ELECTRON RING INSTABILITIES IN A WAVEGUIDE

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Abstract

The electron ring instabilities caused by radiative and resistive processes in a waveguide in the linear approximation are considered. The resistive processes play the essential role for frequencies below cut-off value and near waveguide resonancies.

I. The hydrodynamical equations of small coherent oscillations of a ring beam in the coordinate system moving with the ring are [1,2]:

$$\begin{aligned} &\ddot{\xi}+\Omega^2(Q_r^2-\gamma^2)\xi-\gamma^2\Omega\dot{\eta}=\frac{e}{m\gamma}(E_r+\beta B_z),\\ &\gamma^2(\ddot{\eta}+\Omega\dot{\xi})=\frac{e}{m\gamma}E_\theta,\\ &\ddot{\zeta}+\Omega^2Q_z^2\zeta=\frac{e}{m\gamma}(E_z-\beta B_r), \end{aligned} \tag{1}$$

where ξ,η,ζ are components of deviation vector $\vec{\Delta}(\theta,t)$ in the cylindrical coordinate system (r, θ, z) with Z-axis being along the ring symmetry axis, $\vec{E}$ and $\vec{B}$ are perturbation fields averaged over the ring cross-section, $\Omega=\beta c/r$ - angular frequency of particle rotation, βc - linear velocity, $Q_{r,z}\Omega$ - betatron frequencies caused by external focusing, dot means $d/dt=\partial/\partial t+\Omega\,\partial/\partial\theta$. In linear approximation the right side of equations (I) may be presented as $\hat{A}\vec{\Delta}$ where $\hat{A}$ is a certain linear operator. If $\vec{\Delta}(\theta,t)\sim\exp\{in\theta-i\omega t\}$ and the system is characterised by azimuthal symmetry [x)] then the action of operator $\hat{A}$ is equivalent to multiplication by a matrix $\hat{A}_n(\omega)=\Omega^2 a_{ik}$ and the system of equations (1) runs as follows:

$$\hat{L}_n(\omega)\vec{\Delta}=\hat{A}_n(\omega)\vec{\Delta}, \tag{2}$$

where $\hat{L}_n(\omega)$ is the left side operator of the system (I).

The dispersion equation for monoenergetic beam is derived directly from (2):

$$Det\,\|\hat{L}_n(\omega)-\hat{A}_n(\omega)\|=0. \tag{3a}$$

The particle energy spread is taken into consideration by a well known method [3]. In this case the dispersion equation is presented in the form

$$1+\int_{-\infty}^{\infty}\left\{\left[a_{rr}+\frac{i\Omega}{n\Omega-\omega}(a_{r\theta}-a_{\theta r})+\frac{\Omega^2}{(n\Omega-\omega)^2}a_{\theta\theta}\right]\frac{\Omega^2}{(n\Omega-\omega)^2-Q_r^2\Omega^2}+\frac{1}{\gamma^2}a_{\theta\theta}\frac{\Omega^2}{(n\Omega-\omega)^2}\right\}f(w)\,dw=0, \tag{3b}$$

$$1+\int_{-\infty}^{\infty}a_{zz}\frac{\Omega^2}{(n\Omega-\omega)^2-Q_z^2\Omega^2}f(w)\,dw=0,$$

where $w=\gamma-\gamma_0$ and $f(w)$ is a function of particle energy distribution. Let us consider a monoenergetic beam with $\gamma=\gamma_0$. Away from resonances the solutions of the equation (3a) are:

for azimuthal oscillations

$$p_\theta=\pm\sqrt{(1/Q_r^2-1/\gamma_0^2)a_{\theta\theta}}, \tag{4a}$$

for radial and axial oscillations

$$p_r=\pm Q_r\mp\frac{1}{2Q_r}\left[a_{rr}\mp\frac{i}{Q_r}(a_{r\theta}-a_{\theta r})+\frac{1}{Q_r^2}a_{\theta\theta}\right], \tag{4b}$$

