The maximum of retarding force along the trajectory for relativistic energies is weakly dependent on γ , but the period of oscillation increases proportionally to γ .

We may find the distribution of retarding force at large distances from the cavity ($z/a \gg 1$) if we evaluate the integral (2) by the stationary phase method. For the exit waveguide ($z>d$) the stationary points Ω_n are determined by $\frac{dh_n}{d\omega}=\frac{1}{v}$ and hence

$$\Omega_n=\frac{\nu_n}{a}\gamma c \tag{4}$$

The stationary phase condition means that the essential part of ring interaction with radiated field takes place for frequencies on which the group velocity of waveguide modes and the ring velocity are equal. If we take into account only the main term (n=1) in asymptotic expansion of the integral (2) then

$$T(z)\approx Qc J_0\left(\frac{\nu_1}{a}\rho\right)\left(\frac{\pi\nu_1}{az}\right)^{1/2}(2\gamma\beta)^{3/2}\,\mathrm{Re}\left\{D_1 e^{i\frac{3\pi}{4}+i\frac{\nu_1}{\gamma\beta}\frac{z}{a}}\right\} \tag{5}$$

It is clear that the retarding force oscillates along the source trajectory with a period of $z_0=2\pi\beta\gamma a/\nu_1$ and decreases as $z^{-1/2}$

For the entrance waveguide ($z<-d$) stationary points are absent and the asymptotic value of T(z) is equal to zero. The obtained distribution of retarding force permits to make some judgements about ring interaction with structures consisting of several cavities. The interaction with each of them may be considered independently only if retarding force decreases substantionally along the path between neighbouring cavities. The length of effective interaction increases with energy and the pattern of radiated field is determined by all elements of the structure. If there is a long waveguide tract after the accelerating structure, the latter may be considered as a single scattering obstacle of composite form. In this case the nearly linear dependence of radiation loss is valid, but the total radiation loss is not proportional to the number of cavities.

2. The radiation losses in a channel of periodic structure

The rigorous solution of the boundary problem of electromagnetic field induced by a line charge uniformly moving past an open periodic structure and a crude estimation of the radiative energy loss are given in[3]. In paper [4] numerical calculation was carried out and an analytic asymptotic evaluation of the loss $T\sim\gamma^{-1/2}$ was obtained. It is interesting to obtain the energy loss dependence for a source moving through a channel of a periodical structure. We shall produce the analysis for twodimensional model of the structure as shown in Fig.2. As a source of field we assume a uniformly charged rod with the linear charge density q, which moves with constant velocity along the axis of the structure at a distance b from it.

The periodic structure under examination is supplemental (in the sense of the Babinet principle) to the open comb structure investigated previously in[5], and it can be described by a similar system of dual integral equations for the current density induced at the plates. The formal solution for the periodic structure with the channel based on the factorization method was obtained in the work[6].

The spectral amplitude of the current at the plate z = 0 is represented as Fourier integral

$$j_{\omega 0} = \int_{-\infty}^{\infty} \left[F(w)\cos wy + G(w)\sin wy\right] dw \qquad (6)$$

The unknown functions F(w), G(w) satisfy the set of integral equations

$$\int_{-\infty}^{+\infty} \begin{bmatrix} F(w)\cos wy \\ G(w)\sin wy \end{bmatrix} dw = 0 \qquad |y| < \ell$$

$$\int_{-\infty}^{\infty} \begin{bmatrix} F(w)\cos wy \\ G(w)\sin wy \end{bmatrix} L(w)\,dw = \frac{iq\omega}{2\pi u} e^{-\frac{k|y|}{\gamma\beta}} \begin{bmatrix} \mathrm{sh}\frac{kb}{\gamma\beta} \\ \mathrm{ch}\frac{kb}{\gamma\beta} \end{bmatrix} \qquad |y| > \ell \qquad (7)$$

where the kernel L(w) may be written in the form:

$$L(w) = \frac{v \sin va}{\cos va - \cos(ka/\beta)} = \frac{2}{a}\,\frac{L_1(w)\cdot L_1(-w)}{w^2 + k^2/\gamma^2\beta^2} \qquad (8)$$

