SELF FIELD EFFECTS DURING THE MAGNETIC EXPANSION OF AN ION LOADED ELECTRON RING

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Abstract

In the electron ring accelerator the inherent self fields of an electron ring are used to keep and accelerate ions. For dense electron rings self field phenomena are important beyond the property of merely dragging the ions. So far self field effects mostly have been investigated for electron rings which are axially at rest. LAWSON[1] and MERKEL[2] for instance have calculated a radial blow up of the ring due to the space charge repulsion of opposite sections of the ring. In addition the internal flux due to the magnetic self fields has been taken into account. In the following the space charge influence on the expansion kinematics in an axially decreasing field is calculated. This article is a brief summary of another paper of the author[3].

1. The Kinematical Equations

The calculations are performed in a cylindrical system of coordinates (r,θ,z). We neglect the square of the radial velocity in comparison with the sum of the squares of the axial and azimuthal velocities, which implies the following composition of velocity components:

$$\beta_\theta^2 = \frac{\gamma^2-\gamma_z^2}{\gamma^2\cdot\gamma_z^2} \qquad (1)$$

We assume that the ions do not participate in the azimuthal rotation and that the ring is far away from any metallic or dielectric boundary such that we need not take into account image forces. The external magnetic field has axial symmetry and is supposed to be created by a system of circular currents. Therefore the vector potential has an azimuthal component A_θ only. The components of the external field and the vector potential are given by power series with respect to the radial coordinate r. The coefficients - which are functions of the axial coordinate z - are given by the field fall off on the axis and its derivatives with respect to z [3]. We now operate with a single particle model. To every electron we assign the number $\eta=N_i/N_e$ of ions. In the same way we distribute the total self field energy equally among all of the electrons. We assume that the ring cross section is elliptical with the semi axes a' (radial) and b' (axial) in the frame in which the ring is at rest. The charge density is supposed to be constant. For the moment we assume that the ions are not lost during the expansion process. The validity of this assumption will be checked later on. We then include self field terms in the kinematical equations which are the energy balance, the conservation of the canonical angular momentum (axial symmetry), and the radial force balance. For the calculation of the space charge terms we approximate the elliptical ring cross section by a circular one of radius $(a'+b')/2$. For simplicity we neglect terms of order 1 in comparison with $\ln(16r/(a'+b'))$. The space charge correction terms are given by the self field energy per electron, the magnetic self field flux, and the radially repulsive space charge force per electron respectively. They are calculated in the ring rest frame first. For the calculation of the self field energy in the ring rest frame see for instance [2]. The radially repulsive space charge force is given by the negative radial derivative of the self field energy per electron. The self field flux is determined by the ring dimensions and the ring current. Being calculated in the ring rest frame the self field terms are transformed to the laboratory system with the factors γ_z (self field energy), and $1/\gamma_z$ (transverse space charge force component). The self field flux correction term remains unchanged as longitudinal field components and transverse space vector components remain unchanged. We obtain the following set of equations:

$$m_e c^2\gamma+\eta M_i c^2\gamma_z+\tfrac{1}{2}C_1 m_e c^2\cdot\gamma_z\cdot\tfrac{1}{r}\left[\beta_\theta^2\gamma_z^2+(1-i\eta)\right]\ln\left(\frac{16r}{a'+b'}\right) = E = \text{const.} \qquad (2a)$$

$$m_e\gamma r\cdot\beta_\theta\cdot c+e\cdot rA_\theta+C_1 m_e\cdot c\cdot\beta_\theta\cdot\gamma_z\ln\left(\frac{16r}{a'+b'}\right) = P_\theta = \text{const.} \qquad (2b)$$

$$m_e\cdot\gamma\cdot\tfrac{1}{r}\beta_\theta^2c^2+e\beta_\theta\cdot c\cdot B_z+\tfrac{1}{2}C_1\cdot m_e\cdot c^2\cdot\frac{1}{\gamma_z}\cdot\frac{1}{r^2}\left[\beta_\theta^2\gamma_z^2+(1-i\eta)\right]\ln\left(\frac{16r}{a'+b'}\right) = 0 \qquad (2c)$$

If we eliminate the quantity γ by means of equ. (1) the three equs. (2a), (2b), and (2c) form a complete set of equations to determine the space charge modified kinematical variables r, γ_z and β_θ, provided the transverse ring dimensions a' and b' are known. The transverse ring dimensions are calculated separately using the conservation of the normalized emittance. The focusing is given by a superposition of the external field gradient and the linear space charge focusing (ion focusing)[1]. For the parameter sets investigated here the variables r, γ_z, β_θ and γ are not very different from the corresponding 'space charge free values' r_o, γ_{oz}, $\beta_{o\theta}$, and γ_o which can be obtained from the equations (2a), (2b), and (2c) if the space charge correction terms are neglected. Using this fact it is possible to take the above mentioned 'space charge free values' for the calculation of the transverse ring dimensions. Furthermore the above given system of equations may be linearized in the relative deviations

$$\frac{\delta r}{r_o}=\Delta r,\ \frac{\delta\beta_\theta}{\beta_{o\theta}}=\Delta\beta_\theta \text{ and } \frac{\delta\gamma_z}{\gamma_{oz}}=\Delta\gamma_z$$

from the 'space charge free kinematics'[3].