$$p_z=\pm Q_z\mp\frac{1}{2Q_z}a_{zz}, \tag{4c}$$

where $p=\omega/\Omega_0-n$ and substitutions of $p=0$ for (4a) and $p=\pm Q_{r,z}$ for (4b) and (4c) were done in values a_{ik} . For solving (3a) we assume that

$$\nu/\gamma_0\,\ln(8r_0/a)\ll 1, \tag{5}$$

where $\nu=N/2\pi r_0\cdot e^2/mc^2$, a is the ring cross-section radius, N is the number of particles in the ring, index "0" marks the value for $\gamma=\gamma_0$. The transverse stability is determined by imaginary part of values a_{zz} and $[a_{rr}\mp i/Q_r(a_{r\theta}-a_{\theta r})+\frac{1}{Q_r^2}a_{\theta\theta}]$. As we shall show below this fact is directly connected with the presence of dissipative processes in the system (radiation, energy losses in the metal). The stability of azimuthal oscillations depends on both the imaginary and the real part of $a_{\theta\theta}$.

If $\rho_0(r,z)$ is the charge density in the undisturbed beam then for $|\vec{\Delta}|\ll a$ the charge and current densities take the form

$$\rho=-div(\rho_0\vec{\Delta}),\quad \vec{j}=\rho\vec{V}_0-i(\omega-n\Omega_0)\rho_0\vec{\Delta}, \tag{6}$$

where $\vec{V}_0$ is the undisturbed velocity. Using these expressions we may derive that

$$P=\int\vec{j}^*\vec{E}\,dv=i\omega Nm\Omega_0^2\gamma_0(\vec{\Delta}^*\hat{A}\vec{\Delta}), \tag{7}$$

where (*) means complex conjugation. Placing the eigenvectors $\vec{\Delta}_1=\xi(\vec{e}_r\pm\frac{i}{Q_r}\vec{e}_\theta)$, $\vec{\Delta}_2=\eta\vec{e}_\theta$, $\vec{\Delta}_3=\zeta\vec{e}_z$ ($\vec{e}_r,\vec{e}_\theta,\vec{e}_z$ - unit vectors) in the right side of (7) we obtain the combinations of values a_{ik} which differ from the right side of (4) by an imaginary factor. Thus the direct dependence between $Re\,P$ (that defines the work of the field upon the source) and the coherent beam instability was established: if the energy losses in the system are absent ($Re\,P=0$) then the transverse oscillations are stable ($Im\,p_r=Im\,p_z=0$) and the azimuthal oscillations are stable under condition $Re\,a_{\theta\theta}>0$ ($Im\,a_{\theta\theta}=0$). Below are considered two possible reasons of the energy losses - radiative and resistive ones.

2. The radiative losses take place only at frequencies exeeding the waveguide cut-off frequency. In a perfectly conducting waveguide only azimuthal oscillations should be unstable below cut-off frequency if $Re\,a_{\theta\theta}<0$ (negative mass instability). For $\gamma_0^2\gg 1$ the azimuthal oscillations increment is:

$$Im\,p_\theta=\frac{n}{Q_r}\left(\frac{2\nu}{\gamma_0}\right)^{1/2}Re\sqrt{i Z_n}, \tag{8}$$

where the impedance of the ring in the waveguide

x) The absence of azimuthal symmetry does not change qualitive results.

$$\mathcal{Z}_n=\frac{i}{\gamma_0^2}\ln\frac{L}{a}+2\pi\sum_{j=1}^{\infty}\left[\frac{h_j^{(e)}r_0}{\nu_j^2}\cdot\frac{J_n^2(x_j)}{J_n'^2(\nu_j)}+\frac{y_j^2}{\mu_j^2-n^2}\frac{1}{h_j^{(m)}r_0}\frac{J_n'^2(y_j)}{J_n^2(\mu_j)}\right], \quad (9)$$