We shall assume the Fourier amplitude of odd and even parts of the current (with respect to y) written in the form

$$\begin{bmatrix} F(w) \\ G(w) \end{bmatrix} = \frac{e^{iw\ell}}{L_1(w)} \left\{ \begin{bmatrix} K \\ M \end{bmatrix} + \sum_t \begin{bmatrix} K_t \\ M_t \end{bmatrix} \frac{w - \hat{w}_t}{w + \hat{w}_t} \right\} \pm$$

$$\pm \frac{e^{-iw\ell}}{L_1(-w)} \left\{ \begin{bmatrix} K \\ M \end{bmatrix} + \sum_t \begin{bmatrix} K_t \\ M_t \end{bmatrix} \frac{w + \hat{w}_t}{w - \hat{w}_t} \right\} \qquad (9)$$

$$\hat{w}_t = \sqrt{k^2 - (k/\beta - 2\pi t/a)^2}\,, \quad t = \pm 1, \pm 2, \ldots$$

Here the upper string in brackets corresponds to the odd part of current and the lower to the even part one. The function $L_1(w)$ may be written in the form of the infinite product.

$$L_1(w) = \frac{k}{\gamma\beta}\left(\frac{ka}{2}\cdot\frac{\sin ka}{\cos ka - \cos(ka/\beta)}\right)^{\frac{1}{2}} e^{-i\frac{wa}{\pi}\ln 2} \times$$

$$\times \left(1 + \frac{w}{k}\right) \prod_{m=1}^{\infty} \frac{\left(1 + \frac{w}{w_m}\right)}{\left(1 + \frac{w}{\hat{w}_m}\right)\left(1 + \frac{w}{\hat{w}_{-m}}\right)} \qquad (10)$$

$$w_m = \sqrt{k^2 - (\pi m/a)^2}\,, \quad m = 1, 2, 3, \ldots$$

The constant coefficients K, M, K_t, M_t satisfy the system linear algebraic equations

$$\begin{bmatrix} K_m \\ M_m \end{bmatrix} = \pm R_m \left(\begin{bmatrix} K \\ M \end{bmatrix} + \sum_t \begin{bmatrix} K_t \\ M_t \end{bmatrix} \frac{\hat{w}_m - \hat{w}_t}{\hat{w}_m + \hat{w}_t} \right),$$

$$\pm\left(\begin{bmatrix} K \\ M \end{bmatrix} + \sum_t \begin{bmatrix} K_t \\ M_t \end{bmatrix} \frac{i\Gamma - \hat{w}_t}{i\Gamma + \hat{w}_t} \right) \frac{k^2 e^{-2\Gamma\ell}}{\beta^2 L_1^2(i\Gamma)} + \qquad (11)$$

$$\left(\begin{bmatrix} K \\ M \end{bmatrix} + \sum_t \begin{bmatrix} K_t \\ M_t \end{bmatrix} \frac{i\Gamma + \hat{w}_t}{i\Gamma - \hat{w}_t} \right) = \frac{qka\Gamma e^{-\Gamma\ell}}{4\pi^2\beta L_1(i\Gamma)} \begin{bmatrix} i\,\mathrm{sh}\,\Gamma b \\ -\mathrm{ch}\,\Gamma b \end{bmatrix}$$

where

$$R_m = -\frac{\hat{v}_m^2\,(\hat{w}_m^2 + \Gamma^2)}{4\hat{w}_m^2\,L_1^2(\hat{w}_m)}\, e^{2i\hat{w}_m \ell},$$

$$v_m = \frac{k}{\beta} - \frac{2\pi m}{a}\,, \quad \Gamma = \frac{k}{\gamma\beta}$$

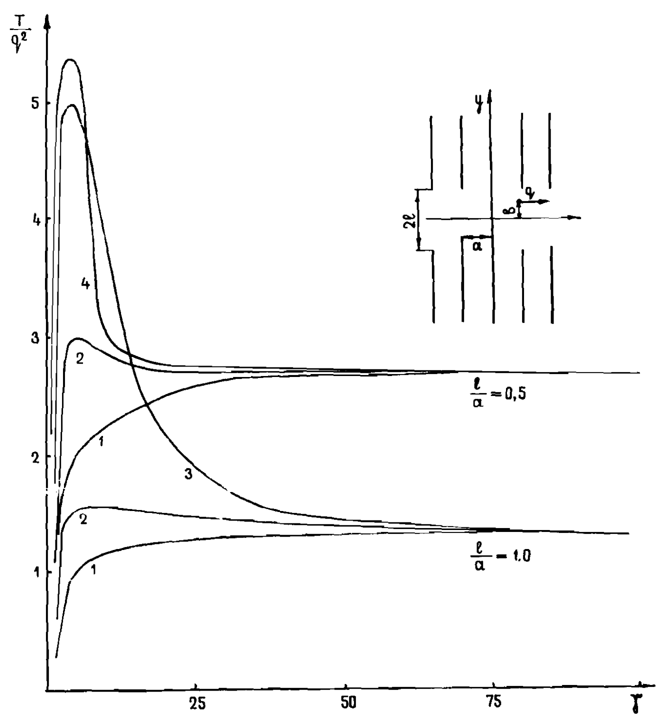

Fig.2. Energy loss per period as a function of the relativistic factor . Curves are presented for various values of b/ℓ: 1.b/ℓ=0; 2.b/ℓ=0.75; 3.b/ℓ=1; 4.b/ℓ=0.95