2. Results

Performing the calculations we get the following two general results:

I. The circulation in the ring rest frame is not modified by space charge effects ($\Delta\beta_\theta = -\Delta\gamma_z$). The total decrease in self field energy during the expansion process (given by the decrease of the space charge correction term in equ. (2a)) is converted to the axial motion of the collective.

II. The relative radial blow up Δr of the ring due to space charge effects is a constant of the motion during the expansion.

Using some further slight approximations these two general results can be derived directly from the two equs.(2b) and (2c) in the following way:

In the space charge correction terms of the equs. (2b) and (2c) we neglect the variation of the logarithmic term and the variation of the product $\beta_\theta\cdot\gamma_z$. Then the space charge correction term in equ. (2b) - which is proportional to the self field flux - is a constant of the motion. If we multiply equ. (2c) by $r^2/\beta_\theta\cdot c$ the space charge correction term of this equation is a constant of the motion also. In addition these two equations now can be combined to a type of equation that is also valid if we neglect the space charge corrections, namely

$$B_z\cdot r^2 - rA_\theta = \text{const.} \qquad (3)$$

Only the constant is different in the two cases. Now we take into account only linear terms in the deviation δr from the 'space charge free-value' r_o and neglect the transverse variation of the external field B_z in the small ring shaped zone between r_o and $r_o+\delta r$. It is possible then to derive the equation

$$B_{zo}\cdot r_o^2\cdot\left(\frac{\delta r}{r_o}\right) = \text{const.} \qquad (4)$$

from equation (3). Equ.(4) implies two things:

a) The additional flux of the external magnetic field through the particle orbit which is due to the space charge induced radius increase δr is a constant of the motion. This together with the constancy of the self field flux is a proof for result No. I.

b) We obtain result No. II if we use the adiabatic invariant $B_{zo}\cdot r_o^2$ = const. for the 'space charge free case'.

In the following two examples from the large variety of parameter sets are given. The kinematical variables are plotted as a function of the axial coordinate **z** in the expansion column. For reference purposes the upper plot of every parameter set gives the kinematics, for which the space charge influence is neglected. The lower plot gives the relative space charge modifications Δr and $\Delta\gamma_z$ together with the transverse ring dimensions. In addition ΔK gives the fraction of the holding power that is not used for acceleration. The quantity ΔK can be calculated from the ring dimensions and γ_z. This is the announced check whether the acceleration rate is such that the ions are not lost during the expansion process. The first parameter set (fig. 1) reveals only very modest self field effects. The quantities Δr and $\Delta\gamma_z$ have a magnitude of only a few percent. The transverse space charge focusing is very small also. Therefore the very pronounced difference in the radial and axial focusing of the external field leads to a large difference of the ring cross section semi axes. The second example (fig. 2) is calculated for a very high particle density. The self field modifications are very strong and such that the linearization in the space charge corrections becomes questionable. The transverse space charge focusing is very strong also such that the differences in the external focusing strengths are compensated. The ring cross section is nearly circular.

References

1) J.D. Lawson: 'Some Limitations in Relativistic charged Particle Beams', Particle Accelerators, 1970, Vol. 1, p. 41 ff.

2) P. Merkel: 'Zur Bestimmung des Radius relativistischer Elektronenringe', internal report, Institut für Plasmaphysik, Garching, 1968.

3) P. Kappe, 'Magnetfeldtoleranzen und Eigenfeldeffekte beim Elektronenringbeschleuniger', Kernforschungszentrum Karlsruhe, Externer Bericht 3/71-05, Mai 1971.