$J_n(x)$ and $J_n'(x)$ are Bessel-function and its derivative, ν_j and μ_j are roots of the equations $J_n(x)=0$ and $J_n'(x)=0$ respectively, $x_j=\nu_j r_0/b$, $y_j=\mu_j r_0/b$, b - waveguide radius, $h_j^{(e)}=(k^2-\nu_j^2/b^2)^{1/2}$, $h_j^{(m)}=(k^2-\mu_j^2/b^2)^{1/2}$, $k=\omega/c$ and the signs of $\mathrm{Im}\,h_j^{(e,m)}$ and $\mathrm{Re}\,\omega$ are the same. The first term of the impedance (9) is important for the ring being near the waveguide wall in this case $L=2(b-r_0)$. On Fig.I,2 values $\mathcal{Z}_n$ and $\mathrm{Im}\,p_\theta$ are plotted versus b/r_0. The peaks of $\mathrm{Im}\,p_\theta$ corresponde to waveguide resonances $\omega_\ell=\mu_\ell c/b$ and the breaks are connected with nonresonant E-waves. According to Fig.Ia the azimuthal oscillations are unstable when the beam is near the wall but there always exists a value δ_n such that the oscillations become stable for $b/r_0>\delta_n$. Because of $\mathrm{Re}\,\mathcal{Z}_n\neq 0$ for $b/r_0>\mu_1/n\beta_0$ (when the cut-off frequency is exeeded) the azimuthal oscillations are unstable. The stability interval increases with γ_0 and with ring thickness. The whole frequency interval below the cut-off value will be stable for sufficiently large γ_0 and a.

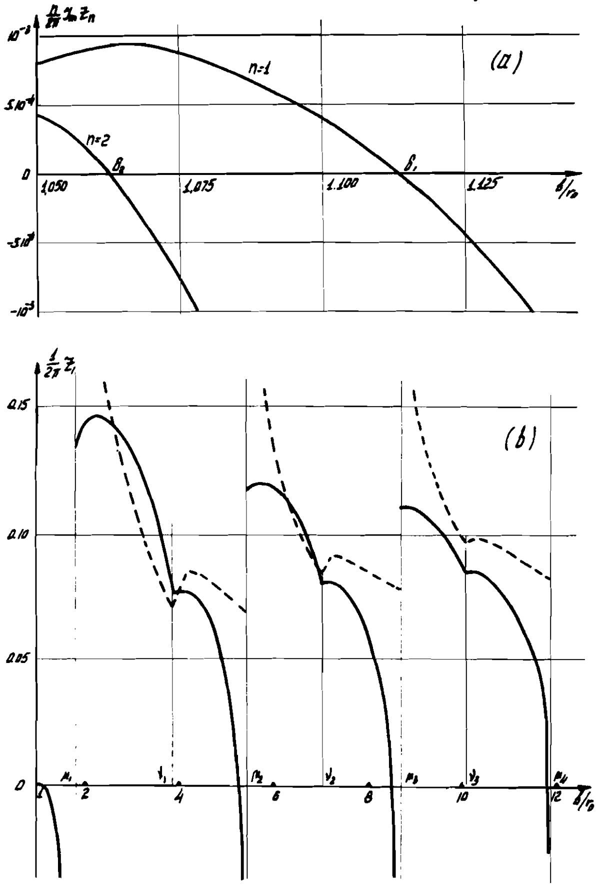

Fig.I. Ring impedance in the circular waveguide. Solid curve - imaginary part of the impedance, dashed curve - real part. (a) - the impedance for frequencies below cut-off value, (b) - the impedance for frequencies above cut-off value.