It can be seen from (11) that the coefficients K_m, M_m become exponentially small when the number m is increased. Hence, the infinite system of equations may be replaced by a finite one. The radiation loss from a charged rod per period may be calculated as a work of the radiated field on the source and may be represented as

$$T=-\frac{16\pi^2 q}{u}\operatorname{Re}\int_0^\infty\Big[\Big(K+\sum_t K_t\frac{i\Gamma-\hat{w}_t}{i\Gamma+\hat{w}_t}\Big)\operatorname{sh}\Gamma b \qquad (12)$$

$$-i\Big(M+\sum_t M_t\frac{i\Gamma-\hat{w}_t}{i\Gamma+w_t}\Big)\operatorname{ch}\Gamma b\Big]\frac{d\omega}{L_1(i\Gamma)}$$

The spectral distribution of losses for various parameters of the problem and total radiative losses as a function of the relativistic factor γ have been numerically evaluated by means of a computer. An example of the radiation loss dependence versus energy is given in Fig.2. It can be seen that for low γ the radiation loss increases rather quickly with source energy and then approaches a certain limit value which depends on the transverse dimensions of the structure channel. For small values of the distance b from the structure axis the radiation loss was found to increase monotonically with γ but for bigger b there is a specific maximum at low energies. It can be noted that the most part of the radiation for the relativistic case is provided by the component of the induced current, antisymmetric with respect to y. The limit value of the energy loss was found to tend to 0 with increasing the transverse dimensions of the structure channel; it corresponds to the results given in paper[3,4]. The γ - independent character of the radiation loss for ultrarelativistic case is in agreement with calculations for a more real model of periodic structure[7].

References

1. G.V.Voskresensky, V.N.Kurdjumov. Proc. of the VII International Conference on Accelerators, vol.II. Yerevan, 1970. Nautchniye Trudy RAIAN SSSR, n.6, p.16, 1971.
2. O.A.Kolpakov, V.I.Kotov. Jurnal Teoreticheskoy Fiziki, 34, n.8, p.1387, 1964.
3. B.M.Bolotovsky, G.V.Voskresensky. Doklady Akademii Nauk SSSR, 156, n.4, p.770, 1964; Uspekhi Fizicheskikh Nauk, 94, n.3, p.377, 1968.
4. R.D.Hazeltine, M.N.Rosenblute, A.M.Sessler. Journ. Math. Phys., 12, n.3, p.502, 1971.
5. E.V.Avdeev, G.V.Voskresensky. Radiotekchnika i Elektronika, 12, n.3, p.469, 1967; 12, n.5, p.839, 1969.
6. V.A.Berezhnoj, A.I.Plis. Kratkije Soobstschenija po Fizike, n.5, p.81, May,1970; Nautchniye Trudy RAIAN SSSR, n.6, p.41, 1971.
7. E.Keil. Proc. of the VII International Conference on Accelerators, vol.II, Yerevan, 1970.

Table

Cavity shape factor b/d	4				2				1			
Relativistic factor	2	5	10	20	2	5	10	20	2	5	10	20
Radiation to the entrance waveguide	0.02	0.25	0.53		0.03	0.30	0.67		0.07	0.50	1.01	
Radiation to the exit waveguide	0.03	0.30	0.74		0.04	0.40	1.01		0.07	0.77	1.64	
Continuous spectrum losses	0.05	0.55	1.27	2.25	0.07	0.70	1.67	2.95	0.13	1.27	2.65	4.71
Discrete spectrum losses	0.60	0.82	0.85	0.85	0.82	1.33	1.41	1.43	0.63	1.67	1.83	1.87
Total losses	0.65	1.37	2.12	3.11	0.89	2.04	3.08	4.38	0.76	2.94	3.48	6.57

Notes: 1. Radiation loss values are normalized by Q^2/a;
2. Relative waveguide size is a/b = 0.5 .