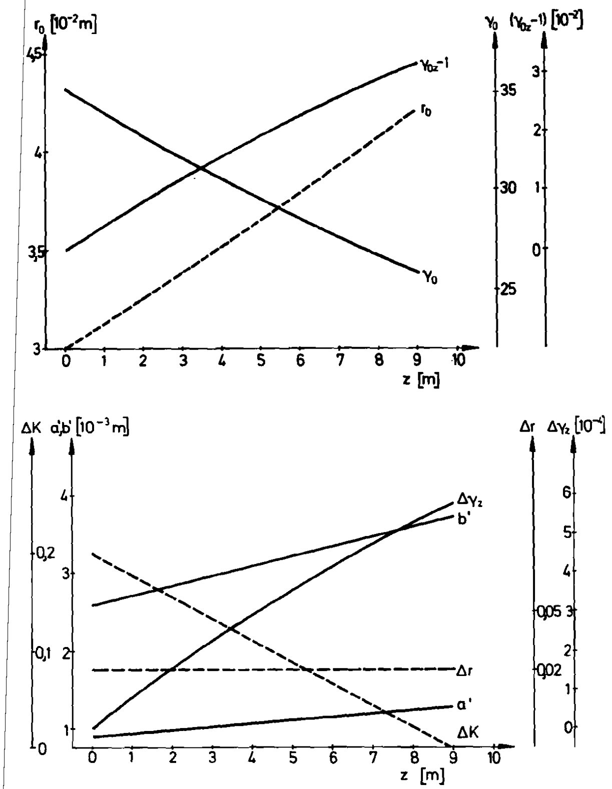

Fig. 1

Expansion kinematics for the parameter set (N_e=5•10^{12}, η=0,01, A=16, i=2, ε'=10^{-3}mrad, r_o(0)=3•10^{-2}m,γ_z(0)=1.0)
The field fall off on the axis is given by the function
$B_z(z,r=0)[\text{Tesla}] = \exp(\frac{-z[m]}{8\ m}) + \exp(\frac{-z[m]}{24m})$

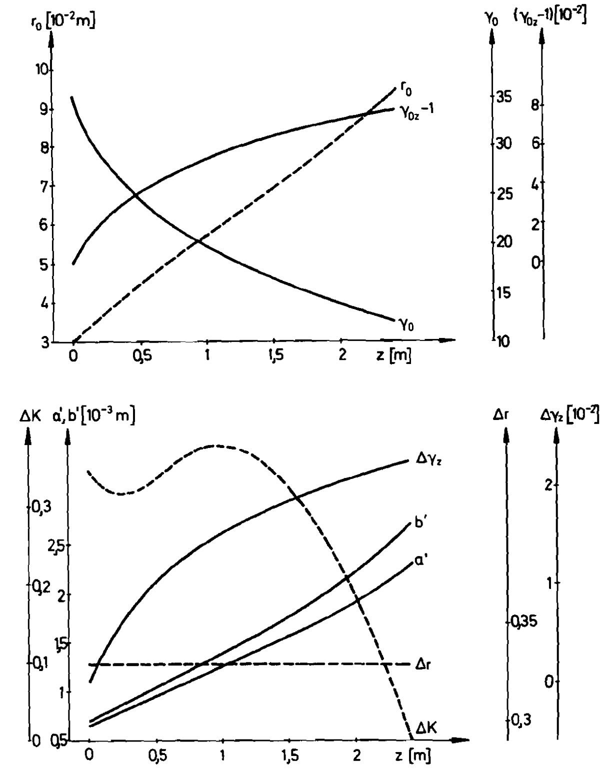

Fig. 2

Expansion kinematics for the parameter set (N_e=1•10^{14}, η=0,01, A=16, i=2, ε'=10^{-3}mrad, r_o(0)=3•10^{-2}m,γ_z(0)=1.0)
The field fall off on the axis is given by the function
$B_z(z,r=0)[\text{Tesla}] = \exp(\frac{-z[m]}{0,3\ m}) + \exp(\frac{-z[m]}{1,5\ m})$

List of symbols, which are not explained in the text

An index 0 corresponds to the kinematics, for which space charge effects are neglected (for instance r_o).

- e = charge of the electron
- m_e = electron rest mass
- c = vacuum velocity of light
- A = atomic number of ions
- M_i = rest mass of ions ∼$A \cdot M_p$, where M_p is the proton rest mass
- N_e = number of electrons in the ring
- N_i = number of ions in the ring ($\eta = \frac{N_i}{N_e}$)
- i = degree of ionization
- r = ring radius
- $\vec{v}$ = particle velocity
- $\vec{\beta} = \frac{\vec{v}}{c}$ with the components
- $\beta_\theta = \frac{r\dot{\theta}}{c}$, $\beta_z = \frac{\dot{z}}{c}$
- $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\gamma_z = \frac{1}{\sqrt{1-\beta_z^2}}$
- B_z = axial component of the external magnetic field at the particle orbit
- $C_1 = \frac{N_e \cdot e^2 \cdot \mu_o}{4\pi^2 \cdot m_e}$ (μ_o = vacuum permeability)
- ε' = normalized emittance