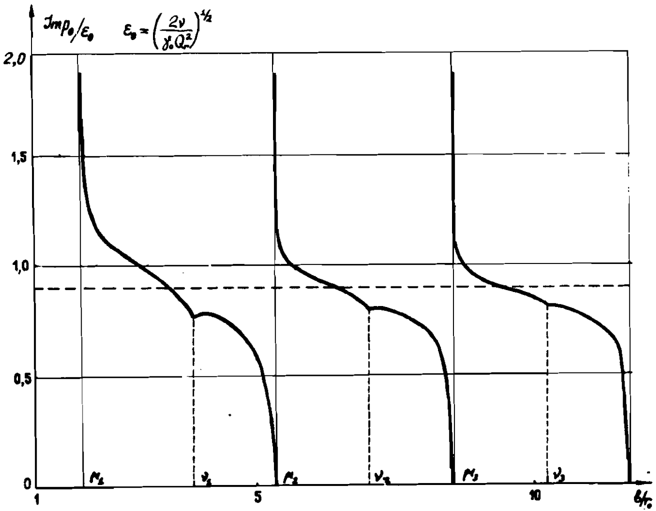

Fig.2. Azimuthal increment versus b/r_0 for n=1

The transverse oscillations are unstable only for $b/r_0>\mu_1/(n-Q)\beta_0$. The transverse increments are given by

$$\mathrm{Im}\,p_r=\frac{2\pi\nu}{Q_r\gamma_0}\sum_j\left\{\frac{h_j^{(e)}r_0}{\beta_0^2\nu_j^2J_{n-1}^2(\nu_j)}\left[n\frac{1\pm Q_r}{Q_r}J_n(x_j)\pm x_jJ_{n-1}(x_j)\right]^2+\right.$$
$$\left.+\frac{1}{h_j^{(m)}r_0(\mu_j^2-n^2)J_n^2(\mu_j)}\left(\frac{n\pm Q_r}{Q_r}\right)^2\left[y_jJ_{n-1}(y_j)-\left(\frac{Q_ry_j^2}{n\pm Q_r}\mp n-nQ_r\right)J_n(y_j)\right]^2\right\}, \quad (10)$$

$$\mathrm{Im}\,p_z=\frac{2\pi\nu}{Q_z\gamma_0}\sum_j\left\{\frac{1}{h_j^{(e)}r_0}\left[\frac{(h_j^{(e)}r_0)^2+\beta_0^2n(n\pm Q_z)}{\beta_0\nu_jJ_{n-1}(\nu_j)}J_n(x_j)\right]^2+\right.$$
$$\left.+\frac{h_j^{(m)}r_0}{\mu_j^2-n^2}\left[\frac{nJ_n(y_j)-y_jJ_{n-1}(y_j)}{J_n(\mu_j)}\right]^2\right\}, \quad (11)$$

where $(\pm)$ corresponds accordingly to modes $\omega=(n\pm Q_{r,z})\Omega_0$ and summation extends over those terms for which the radiation condition $[h_j^{(e,m)}]^2>0$ is satisfied. Only slow waves with $\omega=(n-Q_{r,z})\Omega_0$ for $n>Q_{r,z}$ and $\omega=(n+Q_{r,z})\Omega_0$ for $n<-Q_{r,z}$ are unstable. On Fig.3 and 4 transverse increments are plotted versus b/r_0 for n=I and Q=0.5. There exist the radial oscillation resonances for $b/r_0=\mu_\ell/(n-Q_r)\beta_0$ and the axial ones for $b/r_0=\nu_\ell/(n-Q_z)\beta_0$. The breaks on the plots correspond to arising of nonresonant E-waves for radial oscillations and H-waves for axial ones.

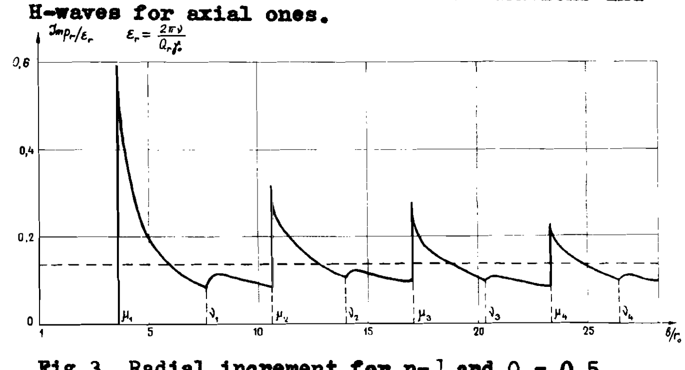

Fig.3. Radial increment for n=1 and Q_r= 0.5.

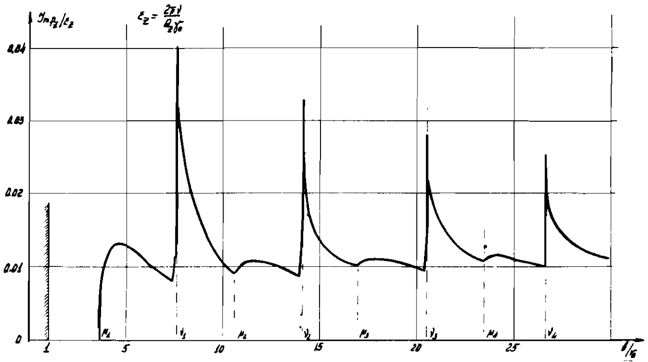

Fig.4. Axial increment for n=1 and Q_z=0.5

In the limit case when $n\ell/r_\circ \to \infty$ the expressions (8-II) turn into corresponding expressions given in ref. 4) for a ring in free space.

3. Considering the resistive effects we assume that the wall conductivity is sufficiently high and the skin-depth is small in comparision with the wave length. In this case the corresponding perturberations of electromagnetic fields are derived as solutions of the homogeneous Maxwell's equations with boundary condition on the conducting wall

$$[\vec{n}\times\vec{E}_1] = (1-i)\sqrt{\frac{\omega}{8\pi\sigma}}\,[\vec{n}\times[\vec{n}\times\vec{B}_\circ]], \qquad (12)$$

where $\vec{E}_1$ is the electric field caused by resistive perturberation, $\vec{B}_\circ$ is the magnetic field for perfectly conducting waveguide, σ - conductivity, $\vec{n}$ - the outward normal of the conducting boundary. This approach is correct when the condition

$$\delta/\ell \ll (\omega-\omega_{res})/\omega_{res} \qquad (13)$$

holds, where $\delta = c/\sqrt{2\pi\sigma\omega}$ is the skin-depth, ω_{res} - the cut-off frequency. The resistive perturberations being small, we take into consideration their influence only below the cut-off frequency when the oscillations are stable in the ideal waveguide. Let us consider the ring moving along the waveguide with velocity $\beta_z c$. The boundary conditions for $\sigma=\infty$ on the conductor at rest and on the conductor moving parallel to its boundary are the same. Therefore the movement of the ring was not essential for the above radiative instability analysis. The increments are expressed in number of turnes and thus are independent of the kind of a reference coordinate system. As it will be seen below the resistive effects depend essentially on the ring movement.

The ring impedance which defines the azimuthal stability may be presented in the form $\mathcal{Z}_n+\Delta\mathcal{Z}_n$ where $\mathcal{Z}_n$ is the reactive impedance for a perfect waveguide (9),

$$\Delta\mathcal{Z}_n = \left(\frac{\omega}{8\pi\sigma\gamma_z}\right)^{1/2}\frac{r_\circ}{n\ell}\frac{1}{(kr_\circ)^{3/2}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{|kr_\circ+\beta_z x|}}\left\{\left[kr_\circ\frac{J_n'(y)}{J_n'(\frac{\ell}{r_\circ}y)}\right]^2 + \frac{\gamma_z^2}{y^2}\left[x(kr_\circ+\beta_z x)\frac{J_n(y)}{J_n(\frac{\ell}{r_\circ}y)} - \frac{r_\circ}{\ell}kr_\circ(x+\beta_z kr_\circ)\frac{J_n'(y)}{J_n'(\frac{\ell}{r_\circ}y)}\right]^2\right\}dx, \qquad (I4)$$

ω is oscillation frequency in the coordinate system moving with the ring, $y=\sqrt{(kr_\circ)^2-x^2}$, $\gamma_z=(1-\beta_z^2)^{-1/2}$. If $k(\ell-r_\circ)\ll 1$ (the ring near the wall) it may be found that

$$\Delta\mathcal{Z}_n \cong \begin{cases} \sqrt{\dfrac{c}{8\pi\sigma r_\circ}}\left(\dfrac{\ell}{r_\circ}-1\right)^{-1} & \text{for } \beta_z\ll 1, \\ \sqrt{\dfrac{c}{8\sigma r_\circ}}\,\gamma_z^{3/2}\left(\dfrac{\ell}{r_\circ}-1\right)^{-1/2} & \text{for } 1-\beta_z\ll 1. \end{cases} \qquad (15)$$

If the frequency is near the cut-off value but the condition (I3) is satisfied we obtain

$$\Delta\mathcal{Z}_n \cong \frac{\beta_\circ^2}{4}\sqrt{\frac{\pi\Omega_\circ}{\sigma\gamma_z}}\,\frac{\mu_1^2+\beta_z^2\gamma_z^2}{\mu_1^{7/2}}\left[\frac{J_n'(n\beta_\circ)}{J_n''(\mu_1)}\right]^2\left(\frac{\mu_1}{n\beta_\circ}-\frac{\ell}{r_\circ}\right)^{-3/2}. \qquad (16)$$

Numerical calculations show that the sum of expressions (15) and (16) is a good approximation (error is less than I0%) for $\Delta\mathcal{Z}_n$ for all frequencies below cut-off value. The azimuthal increment is derived from formula (8) and equals

$$\mathcal{J}m\,p_\theta = n\left[\frac{2\nu}{\beta_\circ\gamma_\circ}\left(\frac{1}{Q_r^2}-\frac{1}{\gamma_\circ^2}\right)/|\mathcal{Z}_n|\right]^{1/2}\frac{\Delta\mathcal{Z}_n}{2/\mathcal{Z}_n/}. \qquad (17)$$

Values of $\mathcal{J}m\,p_\theta$ versus $\ell/r_\circ$ are plotted on Fig.5 for n=1,2,3. The same analysis of the transverse oscillations shows that only the slow waves are unstable similarly to the ideal waveguide case. For the ring being near the wall the radial increment equals

$$\mathcal{J}m\,p_r \cong \begin{cases} \dfrac{\nu}{2Q_r\beta_\circ\gamma_\circ}\sqrt{\dfrac{\Omega_\circ}{8\pi\sigma(n-Q_r)}}\left(\dfrac{\ell}{r_\circ}-1\right)^{-3} & \text{for } \beta_z\ll 1, \\ \dfrac{3}{32}\dfrac{\nu}{Q_r\beta_\circ^2\gamma_\circ}\sqrt{\dfrac{c}{\sigma r_\circ}}\,\gamma_z^{3/2}\left(\dfrac{\ell}{r_\circ}-1\right)^{-5/2} & \text{for } 1-\beta_z\ll 1. \end{cases} \qquad (18)$$

If the frequency is near the first resonance value ($\ell/r_\circ=\mu_1/(n-Q_r)\beta_\circ$) but the condition (13) is satisfied we obtain

$$\mathcal{J}m\,p_r \cong \frac{\nu(kr_\circ)^2}{4Q_r\beta_\circ^2\gamma_\circ}\sqrt{\frac{\pi c}{\sigma r_\circ\gamma_z}}\cdot\frac{\mu_1^2+n^2\beta_z^2\gamma_z^2}{\mu_1^{7/2}}\times \left[\frac{(n-\beta_\circ^2\gamma_\circ^2Q_r)J_n(kr_\circ)-J_n'(kr_\circ)}{\gamma_\circ^2Q_rJ_n''(\mu_1)}\right]^2\left(\frac{\mu_1}{kr_\circ}-\frac{\ell}{r_\circ}\right)^{-3/2}, \qquad (19)$$

where $kr_\circ=(n-Q_r)\beta_\circ$. The sum of expressions (18) and (I9) is a good approximation for $\mathcal{J}m\,p_r$ for all frequencies below cut-off value. For axial oscillations the cut-off value is not resonance frequency therefore approximate expression for $\mathcal{J}m\,p_z$ becomes

$$\mathcal{J}m\,p_z \cong \begin{cases} \dfrac{\nu}{4Q_z\gamma_\circ}\sqrt{\dfrac{c}{2\pi\sigma r_\circ}}\dfrac{1}{\sqrt{kr_\circ}}\left[1+\left(2k^2r_\circ^2-2Q_z^2-n^2-\dfrac{3}{4}\right)\left(\dfrac{\ell}{r_\circ}-1\right)^2\right]\left(\dfrac{\ell}{r_\circ}-1\right)^{-3} & \text{for } \beta_z\ll 1, \\ \dfrac{3}{32}\dfrac{\nu}{Q_z\beta_\circ^2\gamma_\circ}\sqrt{\dfrac{c}{\sigma r_\circ}}\,\gamma_z^{3/2}\left[1-\dfrac{2}{3}\beta_\circ(5Q_z-n)\left(\dfrac{\ell}{r_\circ}-1\right)\right]\left(\dfrac{\ell}{r_\circ}-1\right)^{-5/2} & \text{for } 1-\beta_z\ll 1, \end{cases} \qquad (20)$$

where $kr_\circ=(n-Q_z)\beta_\circ$. If the ring is near the wall ($k(\ell-r_\circ)\ll 1$) then the expressions (18) and (20) for $\beta_z\ll 1$ are similar to those derived in 5).

The resistive perturberations of fields increase with γ_z and for $\gamma_z \gtrsim 100$ they are no more small in comparision with the perfect waveguide

fields therefore our approximation becomes invalid in the case.

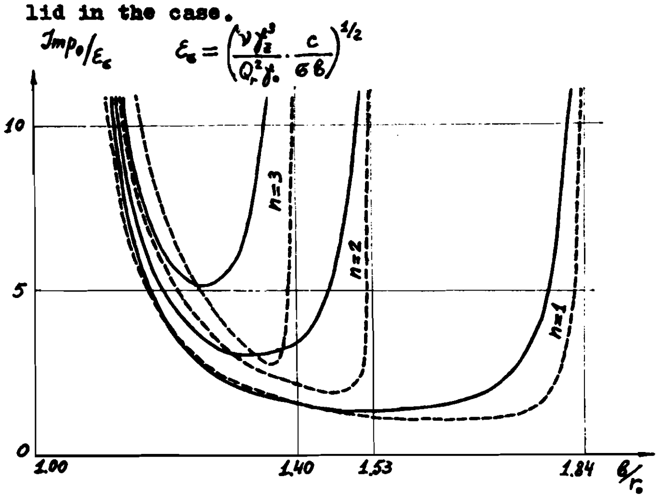

Fig.5. Azimuthal increment for resistive instability. Solid curve $- \beta_z \ll 1$, dashed curve $- \gamma_z \gg 1$.

4. Now we consider the resonant **cases when the** opposite of condition (13) is satisfied. The **azimuthal equation near the resonance must be written in the form**

$$p^2 = i\,\frac{2\nu n^2}{\beta_0 \gamma_0}\left(\frac{1}{Q_r^2} - \frac{1}{\gamma_0^2}\right)\left(Z_n' + \frac{f_\ell}{\sqrt{p + n - \omega_\ell/\Omega_0}}\right), \qquad (\quad)$$

where Z_n' **is the impedance (9) without** resonant term **and the resonance frequency is**

$$\omega_\ell = \mu_\ell \frac{c}{b} + i\Omega_0 \chi_\theta\,, \quad \chi_\theta = n\beta_0 \frac{\mu_\ell^2}{\mu_\ell^2 - n^2}\frac{\delta}{b}\,. \qquad (22)$$

The meaning of f_ℓ **is clear by comparision with the expression (9). The solution of equation (21) for the first resonances when** $(\nu/\gamma_0)^{4/5}|Z_n'| \ll f_\ell^{4/5}$ **runs as follows**

$$\Im m p_\theta = \begin{cases} \left[\frac{2\nu}{\beta_0\gamma_0}\left(\frac{1}{Q_r^2} - \frac{1}{\gamma_0^2}\right) f_\ell\right]^{2/5} n \sin\frac{2\pi}{5} = \Im m p_\theta^{(0)} \\ \qquad\qquad \text{for } \chi_\theta \ll \Im m p_\theta^{(0)}, \\ \Im m p_\theta^{(0)}\left[\frac{\Im m p_\theta^{(0)}}{\chi_\theta}\right]^{1/4} \text{ for } \chi_\theta \gg \Im m p_\theta^{(0)}. \end{cases} \qquad (23)$$

The resonances of sufficiently big numbers ($(\nu/\gamma_0)^{4/5}|Z_n'| \gg f_\ell^{4/5}$) play no role and the expression for $\Im m p_\theta$ **turns into** the corresponding expression of **free space. By means of similar analysis of the transverse oscillations for the first resonances we shall obtain that** $\Im m p_{r,z} = \Im m p_{r,z}^{(0)} \sim (\nu/\gamma_0)^{2/3}$ **when** $\chi_{r,z} \ll |\Im m p_{r,z}^{(0)}|$ **and** $\Im m p_{r,z} \sim \nu/\gamma_0 \cdot (\Omega_0/\sigma)^{1/4}$ **when** $\chi_{r,z} \gg |\Im m p_{r,z}^{(0)}|$ **where** $\chi_z = kr_0\delta/2b$, $\chi_r = \mu_\ell^2 \chi_z/(\mu_\ell^2 - n^2)$ **. The resonan**ces **of big number are ignorable too and the increments become equal to the expressions describing the case of free space** [4]**.**

5. It is easy to take into account the energy spread. As to azimuthal oscillations ref. [6] shows **that for resonance frequencies with energy spread under consideration the instability has no threshold. Away from resonances the instability is supressed when the** following condition is satisfied

$$\left(\frac{\Delta\Omega}{\Omega_0}\right)^2 \gtrsim \left|\left(\frac{1}{Q_r^2} - \frac{1}{\gamma_0^2}\right) a_{\theta\theta}\right|,$$

that is the threshold value $\Delta\Omega \sim (\nu/\gamma_0)^{1/2}$**. The analysis of the transverse oscillations instability shows that both away from resonances and near them there always exists a threshold value of the energy spread. The instability suppression condition is the following**

$$\frac{\Delta\Omega}{\Omega_0} \gtrsim |p_{r,z}^{(0)} \pm Q_{r,z}|$$

where $p_{r,z}^{(0)}$ **is the solution in case of the monoenergetic beam.**

6. The particular case not mentioned above is the absence of axial focusing $Q_z = 0$ **. Then we obtain**

$$p_z = \pm(-a_{zz})^{1/2} \sim (\nu/\gamma_0)^{1/2}$$

that is the axial and azimuthal increments have the same order of magnitude. Specific **features of the axial oscillations in this case are the absence of resonances, so far** as $f_z = 0$ **, and the absence of frequency regions of stability in the waveguide.**

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References

1. R.J.Briggs, V.K.Neil. Plasma Phys.,9,2, 209 (1967)
2. I.N.Ivanov. Proc.of the Electron Ring Accelerator Symposium, Lawrence Radiation Laboratory 1968, p.467 and Journal Teknicheskoy Fiziki, 39, 6, 976 (1969)
3. J.E.Drummond. Plasma Physics. McGraw-Hill, 1961, p.19
4. I.L.Korenev, L.A.Yudin. Radiofizika, I4, n.8 (1971)
5. V.I.Balbekov. Avtoreferat dissertacii. Serpukhov, 1969.
I.L.Korenev, L.A.Yudin. Report on the VII International Conference on Accelerators, Yerevan, USSR, 1969.
6. A.G.Bonch-Osmolovsky, E.A.Perelshtein. Radiofizika, 13, n.7, 1080 (1970).